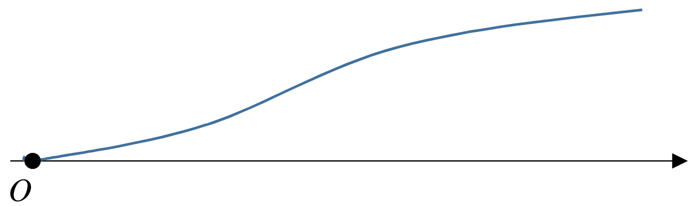
ANNEX B

TPJC H2 Math JC2 Preliminary Examination Paper 1

QN	Topic Set	Answers
1	Equations and Inequalities	$x < -3$
2	Graphs and Transformation	(i) $-1 < y \le 1$ (iii) Translation by 4 units in the positive x-direction, followed by -Stretch of factor 2 parallel to the x-axis. **Alternative Answers:** Stretch of factor 2 parallel to the x-axis, followed by Translation by 8 units in the positive x-direction
3	Functions	(i) $f^{-1}(x) = -\sqrt{x} + k$ $D_{f^{-1}} = (0, \infty)$ (ii) $R_g = [-1, 4]$ $D_f = (-\infty, k)$ Since $k > 5$, $R_g \subseteq D_f$. Thus fg exists. (iii)(a) $fg(-1) = f(0) = k^2$ (b) $R_{fg} = \left[(4-k)^2, (-1-k)^2\right]$ $= \left[(4-k)^2, (1+k)^2\right]$
4	Complex numbers	(i) $\therefore$ smallest positive integer $n = 5$. (ii) $\lvert w \rvert = 2$, $\arg(w) = \frac{13\pi}{6}$ (iii) **Hence Method:** $\arg(z-w) = -\left[\pi - \frac{\pi}{6} - \frac{\pi}{12}\right]$ $= -\left[\frac{5\pi}{6} - \left(\frac{1}{2}\left\{\pi - \frac{5\pi}{6}\right\}\right)\right]$ $= -\frac{3\pi}{4}$ (exact) **Otherwise Method:** $z - w = \left(-1-\sqrt{3}\right) + \left(-1-\sqrt{3}\right)i$ $\arg(z-w) = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$

5	Differentiation & Applications	$V=\frac{128\pi}{9}$ $\frac{\mathrm{d}V}{\mathrm{d}t}=0.12\pi \quad \mathrm{cm}^3\mathrm{s}^{-1}$
6	AP and GP	(a)(i) $d=15$ (ii) $S_{20}=4150$ cm (b)(i) $k=9$ (ii) $n=6$, Length = 235 cm
7	Sigma Notation and Method of Difference	(ii) $\frac{1}{4}-\frac{1}{2(n+1)(n+2)}$ (iii) As $n\to\infty$, $\frac{1}{2(n+1)(n+2)}\to 0.$ $\frac{1}{4}-\frac{1}{2(n+1)(n+2)}\to\frac{1}{4}$ Sum to infinity $=\frac{1}{4}$ (iv)13
8	Differential Equations	(i) $x=\frac{e^{3kt}-1}{e^{3kt}+2}$ (ii)1.45 hours (iii) $x=\frac{1}{2}t-\frac{1}{2}\sin t$ (iv)  The graph shows that as time increases, the drug concentration still continue to increase / the curve shows a strictly increasing function beyond the maximum level of drug concentration.
9	Application of Integration	(i) 64π (iv) The reflected light from the bulb produces a horizontal beam of light/ produces a beam of line parallel to x-axis.

