NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS **9740/02**

Paper 2 **17 Sep 2012**

3 hours

Additional Materials: Answer Papers
List of Formulae (MF15)
Graph paper

READ THESE INSTRUCTIONS FIRST

Write your name and class on every script you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **7** printed pages.

Section A [40 marks]

1 **(a)** The complex numbers z and w are such that

$$z = 1 + \mathrm{i}a, \quad w = -b - \mathrm{i}$$

where a and b are real and positive.

Given that $zw = 6 - 11\mathrm{i}$, find the exact values of a and b. **[4]**

(b) The conjugate of a complex number u has modulus 2 and argument $\frac{2}{3}\pi$ and another complex number v has modulus 5 and argument $\frac{3}{4}\pi$. Find the exact values of the modulus and argument of $\frac{v}{u^2}$. **[2]**

Hence, find the smallest positive integer n such that the point representing $\left(\frac{v}{u^2}\right)^n$ lies on the negative imaginary axis. **[2]**

2 The function f is defined by $\mathrm{f} : x \mapsto x^2 - 4x,\ x \in \mathbb{R},\ |x| < 1$.

(i) Show that f^{-1} exists and find f^{-1} in a similar form. **[4]**

The function g is defined by $\mathrm{g} : x \mapsto 1 + \frac{1}{x+3},\ x \in \mathbb{R}, x \neq -3$.

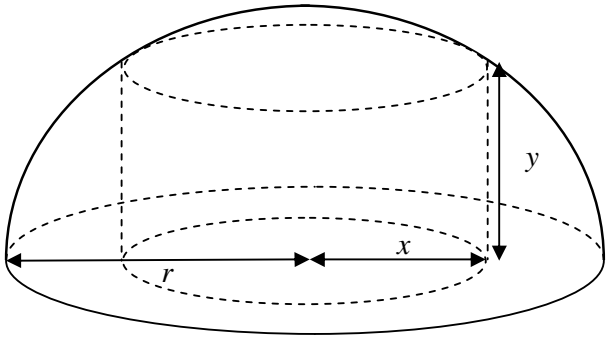
(ii) Show that the composite function gf exists. **[1]**

(iii) Find an expression for gf(x). **[1]**

The function h is defined by $\mathrm{h} : x \mapsto \mathrm{gf}(x),\ x \in \mathbb{R},\ |x| < 1$.

(iv) Using differentiation, determine whether h is strictly increasing or strictly decreasing. Hence or otherwise, find the range of h. **[5]**

3 In a futuristic movie, a hemispherical force field dome (an open hemisphere) is created to protect a cylindrical silo under construction to house deadly weapons. The circumference of the top of the silo lies just within the dome as shown in the diagram below. The power supply is such that the radius r of the dome generated is fixed at 0.5 km. Let x km and y km be the radius and height of the silo respectively. Show that the volume of the silo can be expressed as $\pi\left(\frac{1}{4}y - y^3\right)\text{km}^3$. Find the exact maximum volume of the silo which can be constructed. **[5]**



A power surge causes the dome to be enlarged (still retaining its hemispherical shape) such that its volume is increasing at a constant rate of 0.75 km^3s^{-1}. Find the exact rate of increase of surface area of the dome when $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{3}{4\pi}$ kms^{-1}. **[5]**

[The formula for the volume and surface area of a sphere are $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$.]

4 **(a)** The curve C has equation $y = \ln\left(2 - |x|\right)$. The region R is bounded by C, the lines $y = \ln 2$, $x = 1$ and $x = -1$. Find the exact area of R. **[5]**

(b) The curve C is defined parametrically by

$$x = (1+t)^{\frac{2}{3}}, \qquad y = \ln\left(t^2\right) \qquad \text{for } t \le -1.$$

Find the volume of the solid generated when the region enclosed by C, the lines $x = 0$ and $x = 1$ and the x-axis is rotated 2π radians about the x-axis, giving your answer correct to 2 decimal places. **[6]**

Section B [60 marks]

5 The organisers of a conference wish to conduct a survey on 80 participants to gather feedback about their conference experience. The 800 participants who attended the conference come from various educational institutions, as shown in the table below:

Primary school	Secondary school	Junior college	Polytechnic
350	250	150	50

(i) Describe briefly how a sample of 80 participants can be selected by using an appropriate sampling method. **[2]**

(ii) State one advantage and one disadvantage of the sampling method suggested in **(i)**. **[2]**

6 A committee consisting of six people is to be selected from five women and six men. Three of the women are sisters. Find the number of ways the committee can be formed if

(i) the chosen committee will contain exactly two men, **[1]**

(ii) at least one of the sisters are included. **[2]**

The chosen committee consists of two particular sisters together with 3 other men and one other woman. They are seated at a round table meant for six persons. Find the number of possible arrangements if

(iii) one of the men is to be seated between the two sisters, **[2]**

(iv) the two sisters are sitting directly opposite each other. **[2]**

7 The average trade-in value, y thousand dollars, of a particular make of used car depreciates with the age of the car, x years, according to the following table.

