FAIRFIELD METHODIST SCHOOL (SECONDARY)

PRELIMINARY EXAMINATION 2022
SECONDARY 4 EXPRESS / 5 NORMAL (ACADEMIC)

MATHEMATICS   4048/01

Paper 1

Date: 23 August 2022   **Duration: 2 hours**

Candidates answer on the Question Paper.
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READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.

For π , use either your calculator value or 3.142, unless the question requires the answer in terms of π .

The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 80.

For Examiner's Use

Table of Penalties		Question Number		
Presentation	€ 1			
	€ 2		Parent's / Guardian's Signature	**80**
Rounding off	€ 1			

Name: _____ (   )   Class: _____

Setters: Mr Alester Tan

This question paper consists of <u>20</u> printed pages.

Name: _____ ( ) Class: _____

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

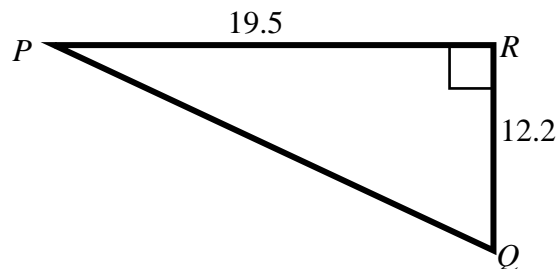$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1   Vivien invested $2500 in a bank at a simple interest rate of 3% per annum. The total value of the investment in the bank after a few years is $2912.50. Calculate the number of years in which she has invested in the bank.

Answer ……………………………years [2]

_____

2

In the triangle, $PR = 19.5$ cm, $RQ = 12.2$ cm and angle $PRQ = 90^o$.
Calculate PQ.

Answer $PQ =$ ….……………………cm [2]

_____

3   A train travels 120 km in 1 hour 45 minutes. During the first 30 minutes, the train travels at an average speed of 96 km/h.
Calculate the average speed of the train during the last 1 hour 15 minutes.

Answer …………………………… km/h [2]

_____

4   A bag contains balls of equal sizes of which 12 are blue, 6 are yellow and the rest are white. The probability of drawing a blue ball from the box is $\dfrac{3}{7}$.

(a)   Find the total number of white balls in the bag.

Answer ……………………… white balls [1]

(b)   Find the number of yellow balls that should be added to the bag so that the probability of drawing a yellow ball is $\dfrac{7}{18}$.

Answer ………………… yellow balls [1]

_____

5   Write as a single fraction in its simplest form $\dfrac{5x}{3} - \dfrac{3(x+4)}{2}$.

Answer ………………………………… [2]

_____

6   **(a)**   Five positive consecutive integers have a mean of 19. The largest integer is x. Find the value of x.

Answer $x =$ …………………………… [1]

       (b)   A set of data is listed as p, p^2, p^3, p^4 and p^5 where p is an integer, $p < 0$ and $p \neq -1$.

Trevor says that p^3 is not the median.

Explain, with mathematical reasoning, why he is correct.

Answer

…………………………………………………………………………………………

…………………………………………………………………………………………

…………………………………………………………………………………………

……………………………………………………………………………………[2]

7   The dimension of a wooden cuboid is 3 cm by 2 cm by 2 cm. It takes 6174 of such wooden cuboids to form a cube.
Find the length of the cube.

Answer ..…………………………… cm [2]

_____

8   Travis wanted to buy a particular pair of sneakers in 2020 but he waited until 2021 when its price was reduced by 13.2%. In 2020, the cost of the pair of sneakers was $510. Calculate the cost of the sneakers in 2021.

Answer $…………………………………… [2]

9 **(a)** Solve $\dfrac{6}{5} \div \dfrac{15x}{2} = 3$

Answer $x =$ ………………………………… [1]

(b) Simplify $3p - 4(6p - 8q)$.

Answer ......………………………………… [2]

_____

10 The table below shows the favourite drinks of a group of students.
A pie chart is to be drawn to show this information.
Calculate the angle of each sector of the pie chart.
Write your answers in the table.

Favourite drink	Number of students	Sector angle
Fruit juice	😊😊😊	
Soft drink	😊😊😊😊😊	
Coffee	😊😊😊	

😊 represents 6 students.

[3]

Name: _____ (   ) Class: _____

11   An interior angle of a regular hexagon is 5 times the size of the exterior angle of another *n*-sided regular polygon. Find the value of *n*.

