CEDAR GIRLS' SECONDARY SCHOOL
Preliminary Examination 2022
Secondary Four

CANDIDATE NAME

CLASS | | CLASS INDEX NUMBER | |

CENTRE/ INDEX NO | | | | | / | | | | |

MATHEMATICS
Paper 2

4048/02
30 August 2022

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer all the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use
100

This document consists of **21** printed pages and **1** blank page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone $= \pi rl$

Surface area of a sphere $= 4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length $= r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the inequality $\frac{x-5}{3}-\frac{2x-1}{2}\leq -1$.

Answer ..... [3]

(b) Express as a single fraction in its simplest form $\frac{1}{x^2-4}-\frac{1}{x^2-x-6}$.

Answer ..... [3]

(c) Ally and Betty buy some pens and notebooks from the same shop.
Ally buys 3 pens and 2 notebooks for $4.80.
Betty buys 5 pens and 4 notebooks for $9.

(i) Form a pair of simultaneous equations to represent this information.

Answer

[1]

(ii) Solve the simultaneous equations to find the cost of a pen and the cost of a notebook.

Answer Cost of pen = $ .....

Cost of notebook = $ ..... [3]

(d) p is inversely proportional to q^3. When q is decreased by 50%, find the percentage change in p.

Answer .....% [3]

2 A fruit juice stall sells regular, medium and large glasses of apple and orange juice. The number of glasses of each type of juice that was sold on a particular morning are summarised in the following table.

	Regular (R)	Medium (M)	Large(L)
Apple Juice	20	30	11
Orange Juice	14	36	8

(a) Represent the data in the table in a 2×3 matrix J.

Answer $\mathbf{J} = \begin{matrix} \\ \text{Apple juice} \\ \text{Orange juice} \end{matrix} \begin{matrix} \text{R} & \text{M} & \text{L} \\ \left(\right. & & \left. \right) \end{matrix}$ [1]

(b) The cost price of each regular, medium and large glass of juice is \$1.50, \$2 and \$3 respectively.

The information can be represented in a 3×1 matrix $\mathbf{C} = \begin{pmatrix} 1.5 \\ 2 \\ 3 \end{pmatrix}$.

(i) Evaluate the matrix $\mathbf{M} = \mathbf{JC}$.

Answer $\mathbf{M} =$ [2]

(ii) State what each element of matrix M represents.

.....

..... [1]

(c) The profit from the sale of each regular, medium and large glass of juice is 80% of its cost price.
Using matrix multiplication, evaluate the total sales of the apple and orange juice of the stall on that particular morning.

Answer ..... [3]

3 The point A is $(0,7)$ and the point B is $(6,9)$.

(a) Express $\overrightarrow{AB}$ as a column vector.

Answer $\overrightarrow{AB} = \begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

(b) The equation of AB is $x + py + q = 0$.
Find the values of p and q.

Answer $p =$ ..... and $q =$ ..... [2]

(c) The point C is $(12,2)$.

(i) M is the point on BC produced such that $\overrightarrow{BM} = 3\overrightarrow{CM}$.
Find the coordinates of M.

Answer (..... , ...) [2]

(ii) Find the length of the line AC.

Answer ..... units [1]

(d) The point D lies on the line AB produced.
The line CD is parallel to the y-axis.

(i) Find the coordinates of D.

Answer (....., ...) [2]

(ii) Express $\overrightarrow{AD}$ in terms of $\overrightarrow{AB}$.

Answer $\overrightarrow{AD}$ = ... [1]

4 The distances travelled per litre of petrol of 50 cars manufactured in Factory X are shown in the table below.

Distance per litre (x km)	$20 < x \leq 21$	$21 < x \leq 22$	$22 < x \leq 23$	$23 < x \leq 24$
Number of cars	9	13	17	11

(a) Calculate an estimate of the mean of the data.

Answer ..... km [1]

(b) Calculate an estimate of the standard deviation of the data.

Answer ..... km [1]

(c) The distances travelled per litre of petrol of 45 similar cars manufactured in Factory Y are tabulated in the same manner.
The estimated mean and the standard deviation of the data are 22.5 km and 0.985 km.