		(v) $y^2 = 4(x-1)$
10	Vectors	(ii) $\left(\frac{3}{2},\ 3,\ \frac{5}{2}\right)$ (iii) $(0,\ 3,\ 2)$ (iv) $\theta = 80.4°,\ 49.8°$ (v) $x+2y-3z=-\frac{\sqrt{14}}{2}$ or $x+2y-3z=\frac{\sqrt{14}}{2}$ (vi) $BD = \frac{\sqrt{6}}{\cos 49.8°} = 3.79$ units (vii) $60°$

H2 Mathematics 2017 Preliminary Exam Paper 1 Solutions

1

$$\frac{3x^2+7x+1}{x+3} < 2x-1$$

$$\frac{3x^2+7x+1}{x+3} - (2x-1) < 0$$

$$\frac{3x^2+7x+1-(2x-1)(x+3)}{x+3} < 0$$

$$\frac{x^2+2x+4}{x+3} < 0$$

$$\frac{(x+1)^2+3}{x+3} < 0$$

Since $(x+1)^2+3>0$ for all real x, the inequality reduces to:

$x+3<0$

$\Rightarrow x<-3$

2

Let $y=\dfrac{1-x^2}{1+x^2},\quad x\in\mathbb{R}$:

$y(1+x^2)=1-x^2$

$(y+1)x^2+(y-1)=0$

Discriminant ≥ 0: $0^2-4(y+1)(y-1)\geq 0$

$-4(y^2-1)\geq 0$

$y^2-1\leq 0$

$y^2\leq 1$

$-1\leq y\leq 1$

Since $y=-1$ is an asymptote, $-1<y\leq 1$

__**Alternative Method:**__

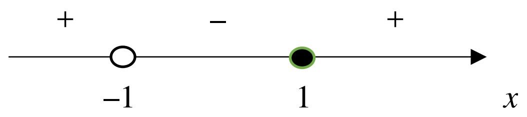
Let $y=\dfrac{1-x^2}{1+x^2},\quad x\in\mathbb{R}$:

$y(1+x^2)=1-x^2$

$(y+1)x^2+(y-1)=0$

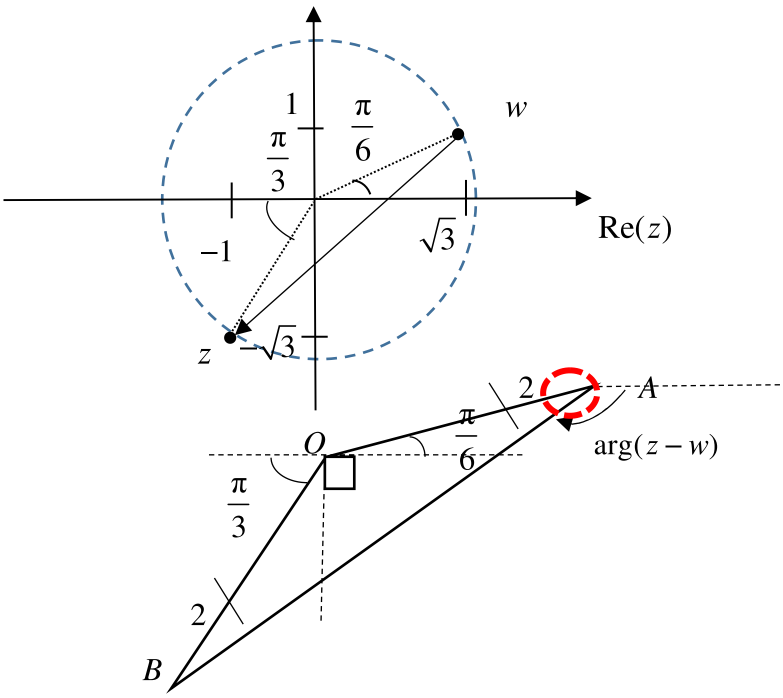
$x^2=\dfrac{1-y}{y+1},\quad y\neq -1$

Since $x^2\geq 0\ \forall x\in\mathbb{R}$, $\dfrac{1-y}{y+1}\geq 0$



$\therefore -1<y\leq 1$

2 (ii)	$\mathrm{p}(-x)=\dfrac{1-(-x)^2}{1+(-x)^2}$ $=\dfrac{1-x^2}{1+x^2}$ $=\mathrm{p}(x)$ for all $x\in\mathbb{R}$ (shown)
2(iii)	Graph of $\mathrm{q}(x)=\mathrm{p}\left(\dfrac{1}{2}x-4\right),\ x\in\mathbb{R}$ is obtained from the graph of $\mathrm{p}(x)$ by: - Translation by 4 units in the positive *x*-direction, followed by Stretch of factor 2 parallel to the *x*-axis.
3(i)	Let $y=(x-k)^2$ $x-k=\pm\sqrt{y}$ $x=-\sqrt{y}+k \quad (\because x<k)$ $\mathrm{f}^{-1}(x)=-\sqrt{x}+k$ $\mathrm{D}_{\mathrm{f}^{-1}}=(0,\ \infty)$
3(ii)	$\mathrm{R}_\mathrm{g}=[-1,\ 4]$ $\mathrm{D}_\mathrm{f}=(-\infty,\ k)$ Since $k>5$, $\mathrm{R}_\mathrm{g}\subseteq\mathrm{D}_\mathrm{f}$. Thus fg exists.
3(iii)	$\mathrm{fg}(-1)=\mathrm{f}(0)=k^2$ Using $\mathrm{R}_\mathrm{g}=[-1,\ 4]$, and the fact that f is a strictly decreasing function in the given domain, $\mathrm{R}_{\mathrm{fg}}=\left[(4-k)^2,\ (-1-k)^2\right]$ $=\left[(4-k)^2,\ (1+k)^2\right]$
4(i)	$\lvert z\rvert=\sqrt{1^2+\sqrt{3}^2}=2 \qquad \arg z=-\left[\pi-\tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right)\right]=-\dfrac{2\pi}{3}$ $z=2\mathrm{e}^{\mathrm{i}\left(-\frac{2\pi}{3}\right)}$ $\dfrac{(\mathrm{i}z)^n}{z^2}=\dfrac{\mathrm{e}^{\mathrm{i}\left(\frac{n\pi}{2}\right)}2^n\mathrm{e}^{\mathrm{i}\left(-\frac{2n\pi}{3}\right)}}{2^2\mathrm{e}^{\mathrm{i}\left(-\frac{4\pi}{3}\right)}}$ $=2^{n-2}\mathrm{e}^{\mathrm{i}\left(\frac{n\pi}{2}-\frac{2n\pi}{3}+\frac{4\pi}{3}\right)}$ $=2^{n-2}\mathrm{e}^{\mathrm{i}\left(\frac{(8-n)\pi}{6}\right)}$ $\dfrac{(\mathrm{i}z)^n}{z^2}$ is purely imaginary: $\cos\left(\dfrac{(8-n)\pi}{6}\right)=0$ $\dfrac{(8-n)\pi}{6}=(2k+1)\dfrac{\pi}{2},\ k\in\mathbb{Z}$ $n=5-6k,\ k\in\mathbb{Z}$ **<u>Note:</u>** You may also have alternative form: $\dfrac{(8-n)\pi}{6}=(2k-1)\dfrac{\pi}{2},\ k\in\mathbb{Z}$ $n=11-6k,\ k\in\mathbb{Z}$

	$\therefore$ smallest positive integer $n=5$. **Alternative Method:** $n\arg(\mathrm{i}z)-2\arg(z)=n\arg(\mathrm{i})+n\arg(z)-2\arg(z)$ $=\frac{n\pi}{2}-\frac{2n\pi}{3}+\frac{4\pi}{3}$ $=\frac{(8-n)\pi}{6}$
4 (ii)	$\lvert wz\rvert=4$ $2\lvert w\rvert=4$ $\lvert w\rvert=2$ $\arg\left(\frac{w^*}{z^2}\right)=-\frac{5\pi}{6}$ $-\arg(w)-2\arg(z)=-\frac{5\pi}{6}$ $\arg(w)=\frac{5\pi}{6}-2\left(-\frac{2\pi}{3}\right)$ $=\frac{13\pi}{6}$ Since $-\pi<\arg(w)\le\pi$, $\arg(w)=\frac{\pi}{6}$ (exact).
4(iii)	 $\angle OAB=\frac{1}{2}\left\{\pi-\left[\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+\frac{\pi}{2}+\frac{\pi}{6}\right]\right\}=\frac{\pi}{12}$ <u>**Hence Method:**</u> $\arg(z-w)=-\left[\pi-\frac{\pi}{6}-\frac{\pi}{12}\right]$ $=-\left[\frac{5\pi}{6}-\left(\frac{1}{2}\left\{\pi-\frac{5\pi}{6}\right\}\right)\right]$ $=-\frac{3\pi}{4}$ (exact)

