2022 VJC Prelim H2 P2 Suggested solution

1(a) His explanation is not correct as even though the force that the truck acts on him is equal and opposite to the force that he acts on the truck, the 2 forces act on different bodies [APPLICATION] so they do not cancel out. [CONCLUSION]

As Adam pushes the truck, the truck exerts a friction <u>on the surface of the roadside</u> [be specific: friction at where?]. By Newton's 3rd law [CONCEPT], the surface of the roadside will exert an equal and opposite friction back on the truck. [APPLICATION]

The truck does not move because the friction that the surface of the roadside exerted on the truck and the force exerted on the truck by Adam are acting on the same object and are equal and opposite, therefore there is no net force on the truck, [APPLICATION] and hence by Newton's 1st law [CONCEPT], it doesn't move [CONCLUSION].

(b)(i) The principle of conservation of momentum states that total momentum of the system before the collision is equal to the total momentum of the system after the collision, <u>provided no net external force acts on the system</u>.

(ii) 1. Total initial momentum of the system is 0 since nobody is moving.

By conservation of momentum [CONCEPT],
Taking upwards as positive,
Total initial momentum of system = total final momentum of system

$0 = M_{balloon}V_{balloon} + m_{man}v_{man}$

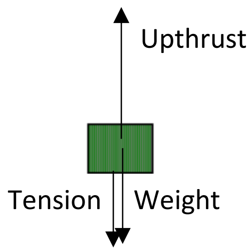
$0 = (320)V_{balloon} + 80(2.5)$

$V_{balloon} = -0.625 \text{ m s}^{-1}$

Since upwards is taken to be positive, -0.625 m s^{-1} would indicate that the balloon is moving in the negative direction, which is downwards.

2. Speed of the balloon is zero [because initial total momentum = 0].

2(a) (i)



(a)(ii) Total upward forces = total downward forces [CONCEPT]

$T + mg = U$

$kx + mg = U$

$L = \frac{V\rho g - mg}{k} = \frac{\frac{2}{3}\left(\frac{5.0}{650}\right)(1000)(9.81) - (5.0)(9.81)}{160}$ where $x = L$

$L = 7.86 \times 10^{-3}$ m

(b)(i) Taking pivot at A, total clockwise moment = total anti-clockwise moment, [CONCEPT]

$(\frac{AC}{2}\cos 65°)(1200) + (AC\cos 65°)(2000) = T(AB)$

$(\frac{AC}{2}\cos 65°)(1200) + (AC\cos 65°)(2000) = T(\frac{3}{4}AC)$

$T = 1.47 \times 10^3$ N

(b)(ii) Since tension exerted on the boom is at an angle, there will be a horizontal component. [APPLICATION]

For the boom to be in equilibrium, there must be a horizontal force to counter this force. [CONCEPT: FORCES IN EQUILIBRIUM] Therefore, the force at the hinge is not vertical but at an angle to provide a horizontal component to counter the horizontal component of the tension. [CONCLUSION]

3(a) Using pV = nRT, [CONCEPT]

Volume $V_f = \dfrac{nRT}{p}$

$= \dfrac{\text{Total mass}}{\text{Molar mass}} \dfrac{RT}{p}$

$= \dfrac{350}{18} \times \dfrac{8.31 \times (273 + 100)}{1.0 \times 10^5}$

$= 0.60271$

$\approx 0.60\ m^3$

(b) Work done, $W = p\Delta V$ [CONCEPT]

$= p(V_f - V_i)$

$= p(V_f - \dfrac{m}{\rho})$

$= 1.0 \times 10^5 (0.6027 - \dfrac{0.350}{1000})$

$= 60235$

$\approx 6.0 \times 10^4$ J

(Note: Volume of a gas at atmospheric pressure is so much larger than its volume as a liquid, so it's OK not to include V_i in the calculation.)

(c) $\Delta U = Q - W_{by}$ [CONCEPT]

$= mL_v - W_{by}$

$= (0.350 \times 2.26 \times 10^6) - 60235$

$= 730770$

$\approx 7.3 \times 10^5$ J

(d) The increase in internal energy takes the form of an increase in potential energy due to intermolecular forces of attraction.

4(a)(i) Since *g* and *r* are constant, so *a* is proportional to *x.*
Negative sign shows that *a* and *x* are in opposite direction. [APPLICATION]
Hence the ball undergoes simple harmonic motion [CONCLUSION]

(ii) $\omega^2 = \frac{g}{r}$ and $\omega = \frac{2\pi}{T}$ [CONCEPT]

$\omega^2 = \frac{9.81}{0.28} = 35$

$T = 1.06$ s

$\tau = 0.53$ s

(b) Sketch:
time period constant (or increases very slightly)
drawn lines always 'inside' given loops, up to given time duration
successive decrease in peak height

5(a)(i) Faraday's law states that the emf induced in a conductor is proportional to the rate of change of magnetic flux linkage.

