Class	Register Number	Name

南洋女子中學校

Nanyang Girls' High School

End-of-Year Examination 2022
Secondary 4

INTEGRATED MATHEMATICS 2

2 hour 30 minutes

Monday 3 October **0845 – 1115**

Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

1. Write your name, register number and class on all the work you hand in.
2. Write in dark blue or black ink.
3. You may use an HB pencil for any diagrams or graphs.
4. Do not use staples, paper clips, glue or correction tape/fluid.
5. Write your answers and working on the separate writing paper provided, unless otherwise stated.
6. Answer **all** questions.
7. Omission of essential working will result in loss of marks.
8. The use of an approved scientific calculator is expected, where appropriate.
9. If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degree to one decimal place. For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.
10. At the end of the examination, fasten all your work securely together.
11. The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\text{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin A + \sin B = 2\sin\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\sin A - \sin B = 2\cos\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

$$\cos A + \cos B = 2\cos\frac{1}{2}(A+B)\cos\frac{1}{2}(A-B)$$

$$\cos A - \cos B = -2\sin\frac{1}{2}(A+B)\sin\frac{1}{2}(A-B)$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\triangle = \frac{1}{2}bc\sin A$$

1 The equation of a curve is $y = 2x^2 - 5x - 7$.

(i) Express $2x^2 - 5x - 7$ in the form $a(x+b)^2 + c$, where a, b and c are constants.

Hence sketch the curve, indicating clearly the turning point and the intercepts. [5]

(ii) Find the range of values of m for which $y = 2x^2 - 5x - 7$ intersects the line $y = mx - 9$ at two distinct points. [3]

2 **(a)** Solve $4^x - 5(2^{x+1}) + 16 = 0$. [4]

(b) The electric power, P watts, is given by $P = I^2R$ where I amperes is the current flowing through the circuit and R ohms is the resistance.

Calculate the resistance of a 72 watts light bulb that draws $\left(\sqrt{3} - 1\right)$ amperes of current. Leave your answer in the form $a + b\sqrt{3}$, where a and b are integers. [3]

3 The expression $(1-p)^6$ is equal to $1 - 6p + 15p^2 - 20p^3 + 15p^4 - 6p^5 + p^6$.

By using this result or otherwise, find the expression of $(1 - x - x^2)^6$ in ascending powers of x up to the term in x^3. [4]

4 It is given that $\dfrac{7x+5}{(x+1)(2x+1)} = \dfrac{A}{x+1} + \dfrac{B}{2x+1}$, where A and B are constants.

(i) Determine the value of A and of B. [4]

(ii) Hence, evaluate $\displaystyle\int_0^2 \frac{7x+5}{(x+1)(2x+1)}\, dx$. [3]

5 **(a)** Solve $3x^3 - 11x^2 + 9x + 2 = 0$, showing your working clearly. [4]

(b) When a polynomial $P(x)$ is divided by $(x-1)$ and $(x-2)$, the remainders are 2 and 3 respectively. Find the remainder when $P(x)$ is divided by $(x-1)(x-2)$.

[4]

6 A fact sheet on caffeine dependence from Johns Hopkins Medical Center states that the half-life of caffeine in the body is between 4 and 6 hours. Half-life is the time taken for the amount of caffeine in the body to decrease by half.

The total amount of caffeine y mg in the body t hours after drinking the coffee can be modelled by the function $y = ab^t$ where a and b are constants.

Rachel drinks a cup of coffee which has 120 mg of caffeine.

Assuming that the half-life of caffeine in her body is 5 hours,

(i) find the value of a and of b, [3]

(ii) calculate the time it will take when there is only 20 mg of caffeine left in her body. [2]

7 It is given that $\sqrt{3}\sin x + 3\cos x = R\sin(x + A)$ where R is a positive real number, the angles x and A are in radians and A is acute.