Answer n = ……………………………… [3]

_____

12   **(a)**   On the Venn Diagram shown in the answer space, shade the set $A \cup B'$.

Answer
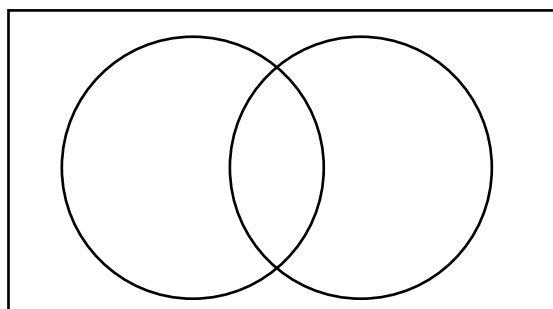
[1]

(b) $\xi = \{x: x \text{ is an integer and } x \geq 1\}$
$P = \{x: x \text{ is a prime number}\}$
$Q = \{x: x \text{ is a perfect square}\}$
$R = \{x: x \text{ is an integer ending in 2}\}$

(i)   Find $n(P \cap R)$.

Answer ……………………………… [1]

(ii)   Find $P \cap Q$.

Answer ……………………………… [1]

13 The figure shows a container with a height of 40 cm and a width of 15 cm. The other dimensions are as shown. The container was initially empty and it takes 35 seconds to fill the container at a constant rate.
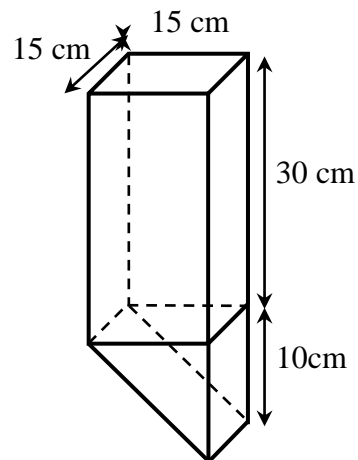
(a) Find the time taken for the water level to reach 10 cm.

Answer ………………………………s [1]

(b) On the grid below, sketch the graph of the height of the water level (d cm) against time (t seconds) for the container.

[2]

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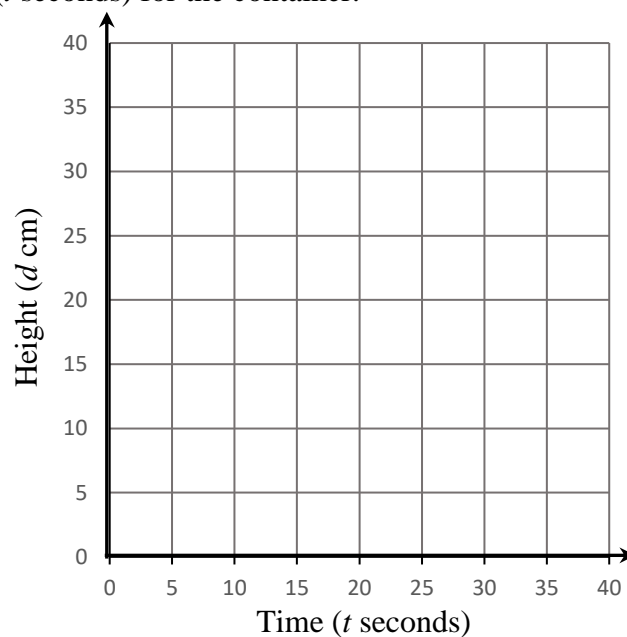
14 Tristan deposited a sum of money. At the end of 2.5 years, the amount of money has grown to $5308.23 at 2.4% per annum compound interest compounded every 3 months. Calculate the sum of money deposited by Tristan at the beginning. Give your answer correct to the nearest dollar.