(ii) The steel string near the permanent magnet gets magnetised (and produces its own magnetic field). [APPLICATION]

When the string vibrates, the magnetic flux density at the location of the coil changes. [APPLICATION]

There is a changing magnetic flux linkage with the coil [APPLICATION] so, according to Faraday's law [CONCEPT], an emf is induced in the coil [CONCLUSION].

(iii) Nylon string cannot be magnetised.

(b) The nodes of the stationary wave are located at the clamps, giving the first harmonic, so the length of 64 cm is equal to half a wavelength of the wave.

$\lambda/2$ = 64 cm
(need to explain properly and not just write the equation)

Frequency of the wave, $f = \frac{v}{\lambda} = \frac{300}{2(0.640)} = 234.4 \text{ Hz}$ [SHOW: GIVE MORE DP]

$\approx$ 230 Hz

(c) maximum vibrational velocity of the string,

$$v_{\max} = \omega x_0 = (2\pi f)x_0 \text{ [CONCEPT]}$$
$$= (2\pi)(230)(1.50\times10^{-2})$$
$$= 21.68 \text{ m s}^{-1}$$

Maximum induced emf, $\varepsilon_{\max} = BLv_{\max}$ [CONCEPT]

$$= (4.50\times10^{-3})(2.00\times10^{-2})(21.68)$$

= 1.95×10^{-3} V or 2.0×10^{-3} V

6(a)(i) As the half-life of X (in years) is very long, it means that the decay constant is very, very small. Thus the fraction of nuclei that would decay during the time of measurement is very low, and the number of nuclei N in the sample remains almost constant. [APPLICATION]

Since activity, A = λN [CONCEPT], then the activity will remain almost constant. [CONCLUSION]

(ii) **1.** Decay constant of Y, $\lambda_Y = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{1.5\times60\times60}$ = 1.283×10^{-4} s^{-1} = 1.3×10^{-4} s^{-1}

2. Equilibrium is reached when the rate of production of Y (from the decay of X) is equal to its rate of decay of Y. Hence, the number of isotope Y in the sample will stabilise at a constant value.

3. Thus, the activity of Y is about 1.1 x 10^7 Bq.

Amount of Y, $N_Y = \frac{A_Y}{\lambda_Y} = \frac{1.1\times10^7}{1.283\times10^{-4}} =$ 8.6 x 10^{10} atoms

b(i) By $N = N_o e^{-\lambda t}$: [CONCEPT]

$$5N = 6Ne^{-\lambda t}$$

Take ln on both sides,

$$\ln 5 = \ln 6 - \lambda t$$

$t = \frac{\ln 6 - \ln 5}{4.95\times10^{-11}} = 3.683\times10^9$ yrs = 3.7 x 10^9 years

(ii) Decay of Th-232 will give rise to a radioactive series where there will be a number of radioactive daughter products before ending up as the stable Pb-208. It is assumed that these intermediate radioactive daughter products have very short half-lives (much shorter than that of Th-232) so the number of intermediate daughter products are insignificant compared to Th-232 and Pb-208.

(iii) If the assumption is not valid, there would have been more Th-232 in the beginning which have decayed and is still in the form of the intermediate daughter products, therefore the fraction of undecayed Th-232 is less than $\frac{5}{6}$. This means that the rock would have been older, as a longer time would have been elapsed, thus answer for **(b)(i)** will be an under-estimate.

7(a) Potential difference used to accelerate the ions, ΔV = 1060 – (–225) = 1285 V

The gain in kinetic energy of the ion = its loss of electric potential energy

$\frac{1}{2}mu^2 = q\Delta V$, where u is the speed of the ion. [CONCEPT]

$$u = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2(1.60\times10^{-19})(1285)}{2.2\times10^{-25}}} = 4.323\times10^4 \text{ ms}^{-1}$$

$$= 4.3\times10^4 \text{ ms}^{-1}$$

(b)(i) The passage says it takes 4 days (or more) to eject 1 kg of xenon.

maximum mass of xenon ejected per second,

$$\frac{\Delta m}{\Delta t} = \frac{1 \text{ kg}}{4 \text{ days}}$$

$\frac{\Delta m}{\Delta t} = \frac{1}{4\times24\times60\times60} = 2.894\times10^{-6} \text{ kg s}^{-1}$ = 2.9 × 10^{-6} kg s^{-1}