(i) Find the value of R and of A. [2]

(ii) Hence or otherwise, solve $\sqrt{3}\sin x + 3\cos x = \sqrt{3}$, where $0 \le x \le 2\pi$. [3]

8 **(a)** Solve $\log_2 \sqrt{x-9} + \log_4 (x-2) = 1\frac{1}{2}$. [4]

(b) Solve $3\cos(180° - 4y) - 2\sin 2y = 5$, where $0° \le y \le 360°$. [5]

9 **(a)** Sketch the graph of $y = \frac{1}{2}\sin x + 3$ for $0 \le x \le 2\pi$, indicating clearly the turning points and the intercept(s). [3]

(b) The graph of $y = \frac{1}{2}\sin x + 3$ is obtained through two transformations from $y = \sin x$. Describe these two successive transformations. [2]

10 **(i)** Express $\frac{1}{2}(\sin 4x + \sin 3x)$ in the form $a\sin bx\cos cx$, where a, b and c are constants. [1]

(ii) Hence, show that $\sin\frac{7x}{2}\cos\frac{x}{2} + \cos\frac{9x}{2}\sin\frac{x}{2} = \frac{1}{2}(\sin 3x + \sin 5x)$. [3]

11 **(i)** Find an expression for $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{x^2+7}+3x\ln x\right)$. [3]

(ii) Hence find the value of $\int_1^2\left(\frac{2}{3}x\mathrm{e}^{x^2+7}+\ln x\right)\mathrm{d}x$, leaving your answer to 2 decimal places. [4]

12 The equation of a curve is $y=-\frac{1}{4}\cos 4x-\frac{1}{2}\sin 2x+\frac{1}{2}$. Point $A(x, y)$ lies on the curve.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) The variables x and y are such that y is decreasing at a rate of 0.05 units per second. Find the rate of change of x when $x = 2.4$. [3]

(iii) Find the equation of the tangent at A when $x=\frac{\pi}{2}$. Leave your answer in the form $y=ax+b\pi+c$, where a, b and c are constants. [4]

13 Cylinders make good drinking cans. The shape can stand upright on tables and fits comfortably into hands.

The base radius of a **closed** cylindrical drinking can is x cm and the height of the cylinder is h cm. V cm^3 and S cm^2 represent the fixed volume and the total surface area of the cylinder respectively.

(i) Show that $S=2\pi x^2+\frac{2V}{x}$. [2]

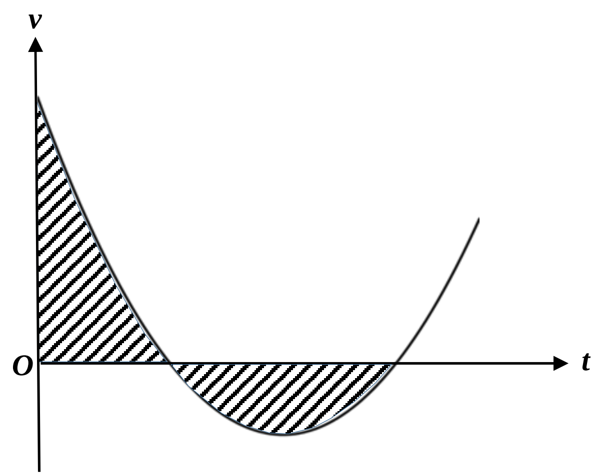
(ii) The volume of a typical cylindrical drinking can is 320 cm^3. Given that x and h can vary, find the value of x and of h for which S has a stationary value and determine whether this value of S is a maximum or a minimum. [5]

(iii) Sam claimed that regardless of the volume, the height of a cylindrical drinking can is twice its radius for the total surface area to have a stationary value. Do you agree? Justify your answer. [3]

[You do not need to determine the nature of the stationary value.]

14 A particle travels in a straight line. The velocity, v m/s, of the particle, t seconds after passing A, is given by $v = t^2 - 5t + 4$. The particle comes to rest momentarily first at point B and then at point C.

The diagram shows part of the velocity-time graph of the particle. This diagram is not drawn to scale.



(i) Find the values of t when the particle is at rest. [2]

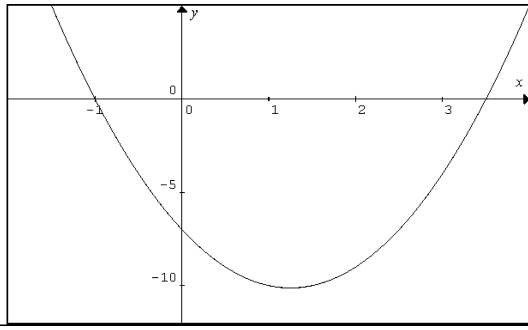
(ii) Find the total shaded area. [4]