Answer $……………………………. [2]

_____

15 In the diagram below, *B* is on *AE* such that *AB* = 6 cm and *BE* = 14 cm. *AD* = 15 cm and *C* is on *AD* such that *BC* = 9 cm, *DE* = 22.5 cm and $\angle ABC = \angle ADE$.
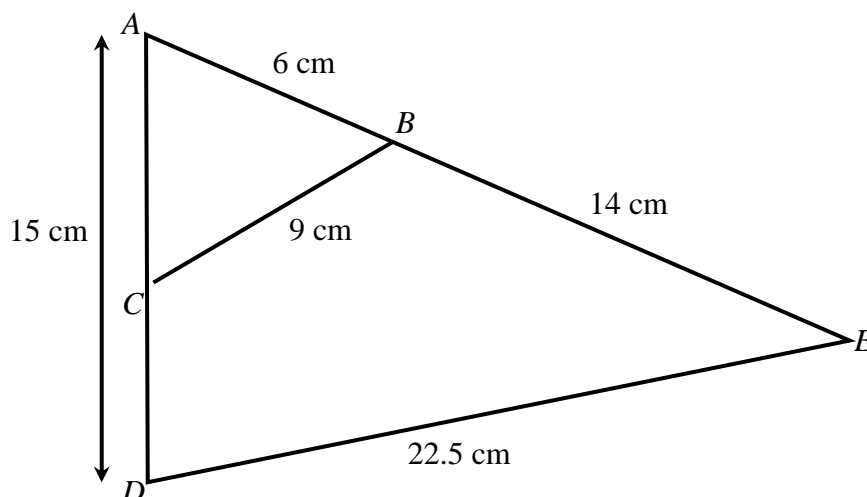
(a) Prove that $\triangle ACB$ is similar to $\triangle AED$.
Answer

[2]

15 **(b)** Show that $CD = 7$ cm.
Answer

[2]

_____

16 **(a)** Simplify $x^2 - (x + y)(x - y)$.

Answer ………………………………. [2]

(b) Hence, write down the value of $1288073407^2 - 1288073405 \times 1288073409$.

Answer ……………………………… [1]

_____

17 Given that p is inversely proportional to the square of q, calculate the percentage change in p when q is increased to 500%.

Answer ……………………………% [2]

18   Simplify $\left(\dfrac{36a^2}{b^4}\right)^{-\frac{3}{2}}$.

Answer …………………………….. [2]

_____

19   **(a)**   Usain Bolt once reached a top speed of 44.72 km/h in a race.
Write this speed in metres per second.

Answer …………………………..m/s [1]

(b)   The heights of two geometrically similar cones are in the ratio of 1 : 25.

(i)   Given that the circumference of the base of the larger cone is 300 cm,
find the circumference of the base of the smaller cone.

Answer …………………………….cm [1]

(ii)   Given that the mass of the larger cone is 7200 g, find the mass of the
smaller cone.

Answer ………………………………g [2]

20
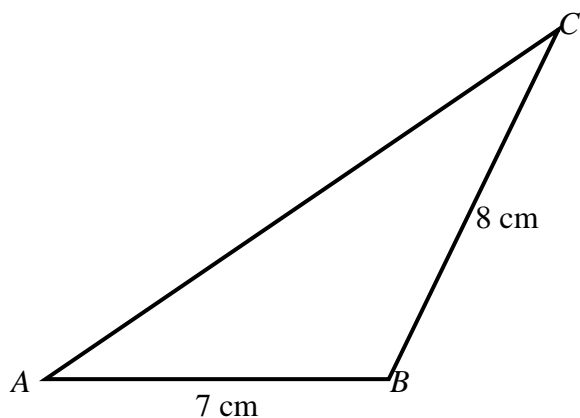
In the diagram, $AB = 7$ cm and $BC = 8$ cm. The area of triangle ABC is 24.249 cm^2.
Given that $\angle ABC$ is an obtuse angle, find the length of AC.

Answer …………………………….cm [5]

21 **(a)** The three bells in a school ring at regular intervals.
The first bell rings every 15 minutes. The second bell rings every 24 minutes.
The third bell rings every 10 minutes. The three bells ring together at 07 00.
Find the next time the three bells ring together again.

Answer ……………………………… [2]

(b) x is a number between 20 and 30.

The highest common factor of x and 540 is 12.

Find the smallest possible value of x.

Answer x =………………………… [2]

22 **(a)** Factorise completely $3a^2 + 4b - 6a - 2ab$.

Answer ………………………………… [2]

(b) Using factorisation, solve $12x^2 - 25x + 12 = 0$.