(i) Make two comparisons between the distance travelled of cars manufactured by Factory X and by Factory Y.

Answer

(1) .....

.....

(2) .....

..... [2]

(ii) Another batch of 5 cars manufactured in Factory Y travelled between 22 km to 23 km on a litre of petrol.
If this information is included in the data for the computation of the mean and standard deviation, state the effect on its

(a) mean,

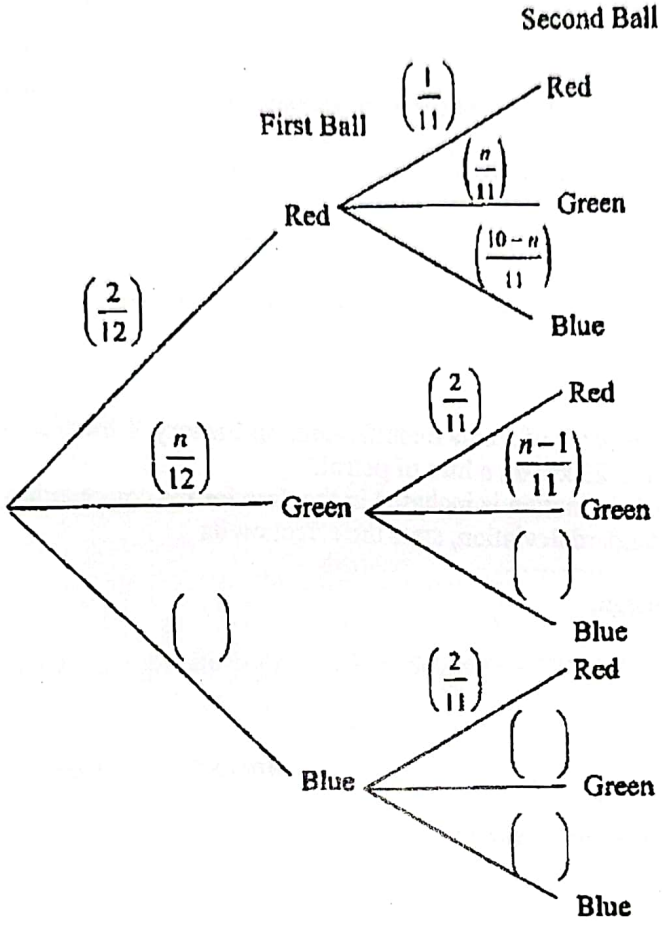
Answer ..... [1]

(b) standard deviation.

Answer ..... [1]

5 A bag contains 12 balls.
2 of the balls are red, n are green balls and the rest are blue balls.
Eunice takes two balls from the bag, at random, without replacement.

(a) Complete the tree diagram.



[2]

(b) The probability that Eunice takes two blue balls is $\frac{5}{22}$.

Write down an equation to represent this information and show that it simplifies to

$$n^2 - 19n + 60 = 0 .$$

Answer

[3]

(c) Solve the equation $n^2 - 19n + 60 = 0$.

Answer $n =$ ..... or ..... [3]

(d) Explain why one of the solutions in part (c) must be rejected.

.....

..... [1]

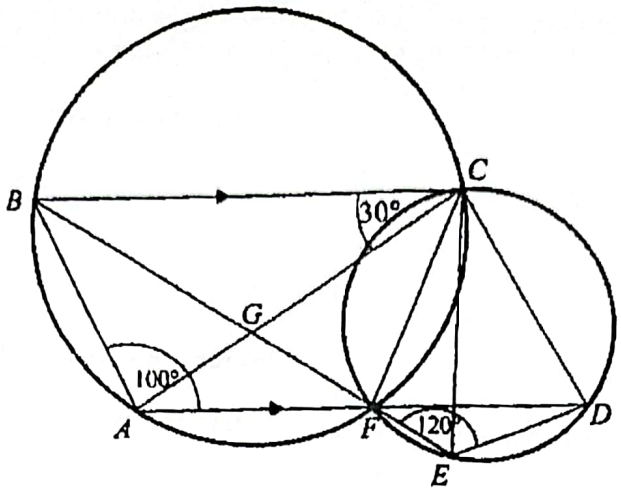
(e) Find, as a fraction, in its simplest form, the probability that Eunice takes balls of different colours.

