INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
in preparation for General Certificate of Education Advanced Level
Higher 2

CANDIDATE NAME	

CIVICS GROUP		INDEX NUMBER	

Mathematics **9740/01**

Paper 1 **22 August 2016**

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

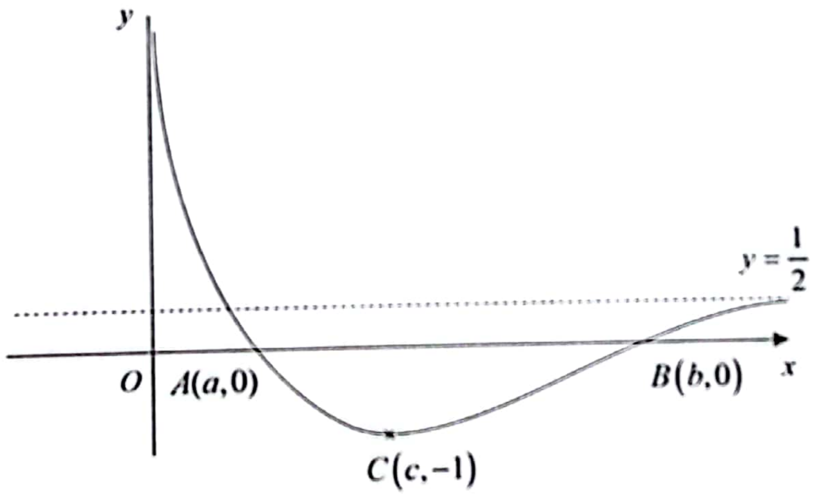
This document consists of **6** printed pages.

1 A theme park sells tickets at different prices according to the age of the customer. The age categories are senior citizen (ages 60 and above), adult (ages 13 to 59) and child (ages 4 to 12). Four tour groups visited the theme park on the same day. The numbers in each category for three of the groups, together with the total cost of the tickets for each of these groups, are given in the following table.

Group	Senior Citizen	Adult	Child	Total cost
A	2	19	9	\$1982
B	0	10	3	\$908
C	1	7	4	\$778

Find the total cost of the tickets for Tour Group *D*, which consists of four senior citizens, five adults and one child. [4]

2



The diagram shows the curve $y=\mathrm{f}(x)$. The curve passes through the point $A(a,0)$ and the point $B(b,0)$, has a turning point at $C(c,-1)$ and asymptotes $y=\frac{1}{2}$ and $x=0$.

Sketch, on separate diagrams, the graphs of

(a) $y=3-|\mathrm{f}(x)|$, [3]

(b) $y=\dfrac{2}{\mathrm{f}(x)}$. [3]

Label the graph in each case clearly and indicate the equations of the asymptotes and the coordinates of the points corresponding to *A*, *B* and *C*.

3 In the triangle ABC, $AB = 1$, $BC = 4$ and angle $ABC = \theta$ radians. Given that θ is a sufficiently small angle, show that

$$AC \approx \left(9+4\theta^2\right)^{\frac{1}{2}} \approx a+b\theta^2 ,$$

for constants a and b to be determined. [5]

4 [It is given that the volume of a pyramid is $\frac{1}{3}\times(\text{base area})\times(\text{height})$.]

A right pyramid of vertical height h m has a square base with side of length $2x$ m and volume $\frac{8}{3}$ m^3.

(i) Express h in terms of x. [1]

(ii) Show that the surface area S m^2 of the pyramid is given by

$$S = 4x^2\left[1+\sqrt{\left(1+\frac{4}{x^6}\right)}\right].$$ [3]

(iii) Use differentiation to find the value of x, correct to 2 decimal places, that gives a stationary value of S. [3]

5 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point C on OA is such $OC : OA = 1 : 3$. The line l passes through the points A and B. It is given that angle $BOA = 60^\circ$ and $|\mathbf{a}| = 3|\mathbf{b}|$.

(i) By considering $(\mathbf{b}-\mathbf{a})\cdot(\mathbf{b}-\mathbf{a})$, or otherwise, express $|\mathbf{b}-\mathbf{a}|$ in the form $k|\mathbf{b}|$, where k is a constant to be found in exact form. [3]

(ii) Find, in terms of $|\mathbf{b}|$, the shortest distance from C to l. [5]

6 A curve has parametric equations

$$x = \cos^2\theta, \quad y = \sin 2\theta, \text{ for } -\frac{\pi}{2} < \theta \le \frac{\pi}{2}.$$

(i) Sketch the curve. [2]

The region enclosed by the curve is denoted by R. The part of R above the x-axis is rotated through 2π radians about the x-axis.

