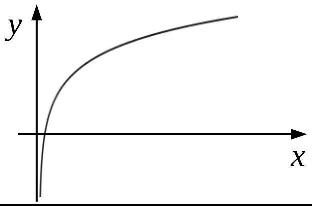
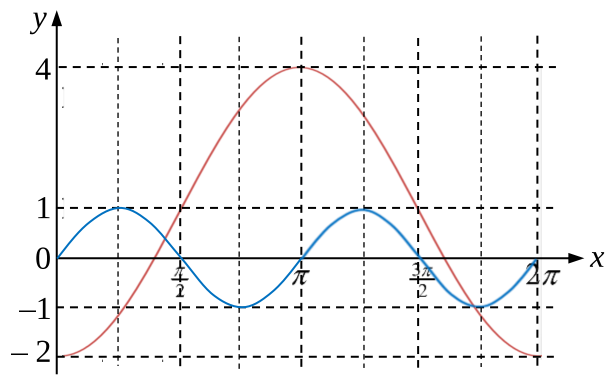
Mark Scheme for 2024 S4E5N Add Math Prelim **Paper 2**				
1	$\frac{1-8x-3x^2}{(x-1)(2x^2+3)}=\frac{A}{x-1}+\frac{Bx+C}{2x^2+3}$	M1		Realising the form of the partial fractions
	$1-8x-3x^2=A(2x^2+3)+(x-1)(Bx+C)$	M1		Realising the need to eliminate denominator
	When $x=1$, $1-8(1)-3(1)^2=A[2(1)^2+3]$			
	$-10=5A\rightarrow\ A=-2$			
	When $x=0$, $1=-2[2(0)^2+3]+(0-1)(C)$			
	$1=-6-C\rightarrow\ C=-7$			
	When $x=2$, $1-8(2)-3(2)^2=-2[2(2)^2+3]+[2B-7]$			
	$-27=-22+2B-7\rightarrow\ B=1$			
	$\frac{-2}{x-1}+\frac{x-7}{2x^2+3}$	A3, 2, 1		-1 for each error inc. final answer
			[5]	
2a	$2x^2-7x-4-c=0$	M1		Eliminates y or x
	$b^2-4ac=(-7)^2-4(2)(-4-c)$	M1		Uses the discriminant
	$49+32+8c>0$			
	$c>-10.125$	A1		Accept $c>-\frac{81}{8}$
	smallest integer $c=-10$	A1		
2b	$y=kx^2+2(2k-5)x+9k$			
	$b^2-4ac=(4k-10)^2-4(k)(9k)$	M1		Uses the discriminant
	$16k^2-80k+100-36k^2<0$			
	$-20k^2-80k+100<0\rightarrow k^2+4k-5>0$			
	$(k+5)(k-1)>0$			
	$k<-5$ or $k>1$	A2		A1 for each
	$k<0\rightarrow\ k<-5$	B1		s.o.i. when $k>1$ rejected
			[8]	
3a	$\frac{1-\cos x}{\sin x-\operatorname{cosec} x+\cot x}$			
	$\frac{1-c}{s-\frac{1}{s}+\frac{1}{t}}$	B1		cosec and cot all correct
	$\frac{1-c}{\frac{s^2-1+c}{s}}\rightarrow\frac{s(1-c)}{s^2-1+c}$	M1		Correct algebra
	$\frac{s(1-c)}{-c^2+c}$	M1		Uses $s^2+c^2=1$
	$\frac{s(1-c)}{c(1-c)}=t$	A1		All correct
3b	$\tan 2x=-3$	M1		Uses part (a) and replaces x with $2x$
	$\alpha=\tan^{-1}(3)$	M1		For finding basic angle
	$2x=180°-\alpha,\ 360°-\alpha$			
	$x=54.2°, 144.2°$	A1		Accept 54.21… and 144.21…
3c	$t=\frac{2t}{1-t^2}$			

	$t - t^3 = 2t$			
	$t^3 + t = 0 \rightarrow t(t^2+1) = 0$			
	$t = 0$ (not valid for $0° < x < 180°$)	B1		Must include and then reject $t = 0$
	or $t^2 = -1$ not possible. No solution	B1		Realises there is no solution to $t^2 = -1$
	Alternative Answer:			
	Correct sketch of $y = \tan x$ and $y = \tan 2x$	B1		
	No point of intersection for $0° < x < 180°$ and so no solution to the equation $\tan x = \tan 2x$	B1		
			[9]	
4a	$\log_2 x + \frac{\log_2 x}{\log_2 8} = \frac{\log_2 2}{\log_2 4}$	M1		Change of base
	$\log_2 x + \frac{\log_2 x}{3} = \frac{1}{2}$	B1		For '$= \frac{1}{2}$'
	$\log_2 x + \frac{1}{3}\log_2 x = \frac{1}{2} \rightarrow \log_2 x = \frac{3}{8}$	M1		Making $\log_2 x$ the subject