Answer x =……………… or …………… [3]

Name: _____ (   ) Class: _____

23   The following table shows the amount of flour, butter and sugar needed in making a sponge cake and a butter cake.

	Flour	**Butter**	**Sugar**
Sponge Cake	250 g	250 g	100 g
Butter Cake	400 g	200 g	90 g

(a)   The amount of ingredients used in making a sponge cake and a butter cake can be represented by the matrix

$$A = \begin{pmatrix} 250 & 250 & 100 \\ 400 & 200 & 90 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

(i)   Evaluate **AB** where

Answer **AB** =……………………………… [2]

(ii)   Explain what the elements in **AB** represent.

Answer

…………………………….……………………………….……….…

……………………….……………………………….…………………….….

……………………….……………………………….…………………….….

……………………….……………………………….…………………….….

……………………………………………………………………….…

…………………………….…………………………………..……… [1]

23 **(b)** The cost of 100 g of flour is 20 cents, 100 g of butter is 48 cents and 100 g of sugar is 15 cents.

     (i) Represent this information in a 3×1 column matrix **D**.

Answer **D** =………………………………. [1]

     (ii) The elements of the matrix **F**, where **F** = **ED**, represent the cost, in cents, of flour, butter and sugar used to make sponge cake and butter cake respectively.

Write down the matrix **E**.

Answer **E** =………………………………. [1]

___

Name: _____ (   )   Class: _____

24   **(a)**   Express $9 - 7x + x^2$ in the form $p + (x+q)^2$.

Answer ……………………………. [2]

(b)   **(i)**   Sketch the graph of $y = 9 - 7x + x^2$ on the axes below.
Indicate clearly the coordinates of the points where the graph crosses the
y-axis and the turning point on the curve.
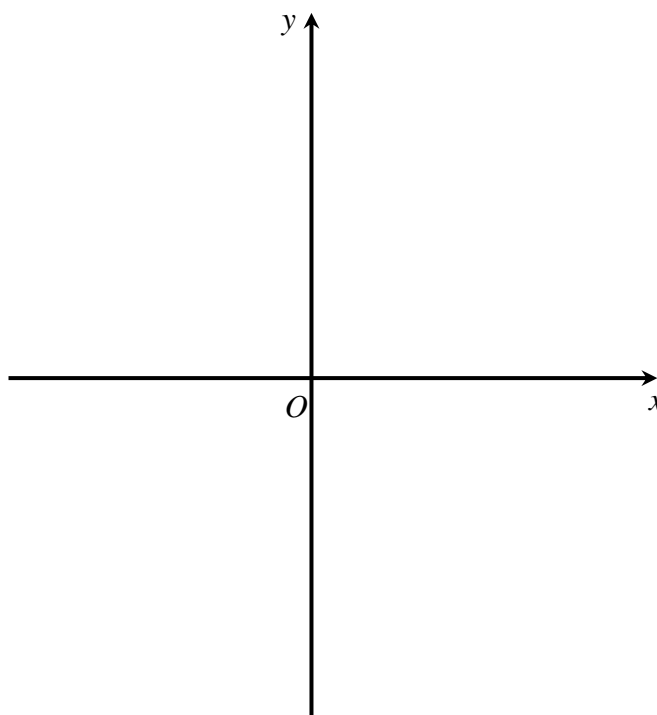
[2]

(ii)   Write the equation of the line of symmetry of $y = 9 - 7x + x^2$.

Answer ………………………………. [1]