(ii) Show that the volume of the solid formed is given by

$$\pi \int_a^b \sin^3 2\theta \, d\theta,$$

for limits a and b to be determined. [3]

Use the substitution $u = \cos 2\theta$ to find this volume, leaving your answer in exact form. [4]

7 The equation of a curve C is given by

$$3y^3 - 8y^2 + 10y = 4 - 5x.$$

(i) Find the equation of the tangent at the point where $x = \frac{4}{5}$. [5]

(ii) Find the Maclaurin series for y, up to and including the term in x^2. [4]

(iii) State the equation of the tangent to the curve C at the point where $x = 0$. [1]

8 **(a)** The complex number w is given by $(\sqrt{3}) + k\mathrm{i}$, where $k < 0$.

Given that w^5 is real, find the possible values of k in the form $k = (\sqrt{3})\tan(n\pi)$, where n is a constant to be determined. [4]

(b) **(i)** If $z = \cos\theta + \mathrm{i}\sin\theta$, where $0 \le \theta \le \frac{\pi}{2}$, show that

$$1 - z^2 = 2\sin\theta(\sin\theta - \mathrm{i}\cos\theta).$$ [2]

(ii) Hence find $|1 - z^2|$ and $\arg(1 - z^2)$ in terms of θ. [3]

9 The function f is defined by

$$\mathrm{f}: x \mapsto \frac{1}{x^2 - x - 6} + 2, \quad x \in \mathbb{R},\ x \neq -2,\ x \neq 3.$$

(i) Explain why the function f^{-1} does not exist. [2]

(ii) Find, algebraically, the set of values of x for which f is decreasing. [3]

In the rest of the question, the domain of f is further restricted to $x \leq \frac{1}{2}$.

The function g is defined by

$$\mathrm{g}: x \mapsto 2 - x,\ x \in \mathbb{R}.$$

(iii) Find an expression for $\mathrm{gf}(x)$ and hence, or otherwise, find $(\mathrm{gf})^{-1}\left(\frac{1}{4}\right)$. [3]

10 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = \frac{1}{2}$ and

$$u_{n+1} = u_n - \frac{n^2 + n - 1}{(n+2)!}, \quad \text{for all } n \geq 1.$$

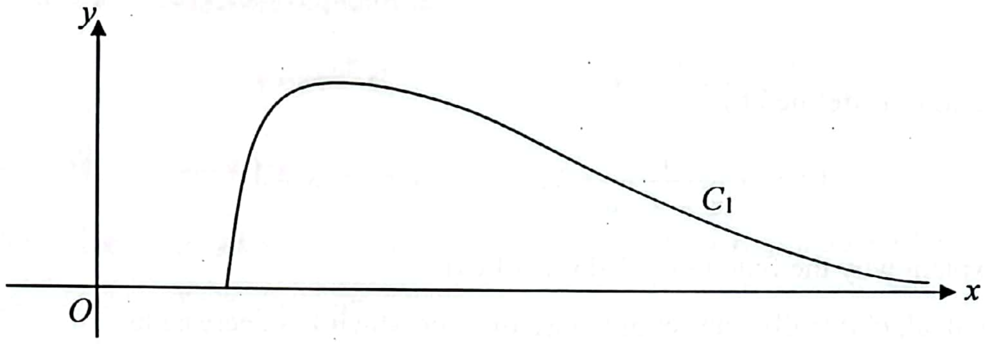
(i) Use the method of mathematical induction to prove that $u_n = \frac{n}{(n+1)!}$. [5]

(ii) Hence find $\displaystyle\sum_{n=1}^{N} \frac{n^2 + n - 1}{(n+2)!}$. [3]

(iii) Explain why $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + n - 1}{(n+2)!}$ is a convergent series, and state the value of the sum to infinity. [2]

[Turn over

11



The diagram above shows the curve C_1 with equation $y = \frac{\ln x}{x^2}$, where $x \geq 1$.

(i) Show that the exact coordinates of the turning point on C_1 are $\left(\sqrt{e}, \frac{1}{2e}\right)$. [3]

(ii) The curve C_2 has equation $\left(x - \sqrt{e}\right)^2 + (2ey)^2 = 1$, where $y \geq 0$. Sketch C_1 and C_2 on the same diagram, stating the exact coordinates of any points of intersection with the axes. [3]

(iii) Write down an integral that gives the area of the smaller region bounded by the two curves, C_1 and C_2, and the x-axis. Evaluate this integral numerically. [4]

12 **(a)** **(i)** Solve the equation

$$z^6 - 2i = 0,$$

giving the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

(ii) Show the roots on an Argand diagram. [2]

(iii) The points A, B, C, D, E and F represent the roots z_1, z_2, z_3, z_4, z_5 and z_6 respectively in the Argand diagram. Find the perimeter of the polygon $ABCDEF$, leaving your answer to 3 decimal places. [2]

(b) The complex number w satisfies the relations

$$|w + 5 - 12i| \leq 13 \text{ and } 0 \leq \arg(w + 18 - 12i) < \frac{\pi}{4}.$$

(i) On an Argand diagram, sketch the region in which the points representing w can lie. [4]

(ii) State the maximum and minimum possible values of $|w + 10|$. [2]