Otherwise Method:

$z-w=\left(-1-\sqrt{3}\right)+\left(-1-\sqrt{3}\right)\mathrm{i}$ $\quad \arg(z-w)=-\left(\pi-\frac{\pi}{4}\right)=-\frac{3\pi}{4}$

5

Using similar triangles: $\frac{r}{4}=\frac{6-h}{6}$

$r=\frac{2}{3}(6-h)$

$$\begin{aligned}V&=\pi r^2 h\\&=\pi\left(\frac{2}{3}(6-h)\right)^2 h\\&=\frac{4\pi}{9}\left(36-12h+h^2\right)h\\&=\frac{4\pi}{9}\left(36h-12h^2+h^3\right)\quad\text{(shown)}\end{aligned}$$

For maximum V, $\frac{\mathrm{d}V}{\mathrm{d}h}=0$:

$\frac{4\pi}{9}\left(36-24h+3h^2\right)=0$

Using GC: $h=2$ or $h=6$ (Rejected as $h = 6$ is height of cone)

Method 1 (1st derivative sign test)

h	2^-	2	2^+
Sign of $\frac{\mathrm{dV}}{\mathrm{dh}}$	+	0	-
slope	/	—	\

Thus, maximum volume $V=\frac{128\pi}{9}$ when $h = 2$ cm.

Method 2 (2nd derivative test)

$\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=\frac{4\pi}{9}(-24+6h)$

When $h=2$: $\frac{\mathrm{d}^2V}{\mathrm{d}h^2}=-\frac{16\pi}{3}<0$

Thus, maximum volume $V=\frac{128\pi}{9}$.

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t}&=\frac{\mathrm{d}V}{\mathrm{d}h}\cdot\frac{\mathrm{d}h}{\mathrm{d}t}\\&=\frac{4\pi}{9}\left(36-24(1.5)+3(1.5)^2\right)(0.04)\\&=0.12\pi\ \ \text{cm}^3\text{s}^{-1}\end{aligned}$$

(Accept: 0.377 cm^3s^{-1})

6(a)(i)

$u_{20}=a+(n-1)d$

$350=65+19d$

$d=15$

6(a)(ii)

$S_{20}=\frac{20}{2}(65+350)$

$\quad=4150$ cm (Accept: 41.5 m)

6(b)(i)	$S_\infty = \dfrac{a}{1-\frac{8}{9}}$ $= 9a$ $\therefore$ integer $k = 9$.
6 (i)	Method 1: Number of ways $= \binom{14}{3} \times 3! = 2184$ Method 2: Number of ways $= 14 \times 13 \times 12 = 2184$
6(b)(ii)	$S_n \le 2000$ $\dfrac{423\left[1-\left(\frac{8}{9}\right)^n\right]}{1-\frac{8}{9}} \le 2000$ $1-\left(\frac{8}{9}\right)^n \le \dfrac{2000}{3807}$ $\left(\frac{8}{9}\right)^n \ge \dfrac{1807}{3807}$ $n \le \dfrac{\ln\left(\frac{1807}{3807}\right)}{\ln\left(\frac{8}{9}\right)}$ $n \le 6.3267$ $\therefore$ Largest integer $n = 6$. Length of shortest plank is $u_6 = 423\left(\dfrac{8}{9}\right)^{6-1}$ $= 235$ cm (3 s.f.)
7(i)	$\dfrac{1}{r^2-1} = \dfrac{1}{2(r-1)} - \dfrac{1}{2(r+1)}$ $\dfrac{1}{r(r^2-1)} = \dfrac{1}{r}\left[\dfrac{1}{2(r-1)} - \dfrac{1}{2(r+1)}\right]$ $= \dfrac{1}{2}\left[\dfrac{1}{r(r-1)} - \dfrac{1}{r(r+1)}\right]$
7 (ii)	$S_n = \dfrac{1}{2\times3} + \dfrac{1}{3\times8} + \dfrac{1}{4\times15} + \ldots + (n\text{th term})$

