SERANGOON JUNIOR COLLEGE

2015 JC2 PRELIMINARY EXAMINATION

MATHEMATICS

Higher 2 **9740/1**

Wednesday **19 Aug 2015**

Additional materials: Writing paper

List of Formulae (MF15)

**TIME : ** 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.
Write in dark or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

Total marks for this paper is 100 marks.

This question paper consists of 6 printed pages (inclusive of this page) and 2 blank pages.

[Turn Over

1 A landscape gardener is tasked to design a garden with a total area of 140 m^2.

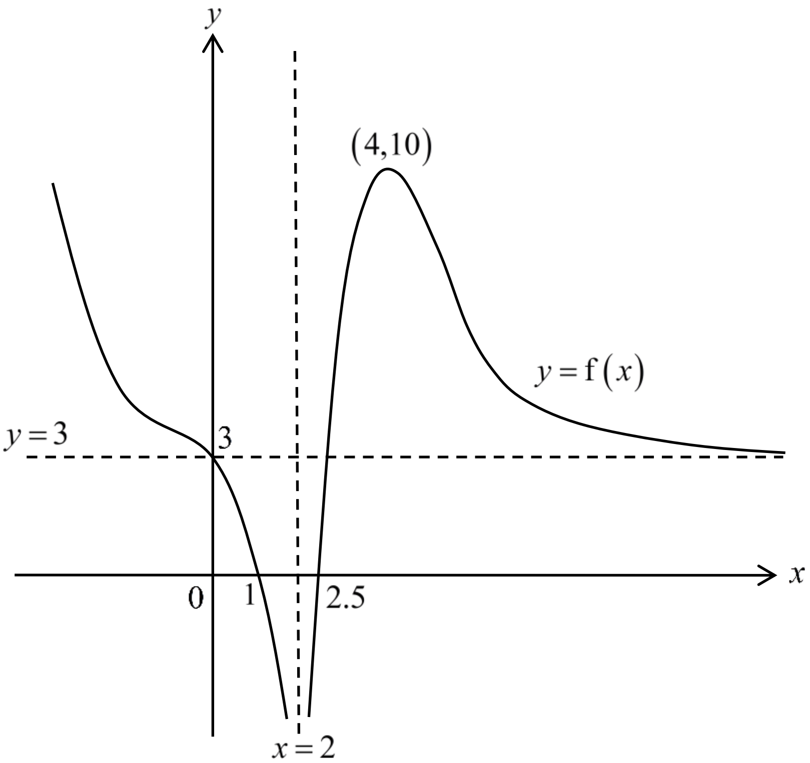
Part of the garden will be decking, part will be flowers and the rest will be grass. Let the area of decking, the area of flowers and the area of grass, be denoted by *d*, *f* and *g* respectively, all measured in m^2.

It is required that the area of grass is 20 m^2 more than the total area of flowers and decking.

Including labour, each square metre of grass, decking and flowers costs \$10, \$21 and \$42 respectively. The landscape gardener has been instructed to come up with a design that will cost \$2900.

Find the values of *d*, *f* and *g* that the landscape gardener should use. [4]

2 The diagram below shows the graph of $y = \mathrm{f}(x)$ with asymptotes $x = 2$ and $y = 3$. The curve cuts the *x*-axis at $x = 1$ and $x = 2.5$, the *y*-axis at $y = 3$ and has only one stationary point at $(4,10)$.



Sketch on separate diagrams, the graphs of

(a) $y = \mathrm{f}(|x|)$, [3]

(b) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

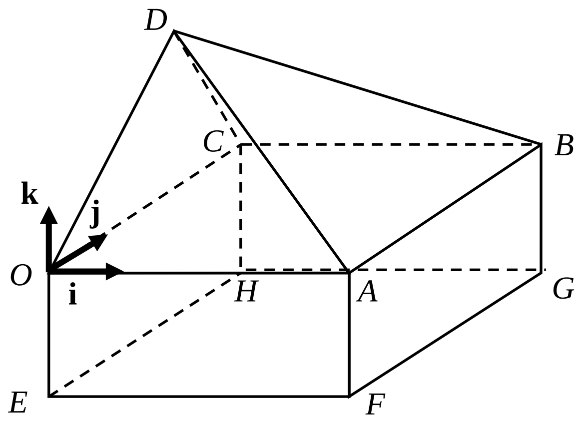
showing clearly any equations of asymptotes, axial intercepts and coordinates of the turning points.

3 Find the exact value of $\int_0^{\frac{\pi}{3}} 3x\sec^2 x \, dx$. [5]

4 A sequence u_n is given by $u_1 = 1$ and $u_{n+1} = u_n + (n+1)\left(\frac{1}{2}\right)^n$, for $n \in \mathbb{Z}^+$.

Prove using mathematical induction that $u_n = 4-(n+2)\left(\frac{1}{2}\right)^{n-1}$. [5]

5 The diagram below shows a figure made up of a pyramid and a cuboid. The pyramid has a square base *OABC* of side 6 units. The vertex *D* is 4 units vertically above *R*, the midpoint of *OC*. The cuboid shares the same square base and is of height 3 units.