Answer ..... [2]

(f) In this round, Eunice takes two balls from the bag, at random, **with replacement**. Find, as a fraction, in its simplest form, the probability that she will take balls of the same colour.

Answer ..... [2]

6 (a)



In the figure, circle $ABCF$ and circle $CDEF$ intersect at C and F.
AFD and BFE are straight lines and BC is parallel to AD.
Angle $ACB = 30°$, angle $BAF = 100°$ and angle $FED = 120°$.

(i) Find, giving reasons for each answer,

(a) angle ACD,

Answer .....° [3]

(b) angle CDA.

Answer .....° [2]

(ii) Prove that triangle BAG is congruent to triangle CFG.
Give a reason for each statement you make.

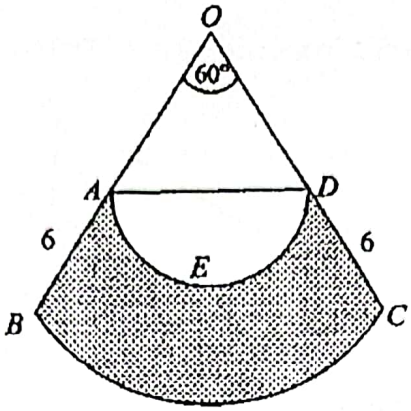
.....

.....

.....

[3]

(b)



In the diagram, OBC is the sector of a circle, centre O, and angle $BOC = 60°$.
A and D are the midpoints of OB and OC respectively, and $AB = DC = 6$ cm.
AED is a semicircle with AD as diameter.

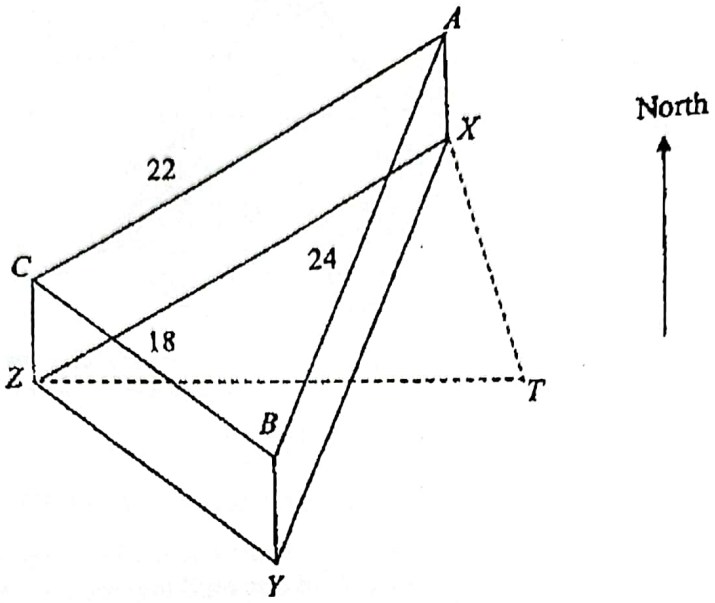
(i) Find the length of the arc BC in the form of $n\pi$ centimetres.

Answer ..... cm [2]

(ii) Find the area of the shaded region.

Answer ..... cm^2 [4]

7



An aircraft waiting to land is flying around a triangular circuit ABC.
A, B and C are vertically above three beacons, X, Y and Z respectively.
T is the control tower at the airport, and T, X, Y and Z lie in a horizontal plane.
$BC = 18$ km, $CA = 22$ km and $AB = 24$ km.

(a) (i) The plane is flying at 200 km/h.
Calculate the time, in minutes and seconds, that the aircraft takes to complete one round of circuit ABC.

Answer ..... min ..... s [2]

(ii) Calculate the largest angle of triangle ABC.

Answer ..... ° [3]

(b) Z is due west of T.
The bearing of X from Z is 042° and the bearing of X from T is 338°.

(i) Find angle ZXT.

Answer .....° [3]

(ii) Calculate the distance of TX.

Answer ..... km [2]

8 The number of bacteria in a colony doubles every hour.
The colony starts with 50 bacteria.
The table below shows the number of bacteria in the colony after time t.