	$= \sum_{r=2}^{n+1} \frac{1}{r(r^2-1)}$ $= \frac{1}{2}\sum_{r=2}^{n+1}\left[\frac{1}{r(r-1)} - \frac{1}{r(r+1)}\right]$ $= \frac{1}{2}\left[\frac{1}{2\times 1} - \frac{1}{2\times 3}\right.$ $+ \frac{1}{3\times 2} - \frac{1}{3\times 4}$ $+ \frac{1}{4\times 3} - \frac{1}{4\times 5}$ ⋮ ⋮ ⋮ $+ \frac{1}{(n-1)\times(n-2)} - \frac{1}{(n-1)\times n}$ $+ \frac{1}{(n)\times(n-1)} - \frac{1}{n\times(n+1)}$ $\left. + \frac{1}{(n+1)\times n} - \frac{1}{(n+1)\times(n+2)}\right]$ $= \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]$ $= \frac{1}{4} - \frac{1}{2(n+1)(n+2)}$
7 (iii)	As $n \to \infty$, $\frac{1}{2(n+1)(n+2)} \to 0.$ $\frac{1}{4} - \frac{1}{2(n+1)(n+2)} \to \frac{1}{4}$ Sum to infinity $= \frac{1}{4}$
7 (iv)	$(0<)\frac{1}{4} - S_n < 0.0025$ $\Rightarrow (0<)\frac{1}{4} - \left[\frac{1}{4} - \frac{1}{2(n+1)(n+2)}\right] < 0.0025$ $\Rightarrow (0<)\frac{1}{2(n+1)(n+2)} < 0.0025$ $\Rightarrow (n+1)(n+2) > 200$ Using G.C. $n < -15.651$ or $n > 12.651$ Since $n \in \mathbb{Z}^+$, Smallest value of $n = 13$

8(i)	**Method 1: Using Partial Fractions**

$$\frac{1}{1+x-2x^2}\frac{dx}{dt}=k$$

$$\int \frac{1}{1+x-2x^2}\ dx=\int k\ dt$$

$$\frac{2}{3}\int \frac{1}{2x+1}\ dx-\frac{1}{3}\int \frac{1}{x-1}\ dx=\int k\ dt$$

$$\frac{1}{1+x-2x^2}=\frac{1}{(1-x)(1+2x)}$$
$$=\frac{2/3}{2x+1}-\frac{1/3}{x-1}$$

$$\frac{1}{3}\ln|2x+1|-\frac{1}{3}\ln|x-1|=kt+C$$

$$\frac{1}{3}\ln\left|\frac{2x+1}{x-1}\right|=kt+C$$

$$\frac{2x+1}{x-1}=Ae^{3kt},\ A=\pm e^{3C}$$

$$x=\frac{Ae^{3kt}+1}{Ae^{3kt}-2}$$

When $t=0,\ x=0$: $\quad 0=\frac{A+1}{A-2}\Rightarrow A=-1$

$$\therefore x=\frac{e^{3kt}-1}{e^{3kt}+2}$$

Method 2: Completing the square

$$\frac{1}{1+x-2x^2}\frac{dx}{dt}=k$$

$$\int \frac{1}{1+x-2x^2}\ dx=\int k\ dt$$

$$\int \frac{1}{-2\left(x-1/4\right)^2+9/8}\ dx=\int k\ dt$$

$$\frac{1}{2}\int \frac{1}{\left(3/4\right)^2-\left(x-1/4\right)^2}\ dx=\int k\ dt$$