25   The diagram shows the speed – time graph of a moving object.
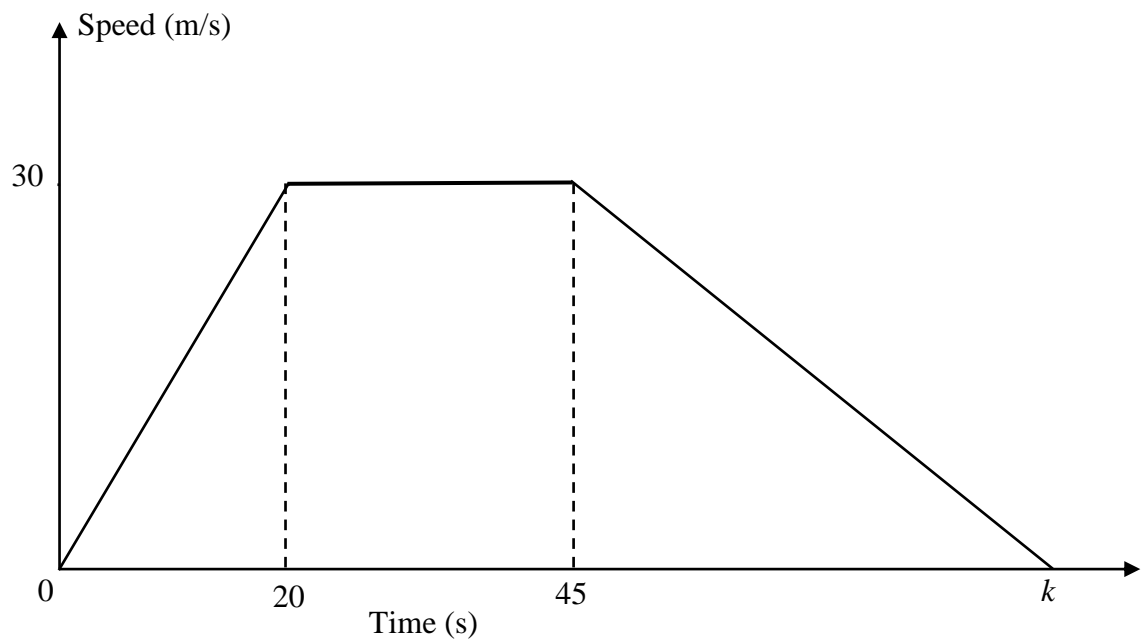
(a)   Calculate the acceleration of the object at time $t = 22$ s.

Answer ………………………… m/s^2 [1]

(b)   Find the speed when $t = 4$ s.

Answer …………………………… m/s [2]

(c)   Find the value of k if the total distance travelled is 1.35 km.

Answer $k =$ …………………………… [2]

25   **(d)**   On the axes in the answer space below, sketch the distance – time graph for the first 45 seconds and indicate clearly, on the vertical axis, the distance travelled at $t = 20$ s and t = 45 s.

Distance (m)

0   20   45

Time (s)
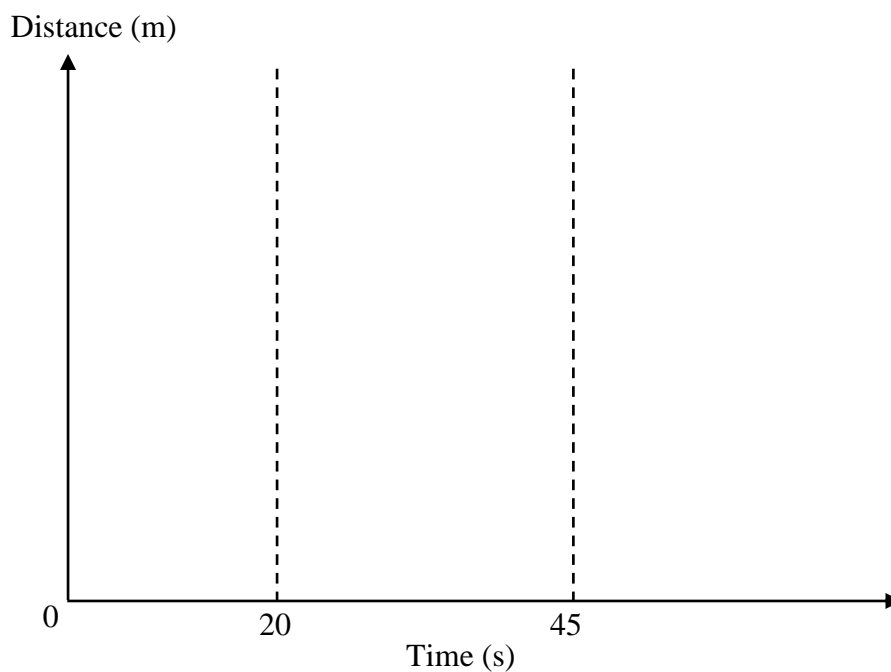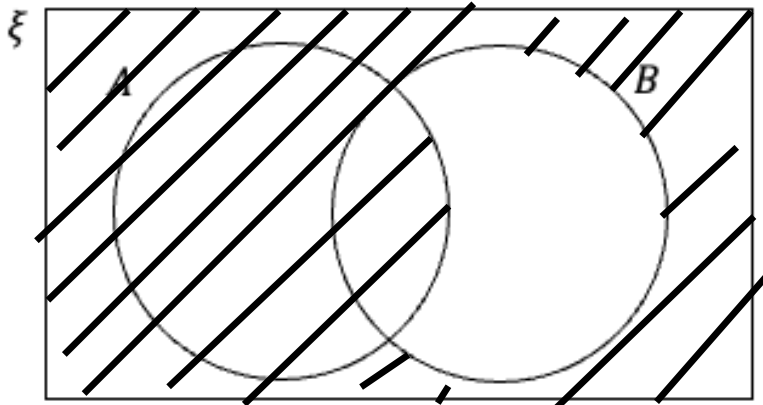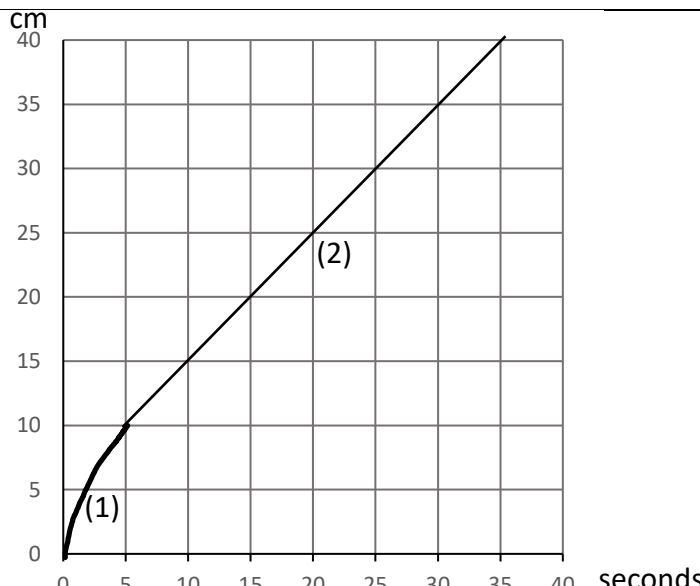
[2]