Time in hours (t)	0	1	2	3	4	5	6	7
Number of bacteria (n)	50	100	200	400	800	1 600	3 200	6 400

(a) On the grid opposite, draw the graph of n against t for $0 \leq t \leq 7$. [3]

(b) (i) By drawing a tangent, find the gradient of the curve at $t = 5.5$.

Answer ..... [2]

(ii) State briefly what this gradient represents.

.....

..... [1]

(c) The number of bacteria in another colony is given by the equation $n = 4\,000 - 500t$.

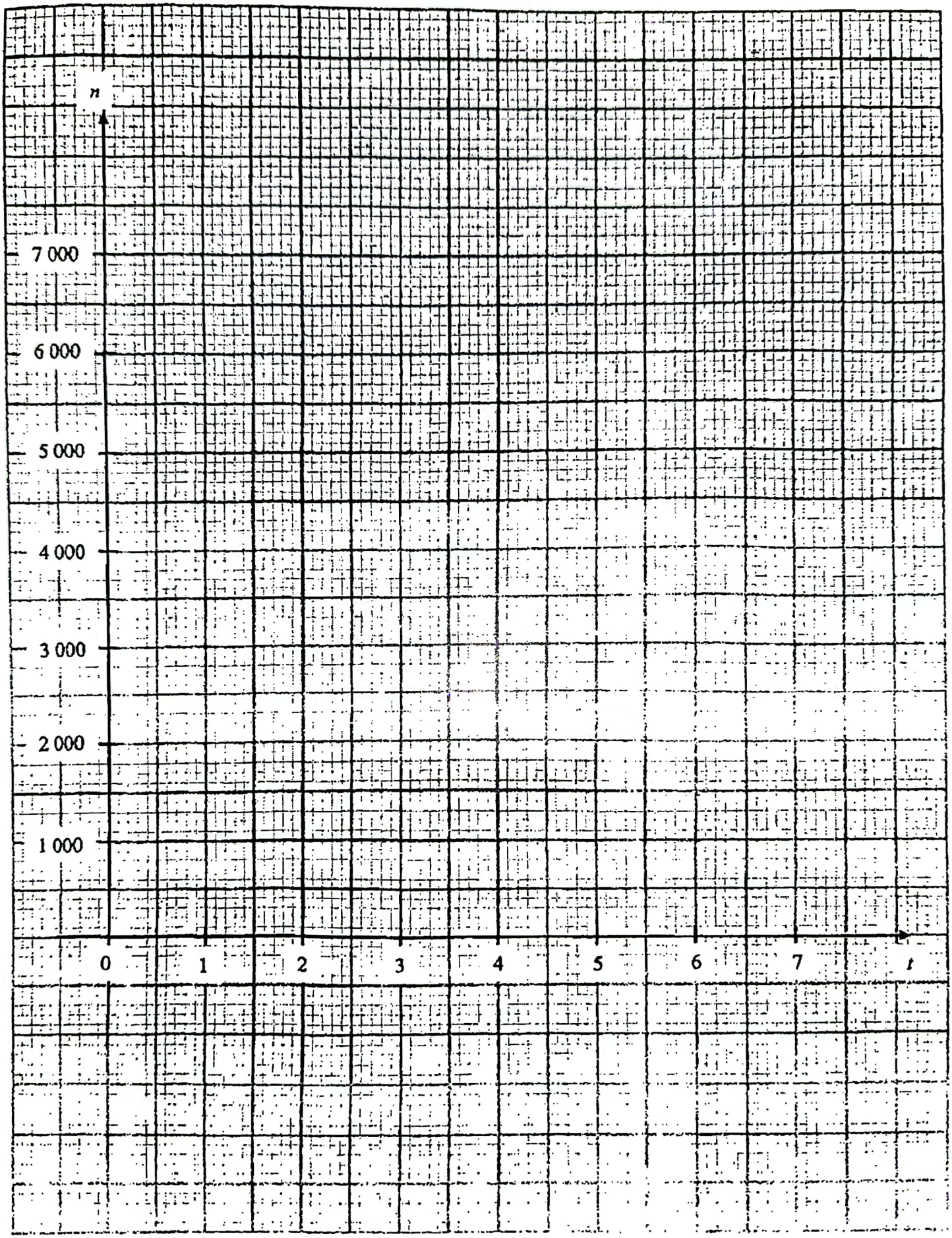
(i) On the grid in part (a), draw a graph to represent the number of bacteria in this colony for $0 \leq t \leq 7$. [2]

(ii) Find the value of t when the numbers in the colonies are equal.