(ii) Maximum force exerted on the ejected xenon,

$$F = \frac{\Delta p}{\Delta t} \text{ [CONCEPT]}$$
$$= \left(\frac{\Delta m}{\Delta t}\right) u$$
$$= \left(2.9\times10^{-6}\right)(4.3\times10^{4})$$
$$= 0.1247 \text{ N}$$

By Newton's 3rd law, the maximum thrust = F = 0.12 N

(c) Assume that, at the start, the mass of Deep Space 1 (M_{DS}) and the mass of the xenon (m_{Xe}) are at rest. So their total initial momentum $p_{initial}$ = 0

Applying the principle of conservation of momentum, [CONCEPT]
their total final momentum p_{final} = $p_{initial}$ = 0

So the momentum of DS must be equal and opposite to that of Xe,

$$M_{DS}v_{DS} = m_{Xe}v_{Xe}$$
$$M_{DS} = \frac{m_{Xe}v_{Xe}}{v_{DS}} = \frac{(74)(4.3\times10^{4})}{(4.3\times10^{3})} = 740 \text{ kg}$$

(d) The positive ions will be attracted to the negatively charged spacecraft even as the ions are ejected away, so they will be ejected at a lower speed than intended.

(e) Any 2 reasonable points:
- Ion engines use fuel more efficiently than chemical rockets, so they can travel larger distances per unit mass of fuel. *[Nasa source: Chemical rockets have demonstrated fuel efficiencies up to 35%, but ion thrusters have demonstrated fuel efficiencies over 90%.]*
- Ion engines can produce a larger increase in speed per unit mass of fuel used, compared to chemical rockets, so they can travel longer distances with less fuel.
- Ion engines use their fuel much more slowly, so the fuel can last for much longer periods of time.
- Ion engines use inert gases (like xenon) for propellant and there is no risk of the explosions associated with chemical rockets, so ion engines are less risky (for long missions).

(f)(i) Energy of a photon, $E = \frac{hc}{\lambda}$ and momentum of a photon, $p = \frac{h}{\lambda}$

where λ is the wavelength of the photon

Substitute $p = \frac{h}{\lambda}$ into $E = \frac{hc}{\lambda}$ gives, $E = \frac{hc}{\lambda} = \left(\frac{h}{\lambda}\right)c = pc$

['Show' question: need to explain your steps and show substitution of equation for p into equation for E]

(ii) Power received by 1 m^2 of sail, power = intensity × area = 1400 W
Energy received by 1 m^2 of sail in 1 s, E = power × time = 1400 J

From $E = pc$, we have

Momentum in 1 s, $p = \dfrac{E}{c} = \dfrac{1400}{3.00\times10^{8}} = 4.667\times10^{-6}\ \text{N s}$

$\approx 4.7\times10^{-6}\ \text{N s}$

(iii) total momentum of the photons incident on 32 m^2 of sail in 1 s,

$p_{total} = p\times32 = (4.7\times10^{-6})(32) = 1.504\times10^{-4}\ \text{N s}$

Change of momentum upon reflection,

$\Delta p = 2p_{total} = 2(1.504\times10^{-4}) = 3.008\times10^{-4}\ \text{N s}$

Force = change of momentum per unit time = $3.008\times10^{-4}\ \text{N}$

By Newton's 3rd law, the force on the whole sail = $3.0\times10^{-4}\ \text{N}$

(iv) If all the light is absorbed, then the final momentum of the light is 0.
So the change of total momentum is half of that in (iii).
Hence, the force on the sail is also halved.

(g) LightSail orbits Earth with an orbital radius of 7020 km.
The centripetal acceleration of LightSail = the gravitational field strength at that orbit.
[CONCEPT]

$a_c = \dfrac{GM}{r^2} = \dfrac{(6.67\times10^{-11})(6.0\times10^{24})}{(7020\times10^{3})^2}$ = 8.12 m s^{-2}

So the centripetal acceleration of LightSail 2 cannot be 0.058 mm s^{-2}.

(h) Advantage: (or some other reasonable point)

- The solar sail will never run out of "fuel" so long as there is light shining on the sail, whereas ion engines can run out of fuel.

Disadvantage: (any 1 or some other reasonable point)

- The thrust of a solar sail ($3.0\times10^{-4}\ \text{N}$) is much lower than that of an ion engine (0.12 N).
- The sail is very thin (4.5 μm) and may be easily damaged by space rocks. (whereas an ion engine is not so fragile)
- The total surface area of a solar sail needs to be very big, but that makes it more susceptible to collisions with space rocks.