Age, x	2	3	4	5	6	7	8
Trade-in value, y	53.9	49.9	44.8	38.0	34.6	32.5	29.8

(i) Draw a scatter diagram to illustrate the data. **[2]**

(ii) Comment on whether a linear model would be appropriate, referring both to the scatter diagram and the context of the question. **[2]**

(iii) Explain why in this context a quadratic model would probably not be appropriate for long-term predictions. **[1]**

(iv) Fit a model of the form $\ln y = ax + b$ to the data, and use it to predict the trade-in value of a $5\frac{1}{2}$-year-old car. **[2]**

8 A bag contains 10 red and 15 green beads which are indistinguishable apart from colour. Beads are drawn from the bag singly and at random, with replacement.

(i) Calculate the probability that the first red bead is obtained on or before the 5th draw. **[2]**

(ii) Calculate the probability of first obtaining a green bead on the 8th draw, given that no green bead has been obtained in the first 5 draws. **[2]**

(iii) Find the probability that exactly r draws are required for beads of both colours to be obtained, leaving your answer in the form $a(b^{r-2} + c^{r-2})$, where a, b and c are of the form $\frac{p}{q}$, $p, q \in \mathbb{Z}^+$. **[2]**

(iv) Calculate the probability of first obtaining beads of different colours after 5 or more draws. **[3]**

9 In a box of cheese rings from a certain manufacturer, the mass of sodium, X mg, is a continuous random variable with mean μ_0. Following a change in production chain, 15 randomly chosen boxes of cheese rings were analysed. The masses of sodium (in mg) in the boxes are summarised by

$$\sum(x-1180)=17.5, \qquad \sum(x-1180)^2=190.5$$

(i) Given that $\mu_0=1183$, test at 5% significance level, whether the mean mass of sodium in a box of cheese rings has changed. You should state clearly any necessary assumption you need to make when carrying out the test. **[6]**

(ii) Given that the standard deviation of mass of sodium in a box of cheese ring is 6.8 mg, find the range of values of μ_0 if the test based on the same set of data leads to the conclusion that the mass of sodium has increased at 5% significance level. Explain briefly if the assumption made in part **(i)** is still necessary. **[3]**

10 Every morning, Peter has to take a bus to school. On average, he reaches the bus stop at 6:30 am each day. His arrival time at the bus stop is normally distributed with a standard deviation of 10 minutes. The first bus will leave the bus stop at 6:40 am sharp and the second bus will leave at 6:50 am sharp. Regardless of the time the buses leave the bus stop, the bus journey will follow a normal distribution with mean 45 minutes and standard deviation 20 minutes. Peter will be late if he arrives at school after 7:30 am or if he misses the 6:50 am bus.

Assuming that the time Peter reaches the bus stop and the time taken for the bus journey are independent, find the probability that

(i) Peter will miss the 6:50 am bus. **[2]**

(ii) Show the probability that Peter will be late for school is 0.442. **[4]**

In a particular five-day week, find the probability that Peter will be late on at least 2 days. **[2]**

Using a suitable approximation, find the probability that in 40 five-day weeks, Peter will be late for at least 2 days on average per week. **[2]**

11 A credit card company has a special 24-hour telephone hotline for customers to either report a lost credit card or to request for an increase in their credit limit. It is found that in any 10-minute period, on average, three calls are received reporting on a lost card and two calls are received requesting for an increase in credit limit. In addition, six "nuisance" calls on average are received every 15 minutes for other customer services (i.e. those unrelated to reporting of a lost card or a credit limit increase). Assuming that the calls received may be modeled with Poisson distributions,

(i) find the probability that in a one-hour period, more than 12 calls are received reporting on a lost credit card, **[2]**

(ii) find the probability that in a given 15-minute interval, at most 12 calls are received, **[3]**

(iii) find the probability that in 24 consecutive one-hour periods, there are at least 20 one-hour periods where more than 12 calls are received per hour reporting on a lost credit card, **[3]**

(iv) use a suitable approximation to find the probability that in a one-hour period, the number of "nuisance" calls exceeds the total number of calls reporting on a lost card or requesting for a credit limit increase. **[4]**

(v) State clearly any two assumptions used in obtaining your answers above. **[2]**

End of Paper