Answer ..... [1]

(d) Given that the equation of the first graph is $n = k2^t$, find the value of k.

Answer $k =$ ..... [1]



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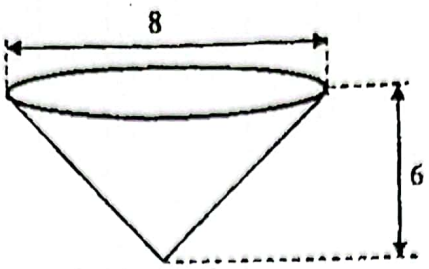
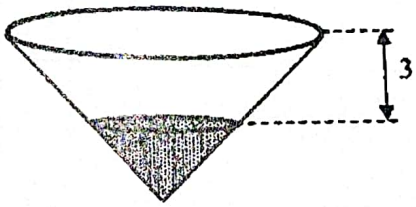


A paper cup is in the shape of an inverted cone.
The diameter of the top of the cup is 8 cm.
The height of the cone is 6 cm.
The thickness of the paper is negligible.

(a) Calculate the curved surface area of the inside of the paper cup.

Answer ..... cm^2 [3]

(b)



Farah pours water into the paper cup.
The surface of the water is 3 cm below the top of the paper cup.

(i) Farah thinks that the paper cup is filled to 50% of its total capacity.
Explain why she is wrong.

.....

..... [1]

(ii) Calculate the percentage of the total capacity of the cup that is filled.

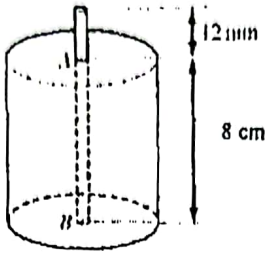
Answer ..... % [1]

(iii) Farah pours the water from this conical cup and it fills a hemispherical bowl completely.
Calculate the diameter of the hemispherical bowl.

Answer ..... cm [3]

10 Gillian wants to make cylindrical soy wax candles as farewell gifts for 50 classmates and friends in school.

The diagram below shows one of the cylindrical candles she intends to make.



A is the centre of the top of the candle and B is the centre of the base of the candle.
The cotton wick runs from B through A and extends 12 mm above A.

(a) How many of these candles can be made using a 10 m length of cotton wick?

Answer ..... [1]

(b) The cotton wick is in the form of a solid cylinder. It has a diameter of 4 mm.
Find the volume of the wick inside the candle from A to B.

Answer ..... cm^3 [2]

Gillian makes one soy wax candle by putting the wick into a cylindrical jar.
She will then pour the melted soy wax into the jar so that it surrounds the wick.
The cylindrical jar has an internal radius of 2.9 cm.

(c) Calculate the volume of soy wax needed to make each cylindrical candle.

Answer ..... cm^3 [2]

Gillian's mum has the tools for candle-making and is willing to provide 50 cylindrical jars for her farewell gifts.

The following tables give online information that Gillian can use to buy the rest of the materials that she needs for making the 50 soy wax candles.

Cost of a box of soy wax	
Weight	Cost
250 g	$4.40
500 g	$8.25
1 kg	$16
5 kg	$64.73
Buy $45 or more to enjoy free shipping. Shipping fee is $1.49. Density of soy wax = 0.9 g/cm^3.	

Cost of 10 m length of cotton wick with a diameter of 4 mm (minimum length of purchase)	$6.32
Shipping fee	$1

Cost of 100 candle wick tab and sticker used to hold down the wick in the jar (fixed minimum number to purchase)	$1.20
Shipping fee	$1

Gillian's mum claims that each farewell gift will not cost Gillian more than $3.

(d) Is Gillian's mum claim correct?
Justify any decisions you make and show your calculations clearly.

.....

..... [5]

End of Paper