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End of paper

Math scheme to 2022 Sec 4/5 Express Prelim P1		
1	$\$2912.50 - \$2500 = \$412.50$ $412.50 = \dfrac{2500 \times 3 \times T}{100}$ $T = \dfrac{412.50 \times 100}{2500 \times 3}$ $T = 5.5 years$	M1 – show $\dfrac{412.50 \times 100}{2500 \times 3}$ A1
2	$Using\ Pythgoras\ Theorem,$ $PQ = \sqrt{19.5^2 + 12.2^2}$ $PQ = \sqrt{529.09}$ $PQ = 23.00195644$ $PQ = 23.0\ cm(3s.f.)$	M1 – show either $\sqrt{19.5^2 + 12.2^2}$ or $\sqrt{529.09}$ A1
3	$96 \times 0.5h = 48km$ $120 - 48 = 72km$ $av\ speed\ = \dfrac{72km}{1.25h}$ $av\ speed\ = 57.6km/h$	M1 – show 72km or 1.25h A1
4a	$\dfrac{3}{7} = \dfrac{12}{28}$ $total\ number\ of\ balls\ =\ 28$ $28 - 12 - 6 = 10\ white\ balls$	B1
4b	$let\ the\ number\ of\ yellow\ balls\ added\ be\ x.$ $\dfrac{6+x}{28+x} = \dfrac{7}{18}$ $18(6+x) = 7(28+x)$ $108 + 18x = 196 + 7x$ $18x - 7x = 196 - 108$ $11x = 88$ $x = 8$	B1

5	$\dfrac{5x}{3} - \dfrac{3(x+4)}{2}$ $= \dfrac{2 \times 5x}{6} - \dfrac{3 \times 3(x+4)}{6}$ $= \dfrac{10x}{6} - \dfrac{(9x+36)}{6}$ $= \dfrac{10x - 9x - 36}{6}$ $= \dfrac{x-36}{6}$	M1 – form common denominator A1
6a	$x + x - 1 + x - 2 + x - 3 + x - 4 = 19 \times 5$ $5x - 10 = 95$ $5x = 105$ $x = \dfrac{105}{5}$ $x = 21$	B1
6b	p, p^3, p^5 are negative integers. p^2, p^4 are positive integers. The ascending order is p^5, p^3, p, p^2, p^4. p is the median, instead of p^3.	} } either statement B1 } }B1
7	Volume of a cuboid $= 3 \times 2 \times 2$ $= 12 cm^3$ Total Volume of cube $= 12 \times 6174$ $= 74088 cm^3$ Length of the cube $= \sqrt[3]{74088}$ $= 42 cm$	M1 – find total volume of cube A1
8	$510 \times \dfrac{100 - 13.2}{100}$ $= \$442.68$ OR $510 \times \dfrac{13.2}{100}$ $= 67.32$ $510 - 67.32 = \$442.68$	M1 – show 86.8% of $510 A1 M1 – show 13.2% of $510 A1