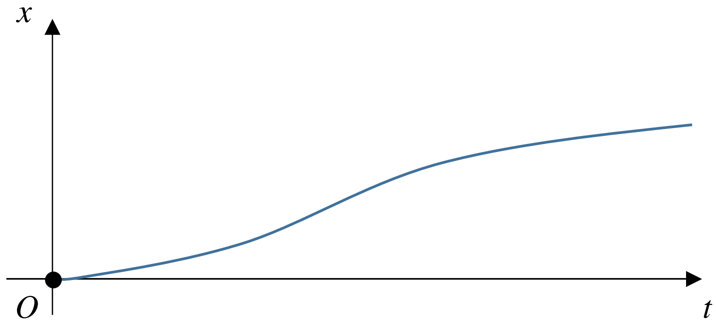
$$\frac{1}{2}\left(\frac{1}{2\left(3/4\right)}\right)\ln\left|\frac{3/4+x-1/4}{3/4-\left(x-1/4\right)}\right|=kt+C$$

$$\frac{1}{3}\ln\left|\frac{1/2+x}{1-x}\right|=kt+C$$

$$\frac{1}{3}\ln\left|\frac{2x+1}{2(1-x)}\right|=kt+C$$

$$\frac{2x+1}{2(1-x)}=Ae^{3kt},\ A=\pm e^{3C}$$

$$x=\frac{2Ae^{3kt}-1}{2\left(Ae^{3kt}+1\right)}$$

	When $t=0,\ x=0$: $0=\dfrac{2A-1}{2(A+1)} \Rightarrow A=\dfrac{1}{2}$ $\therefore x=\dfrac{e^{3kt}-1}{e^{3kt}+2}$
8 (ii)	When $t=1,\ x=\dfrac{3}{4}$: $\therefore \dfrac{3}{4}=\dfrac{e^{3k}-1}{e^{3k}+2} \Rightarrow e^{3k}=10$ $\Rightarrow k=\dfrac{1}{3}\ln 10$ (shown) $\therefore x=\dfrac{10^t-1}{10^t+2}$ When $x=\dfrac{9}{10}$: $\therefore \dfrac{9}{10}=\dfrac{10^t-1}{10^t+2} \Rightarrow 10^t=28$ $\Rightarrow t=\dfrac{\ln 28}{\ln 10}$ = 1.45 hours (3 s.f.) **<u>Also Accept:</u>** 86.8 mins (3 s.f.)
8 (iii)	$\dfrac{dx}{dt}=\sin^2\left(\dfrac{1}{2}t\right)$ $=\dfrac{1}{2}-\dfrac{1}{2}\cos t$ $x=\displaystyle\int \dfrac{1}{2}-\dfrac{1}{2}\cos t\ dt$ $=\dfrac{1}{2}t-\dfrac{1}{2}\sin t+C$ When $t=0,\ x=0$: $C=0$ $\therefore x=\dfrac{1}{2}t-\dfrac{1}{2}\sin t$
8(iv)	 The graph shows that as time increases, the drug concentration still continue to increase / the curve shows a strictly increasing function beyond the maximum level of drug concentration.
9(i)	$y^2=(4t)^2=16t^2$ $=8(2t^2)$ $=8x$ (shown)

	$\text{Volume} = \pi \int_0^4 8x \, dx$ $= \pi \left[4x^2 \right]_0^4$ $= 64\pi$
9(ii)	$\frac{dx}{dt} = 4t, \ \frac{dy}{dt} = 4$ $\frac{dy}{dx} = \frac{1}{t}$ Gradient of tangent $TS = \tan\theta$ $\therefore \tan\theta = \frac{1}{t}$ $\cot\theta = t$ (shown
9 (iii)	Gradient of line $QP = \frac{4t-0}{2t^2-2}$ $= \frac{2t}{t^2-1}$ $= \frac{2/\tan\theta}{1/\tan^2\theta - 1}$ $= \frac{2\tan\theta}{1-\tan^2\theta}$ $= \tan 2\theta$ $\tan\phi = \tan 2\theta \Rightarrow \phi = 2\theta$ (shown) $\angle QPR = 180° - \phi$ (interior angles) $= 180° - 2\theta$ (by earlier results) $\angle TPQ + (180° - 2\theta) + \theta = 180°$ $\therefore \angle TPQ = \theta$ (shown)
9 (iv)	The reflected light from the bulb <u>produces a horizontal beam</u> of light/ produces a beam of line parallel to x-axis
9 (v)	Midpoint $M = \left(\frac{2+2t^2}{2}, \frac{4t+0}{2} \right)$ $= (1+t^2, \ 2t)$ $\begin{cases} x = 1+t^2 \\ y = 2t \Rightarrow t = \frac{y}{2} \end{cases}$ Locus of midpoint M is: $x = 1 + \frac{y^2}{4}$ $y^2 = 4(x-1)$