With *O* as the origin and using the unit vectors **i, j** and **k** given in the diagram,

(i) show that the position vector of point *P* is $4\mathbf{i}+\mathbf{j}+\frac{4}{3}\mathbf{k}$, where *P* lies on *AD* such that $AP:PD = 1:2$, [3]

(ii) find the position vector of point *Q* in terms of **i, j** and **k**, where *Q* is the midpoint of *FG*. Hence, find the area of triangle *OPQ*. [4]

6 Given that $y = \sin^{-1} x$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

By further differentiation of this result, show that, up to and including the term in x^3, the Maclaurin's series for $\sin^{-1} x$ is $x + kx^3$, where k is a constant to be determined. [5]

Hence, determine the series expansion of $\sqrt{1+\sin^{-1} x}$, up to and including the term in x^3. [2]

[Turn Over

7 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_n = \dfrac{2}{n^3}$ and $u_{n+1} = u_n - \dfrac{6n^2+6n+2}{n^3(n+1)^3}$, for $n \geq 1$.

(i) Find $\displaystyle\sum_{n=1}^{N} \frac{3n^2+3n+1}{n^3(n+1)^3}$ in terms of N. [3]

Using your answer in part **(i)**,

(ii) find $\displaystyle\sum_{n=3}^{N} \frac{3n^2-3n+1}{n^3(n-1)^3}$ in terms of N, [3]

(iii) deduce the value of the following sum to infinity

$$\frac{7}{1^3(2^3)} + \frac{19}{2^3(3^3)} + \frac{37}{3^3(4^3)} + \ldots.$$ [1]

8 The equations of three planes p_1, p_2 and p_3 are

$$2x + 5y - z = 3$$
$$3x + y + 5z = -2$$
$$5x + \alpha y + 4z = \beta$$

respectively, where α and β are constants. When $\alpha = -3$ and $\beta = 1$, find the coordinates of the point where all the 3 planes meet. [1]

The planes p_1 and p_2 intersect in a line l.

(i) Find a vector equation of l. [1]

(ii) Show that the shortest distance from the point $(1,3,2)$ to l is $2\sqrt{3}$. [2]

(iii) Given that all 3 planes meet in the line l, find the values of α and β. [3]

(iv) Given instead that the three planes have no point in common, what can be said about the values of α and β? [2]

(v) If the planes p_1 and p_4 are parallel and the point $(1,3,2)$ is equidistant from these two planes, find the equation of the plane p_4 in scalar product form. [3]

9 **(a)** A curve C is defined by the equation $xy + y^2 - x^3 = 7$.

Find the coordinates of the point where the tangent line to the curve C is parallel to the y – axis and state the equation of this tangent line. [5]

(b) A curve is defined parametrically by the equations $x = at,\ y = \dfrac{a}{t^2}$. The fixed point P on the curve has parameter p.

Find the equation of the normal to the curve at the point P, leaving your answer in terms of a and p. [3]

Given that the normal at point P does not meet the curve again, find exactly the range of values of p. [4]

10 **(a)** An arithmetic progression, A, has first term, a, and a non-zero common difference, d. The first, fifth and fourteenth term of A are equal to the third, second and first term of a geometric progression G respectively.

(i) Show that the common ratio of G is $\dfrac{4}{9}$. [3]

(ii) Determine the ratio of the sum to infinity of G to the sum to infinity of the odd-numbered terms of G. [3]

(b) In January 2001, John borrowed \$29 000 from a bank. Interest was charged at 4.5% per year and was calculated at the end of each year starting from 2001. John planned to pay back a fixed amount of \$$x$ on the first day of each month, starting from the first month after his graduation. John graduated from his study in at the end of 2004 and started payment in January 2005.

(i) Show that the amount owed at the end of 2005 was

$$1.045^5(29000) - 1.045(12x).$$ [1]

Taking 2005 as the first year, find an exact expression for the amount John owed at the end of the n^{th} year, simplifying your answer in terms of n and x. [3]

(ii) If the loan must be repaid within 8 years from John's graduation, find the minimum amount, correct to the nearest dollar, John must pay each month. [2]

11 **(a)** The complex number $-1+\mathrm{i}$ is a root of $3z^3 + 13z^2 + az + b = 0$, where a and b are real constants. Find the values of a and b. [3]

(b) The complex number z is such that $z^4 = z^*$.

(i) Write down all possible value(s) of $|z|$. [2]

(ii) Find all possible values of z in exponential form. [3]

(iii) Given that $0 < \arg(z) < \frac{\pi}{2}$, find the smallest positive real number k for z^k to be purely imaginary. [3]

12 **(a)** A curve C is defined by the parametric equations

$$x = 2t + \cos 2t, \quad y = \sin 2t, \qquad \text{where } -\frac{\pi}{2} \le t \le 0.$$

Sketch the curve C, labelling the x-intercepts. Hence find the exact area of the region bounded by C and the x-axis. [6]

(b) The curve has equation $\mathrm{f}(x) = x(x+2a)$, where a is a positive constant.

(i) Find, in terms of a, the value of $\int_{-a}^{a} |\mathrm{f}(x)| \, \mathrm{d}x$. [2]

(ii) The region R is bounded by the curve $y = \mathrm{f}(x)$ and the x-axis. Find, in terms of a and π, the volume generated when R is rotated through 2π radians about the y-axis. [4]

End of Paper

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