9a	$\dfrac{6}{5} \times \dfrac{2}{15x} = 3$ $\dfrac{12}{75x} = 3$ $12 = 225x$ $x = \dfrac{4}{75}$ or $0.0533\ (3s.f)$	B1
9b	$3p - 4(6p - 8q)$ $= 3p - 24p + 32q$ $= -21p + 32q$	M1 – expansion of -4(6p-8q) A1
10	<table><thead><tr><th>Favourite drink</th><th>Sector angle</th></tr></thead><tbody><tr><td>Fruit juice</td><td>108°</td></tr><tr><td>Soft drink</td><td>162°</td></tr><tr><td>Coffee</td><td>90°</td></tr></tbody></table>	B1 each (deduct 1 mark overall for no unit) Max: B3
11	*interior angle of hexagon* $= \dfrac{(6-2)180}{6}$ $= 120°$ *exterior angle of n − sided polygon* $= \dfrac{120°}{5} = 24°$ *number of sides* $= \dfrac{360}{24}$ $n = 15$ *or* *interior angle of n − sided polygon* $= 180 - 24 = 156°$ $(n-2)180 = 156n$ $180n - 350 = 156n$ $24n = 360$ $n = \dfrac{360}{24}$ $n = 15$	M1 – finding interior angle M1 - $\dfrac{360}{24}$ A1

12a		B1
12bi	$n(P \cap R) = 1$	B1
12bii	$P \cap Q = \emptyset$ or $\{\}$	B1
13a	$\dfrac{1}{7} \times 35 = 5sec$	B1

13b		B1 – (1) t = 0 to t = 5: 0 – 10 cm B1 – (2) for last 30 s: 10 – 40 cm 1) Award 0 m for (1) if Q13a is wrong 2) Award 1 m for (2) if (2) start at 10 cm and end at 40 cm at 35 s

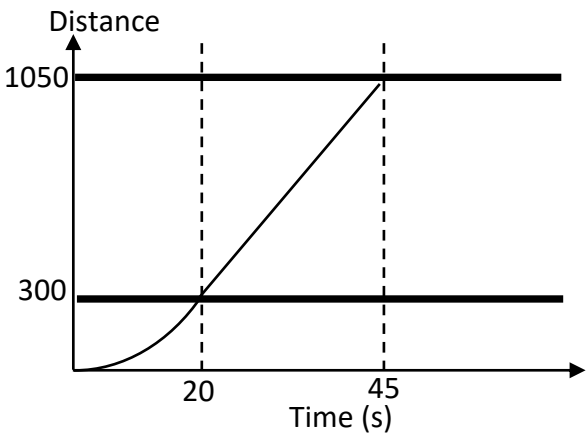| 14	$$5308.23 = P\left(1 + \dfrac{\dfrac{2.4}{4}}{100}\right)^{10}$$ $$P = 5308.23 \div \left(1 + \dfrac{\dfrac{2.4}{4}}{100}\right)^{10}$$ $$P = 4999.999$$ $$P = 5000(nearest\ dollar)$$	M1 – r = 0.6 & n = 10 A1