10(i)	$\overrightarrow{AA'} = \begin{pmatrix} 2-1 \\ 4-2 \\ 1-4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ Since $\overrightarrow{AA'} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \underset{\sim}{n}_1$, $\overrightarrow{AA'}$ is parallel to the normal of p_1, and thus $\overrightarrow{AA'}$ is perpendicular to p_1. **<u>Alternative Method:</u>** Since $\overrightarrow{A'A} = \begin{pmatrix} 1-2 \\ 2-4 \\ 4-1 \end{pmatrix} = -\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = -\underset{\sim}{n}_1$, $\overrightarrow{A'A}$ is parallel to the normal of p_1, and thus $\overrightarrow{A'A}$ is perpendicular to p_1
10 (ii)	Since M is the midpoint of A and A': $\overrightarrow{OM} = \frac{1}{2}\left[\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}\right] = \begin{pmatrix} 3/2 \\ 3 \\ 5/2 \end{pmatrix}$ Coordinates of M are $\left(\frac{3}{2}, 3, \frac{5}{2}\right)$. ← **<u>Note:</u>** Question asks for coordinates form. Since $\frac{3}{2} + 2(3) - 3\left(\frac{5}{2}\right) = -6 + 6 = 0$, M lies in p_1. (shown)
10 (iii)	$\overrightarrow{OB} = \begin{pmatrix} 1+\lambda \\ 2-\lambda \\ 4+2\lambda \end{pmatrix}$ for some $\lambda \in \mathbb{R}$. Since B lies on p_1, $(1+\lambda) + 2(2-\lambda) - 3(4+2\lambda) = 0$ $-7 - 7\lambda = 0$ $\lambda = -1$ $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 2 \end{pmatrix}$ Coordinates of B are $(0, 3, 2)$.

Likewise for part (**vi**).

10 (iv)	$\theta = \cos^{-1}\left\lvert\frac{\overrightarrow{BA}\cdot\overrightarrow{A'B}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{A'B}\rvert}\right\rvert$ $= \cos^{-1}\left\lvert\frac{\begin{pmatrix}1\\-1\\2\end{pmatrix}\cdot\begin{pmatrix}-2\\-1\\1\end{pmatrix}}{\sqrt{6}\sqrt{6}}\right\rvert$ $= \cos^{-1}\left\lvert\frac{1}{6}\right\rvert$ $= 80.4°$ (1 d.p.) Hence, acute angle between the line AB and p_1 $= \frac{180° - 80.4°}{2}$ $= 49.8°$ (1 d.p.) **Note:** You are expected to recognize that $\overrightarrow{A'B} = \overrightarrow{BC}$.
10 (v)	Possible cartesian equations of p_2: $x + 2y - 3z = -\frac{\sqrt{14}}{2}$ or $x + 2y - 3z = \frac{\sqrt{14}}{2}$
10 (vi)	As incident ray AD varies, D is nearest to origin when OD is the shortest. Note that p_1 contains the origin. $AB = \left\lvert\begin{pmatrix}-1\\1\\-2\end{pmatrix}\right\rvert = \sqrt{6}$ $\cos 49.8° = \frac{\sqrt{6}}{BD}$ $\Rightarrow BD = \frac{\sqrt{6}}{\cos 49.8°} = 3.79$ units (3 s.f.)
10 (vii)	Let γ be the required angle of inclination: $\cos^2 60° + \cos^2 45° + \cos^2 \gamma = 1$ $\frac{1}{4} + \frac{1}{2} + \cos^2 \gamma = 1$ $\cos \gamma = \pm\frac{1}{2}$ $\therefore \gamma = 60°$ (since γ is acute)

End of Paper