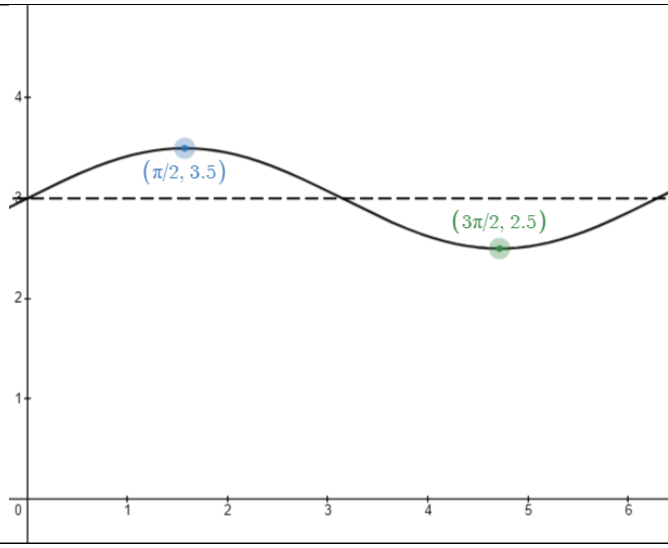
(iii) Explain the significance of your answer obtained in **(ii)**. [1]

(iv) The particle has zero acceleration at the point D. Determine with full working, whether D is nearer to A or to C. [5]

End of Paper

2022 Sec 4 IM2 EOY Exam Answers

1(i)	$2\left(x-\frac{5}{4}\right)^2-\frac{81}{8}$ 
1(ii)	$m<-9$ or $m>-1$
2(a)	$x=1$ or $x=3$
2(b)	$72+36\sqrt{3}$
3	$1-6x+9x^2+10x^3+...$
4(i)	$A=2$ $B=3$
4(ii)	4.61
5(a)	$x=2$ or $x=1.85$ or -0.180
5(b)	$x+1$
6(i)	$a=120$ $b=\left(\frac{1}{2}\right)^{\frac{1}{5}}$ or $2^{-\frac{1}{5}}$
6(ii)	12.9 hours
7(i)	$R=2\sqrt{3}$ $A=\frac{\pi}{3}$
(ii)	$x=\frac{\pi}{2}$ or $\frac{11\pi}{6}$
8(a)	$x=10$ or 1 (rejected)
8(b)	$y=135°$ or $315°$

9(a)	
(b)	Scale along *y*-axis by a factor of $\frac{1}{2}$. Translate in positive *y*-direction by 3 units.
10(i)	$\sin\frac{7x}{2}\cos\frac{x}{2}$
11(i)	$2xe^{x^2+7}+3+3\ln x$
11(ii)	18964.78 (to 2 dp)
12(i)	$\sin 4x-\cos 2x$
12(ii)	the rate of change of x is 0.191 units/s.
12(iii)	$y=x-\frac{1}{2}\pi+\frac{1}{4}$
13(ii)	$x=3.71,\quad h=7.41$ this surface area is a minimum.
13(iii)	When surface area has a stationary value, regardless of the volume, V, $\frac{\mathrm{d}S}{\mathrm{d}x}=4\pi x-\frac{2V}{x^2}=0$ $4\pi x=\frac{2V}{x^2}$ $4\pi x=\frac{2\pi x^2 h}{x^2}$ $4\pi x=2\pi h$ $2x=h$ $\therefore$ I agree with Sam that the height is twice its radius.
14(i)	$t=1$ or $t=4$
14(ii)	$6\frac{1}{3}$ square units
(iii)	The **total distance** travelled by the particle in the **first 4 seconds** is $6\frac{1}{3}$ m.

(iv)	$v = t^2 - 5t + 4$ $a = 2t - 5 = 0$ $t = \frac{5}{2}$ $s = \int_0^{\frac{5}{2}} (t^2 - 5t + 4)\ \mathrm{d}t$ $= -0.41667$ (to 5 sf) Distance between *A* and *D* = 0.41667 m $s = \int_{\frac{5}{2}}^{4} (t^2 - 5t + 4)\ \mathrm{d}t$ $= -2.2500$ (to 5 sf) Distance between *C* and *D* = 2.2500 m Since 2.2500 > 0.41667, *D* is nearer to *A*.