	$x = 2^{\frac{3}{8}} \rightarrow m = \frac{3}{8}$	A1		
4b		B2		B1 for curvature of decreasing gradient observed B1 for graph close to asymptote observed with x-intercept 1
4c	$\log_5(\frac{2x-11}{x-4}) = 1$	M1		Combine to single log
	$\frac{2x-11}{x-4} = 5$			
	$2x - 11 = 5(x-4) \rightarrow x = 3$	A1		
	If $x = 3$, $\log_5(2x-11) = \log_5(-5)$ or $\log_5(x-4) = \log_5(-1)$	M1		Substitute into log expression
	Does not exist $\rightarrow$ No solutions	A1		Correct argument and conclusion
			[10]	
5a	$f'(x) = e^{x^2+x}(2x+1)$	B1		
	Since $x > 0$, $2x+1 > 0$ and $e^{x^2+x} > 0$	M1		Argues correctly
	$f'(x) > 0 \rightarrow$ increasing	A1		$f'(x) > 0$ or $e^{x^2+x}(2x+1) > 0$ must be seen
5bi	$\frac{(x+3)(2x) - x^2(1)}{(x+3)^2} = \frac{x^2+6x}{(x+3)^2}$	B2, 1		B1 for unsimplified
	$\frac{x^2+6x}{(x+3)^2} = 0 \rightarrow x = 0,\ x = -6$	M1		Sets to 0 and solves
	$(0,\ 0)$ and $(-6, -12)$	A1 A1		SR1 answers not in coordinate form
5bii	$\frac{18}{(x+3)^3}$	B1		
	$x = 0 \rightarrow \frac{d^2y}{dx^2} = \frac{2}{3} > 0 \rightarrow$ minimum point	DB1		Correct $\frac{d^2y}{dx^2}$ value and conclusion

	$x=-6 \rightarrow \frac{d^2y}{dx^2} = -\frac{2}{3} < 0 \rightarrow$ maximum point	DB1		Correct $\frac{d^2y}{dx^2}$ value and conclusion
			[11]	
6a	$m_{PQ} = \frac{7-1}{2-(-6)} = \frac{3}{4}$	B1		
	$M_{PQ} = (\frac{2+(-6)}{2}, \frac{7+1}{2}) = (-2, 4)$	B1		
	Gradient of Perpendicular $= -\frac{4}{3}$	M1		Uses $m_1 m_2 = -1$
	$y-4 = -\frac{4}{3}(x-(-2))$			
	$y = -\frac{4}{3}x + \frac{4}{3}$	A1		
6b	$-\frac{4}{3}x + \frac{4}{3} = -2x - 4$	M1		Realises the need to use sim eqns
	$x = -8$			
	$(-8,12)$	A1		
	$\sqrt{(-8-2)^2 + (12-7)^2}$	M1		Uses distance formula correctly with P or Q
	$\sqrt{125}$	A1		
	$(x+8)^2 + (y-12)^2 = 125$	B1√		√ for their radius and centre
6c	$(-8-\sqrt{125}, 12)$	B2√		B1 for each, √ for their radius and centre
			[11]	
7a	$v = -0.3(4-t)^2 + 1.2$			
	$a = 0.6(4-t)$	B1		
	When $v=0 \rightarrow (4-t)^2 = 4$	M1		Sets to 0
	$t=2,\ t=6$	A1		
	$a = 0.6(4-2) = 1.2\ \text{m/s}^2$	A1		
7b	$s = \int -0.3(4-t)^2 + 1.2\ dt$	M1		Realises need to integrate
	$s = 0.1(4-t)^3 + 1.2t \quad (+c)$	A1		
	When $t=1,\ s=2.5,\ c=-1.4 \rightarrow$ $s = 0.1(4-t)^3 + 1.2t - 1.4$			Use $t=1$ and $s=3.9$ to find c
	When $t=0, s=5\text{m}$	A1		
7c	Particle turned during $t=0$ to $t=7$	B1		
	When $t=7$, it only gives distance from O	B1		
7d	When $t=2,\ s=1.8$ or When $t=6,\ s=5$ OR distance $= \left\lvert \int_0^2 v\ dt \right\rvert = 3.2\text{ m}$ or $\int_2^6 v\ dt = 3.2\text{ m}$	M1		For finding displacement at either time it turns
	When $t=7,\ s=4.3$ OR distance $= \left\lvert \int_6^7 v\ dt \right\rvert = 0.7\text{ m}$	M1		For finding displacement at ending time
	distance $= (5-1.8)+(5-1.8)+(5-4.3) = 7.1\text{ m}$	A1		
			[12]	

8a		B3		B1 for two complete sine cycles with correct amplitude and intersections at *x*-axis B1 for one negative cosine cycle B1 for cosine cycle with correct amplitude and intersections at axis of rotation
8bi	$AB = CE - CF = 5\sin\theta - 2\cos\theta$	B1		
	$AD = ED + BF = 5\cos\theta + 2\sin\theta$	B1		
	$\text{Perimeter} = 5\sin\theta - 2\cos\theta + 5\cos\theta + 2\sin\theta + 7$ $\rightarrow 3\cos\theta + 7\sin\theta + 7$	B1		Answer was given – so all working must be correct
8bii	$R = \sqrt{3^2 + 7^2} = \sqrt{58}$	B1		
	$\tan\alpha = \frac{7}{3} \rightarrow \alpha = 66.8°$	M1		For finding α
	$\sqrt{58}\cos(\theta - 66.8°)$	A1		
8biii	$\sqrt{58}\cos(\theta - 66.8°) + 7 = 13$			
	$\cos(\theta - 66.8°) = \frac{6}{\sqrt{58}}$			
	$\alpha = 38.0°$	M1		For finding α
	$\theta - 66.8° = -\alpha,\ \alpha$			
	$\theta = 28.8°,\ 104.8°$ (rejected)	A1		Accept 28.78…
	$6.75\ \text{cm}^2$	A1		
			[12]	
9a	$\frac{3x}{3x+1} = 1 - \frac{1}{3x+1}$	B1		
	$\int \frac{3x}{3x+1}\,dx = x - \frac{1}{3}\ln(3x+1) + c$	B3, 2, 1		−1 for each error in the integration – needs +c
9b	$\frac{dy}{dx} = \ln(3x+1) + x \times \frac{1}{3x+1} \times 3$	M1 B1 A1		Uses product formula. B1 ‘$\frac{1}{3x+1}$’. A1 ‘×3’
	$= \ln(3x+1) + \frac{3x}{3x+1}$			
9c	$\int \ln(3x+1) + \frac{3x}{3x+1}\,dx - \int \frac{3x}{3x+1}\,dx$ $\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$ OR $\ln(3x+1) = \frac{dy}{dx} - \frac{3x}{3x+1}$ $\rightarrow x\ln(3x+1) - \int \frac{3x}{3x+1}\,dx$	M1		Attempts to use the result of part (b)

	$x\ln(3x+1)-\int\frac{3x}{3x+1}\,\mathrm{d}x$ $\rightarrow x\ln(3x+1)-\int 1-\frac{1}{3x+1}\,\mathrm{d}x$	M1		Realises the need to use part (a)
	$[x\ln(3x+1)]_0^4-\left[x-\frac{1}{3}\ln(3x+1)\right]_0^4$	A1		All correct
	$[4\ln 13-0]-\left[4-\frac{1}{3}\ln 13\right]$	M1		Use definite integral formula on $x\ln(3x+1)$ or antiderivative from part (a)
	$\frac{13}{3}\ln 13-4$	A1		For both values
			[12]	