15a	$\angle ABC = \angle ADE \ (given)$ $\dfrac{AB}{AD} = \dfrac{6}{15} = \dfrac{2}{5}$ $\dfrac{BC}{DE} = \dfrac{9}{22.5} = \dfrac{2}{5}$ *Since two pairs of corresponding sides are in the same ratio* *and one pair of corresponding included angles is the same,* $\triangle ACB$ and $\triangle AED$ are similar. OR	M1 – must show the same ratio and common **included** angle A1 – statement must be stated
	$\angle ABC = \angle ADE \ (given)$ $\angle BAC = \angle DAE \ (common \ angle)$ *Since two pairs of corresponding angles are the same,* $\triangle ACB$ and $\triangle AED$ are similar.	M1 – must show the two **pairs of** angles A1 – statement must be stated
15b	$\dfrac{AC}{AE} = \dfrac{AB}{AD}$ $\dfrac{AC}{20} = \dfrac{6}{15}$ $AC = \dfrac{6}{15} \times 20$ $AC = 8cm$ $CD = 15 - 8$ $CD = 7cm$	 M1 AG1
16a	$x^2 - (x+y)(x-y)$ $= x^2 - \left(x^2 - y^2\right)$ $= y^2$	M1- show the expansion A1
16b	$1288073407^2 - 1288073405 \times 1288073409$ $= 1288073407^2 - \left(1288073407 - 2\right) \times \left(1288073407 + 2\right)$ $= 4$	 B1

17	$p = \dfrac{k}{q^2}$	
	$k = pq^2$	
	$new\ p\ =\ \dfrac{pq^2}{(5q)^2}$	M1 – either show $k = pq^2$ or $25q^2$
	$new\ p\ =\ \dfrac{pq^2}{25q^2}$	
	$new\ p\ =\ \dfrac{1}{25}p$	
	$\dfrac{1}{25}p - p = -\dfrac{24}{25}p$	
	$\%change\ =\ -\ \dfrac{24}{25} \times 100$	
	$=-96\%$	A1
18	$\left(\dfrac{36a^2}{b^4}\right)^{-\frac{3}{2}}$	
	$=\left(\dfrac{b^4}{36a^2}\right)^{\frac{3}{2}}$	M1 – apply fractional index, either square root or cube all terms
	$=\left(\sqrt{\dfrac{b^4}{36a^2}}\right)^3$	
	$=\left(\dfrac{b^2}{6a}\right)^3$	
	$=\dfrac{b^6}{216a^3}$	A1
19a	$\dfrac{44.72km}{1h} = \dfrac{44.72 \times 1000}{1 \times 60 \times 60}$	
	$=\dfrac{559}{45}$	
	$=12\dfrac{19}{45}\ or\ 12.4(3sf)$	B1
19bi	$\dfrac{300}{25} = 12cm$	B1

19bii	$\dfrac{mass\ of\ smaller\ cone}{mass\ of\ larger\ cone} = \left(\dfrac{1}{25}\right)^3$ $\dfrac{mass\ of\ smaller\ cone}{7200} = \dfrac{1}{15625}$ $mass\ of\ smaller\ cone = \dfrac{1}{15625} \times 7200$ $= 0.4608g\ or\ 0.461g\,(3sf)$	M1 - $\left(\dfrac{1}{25}\right)^3$ A1
20	$24.249 = \dfrac{1}{2} \times 7 \times 8 \times \sin\angle ABC$ $\angle ABC = \sin^{-1}\left(\dfrac{24.249}{\dfrac{1}{2} \times 7 \times 8}\right)$ $\angle ABC = 60.001°$ $obtuse\ \angle ABC = 180 - 60.001 = 119.999°$ $Appling\ cosine\ rule,$ $AC = \sqrt{7^2 + 8^2 - 2(7 \times 8 \times \cos 119.999)}$ $AC = \sqrt{168.998}$ $AC = 12.999$ $AC = 13.0\,(3sf)cm$	M1 – applying Area of Triangle, show sin-1 M1 - Find obtuse M2 – apply cosine rule A1
21a	$LCM = 2^3 \times 3 \times 5$ $= 120\ minutes$ $= 2\ hours$ $Time = 09\,00\ or\ 9am$	M1 A1
21b	$HCF = 12 = 2^2 \times 3$ $smallest\ possible\ value\ of\ x = 2 \times 2^2 \times 3 = 24$	M1 A1
22a	$3a^2 + 4b - 6a - 2ab$ $= 3a^2 - 2ab - 6a + 4b$ $= a(3a - 2b) - 2(3a - 2b)$ $= (a - 2)(3a - 2b)$	 M1 A1

22b	$12x^2 - 25x + 12 = 0$	M1
	$(4x - 3)(3x - 4) = 0$	
	$4x - 3 = 0 \text{ or } 3x - 4 = 0$	
	$4x = 3 \text{ or } 3x = 4$	
	$x = \dfrac{3}{4} \text{ or } x = \dfrac{4}{3}$	A2
23ai	$AB = \begin{pmatrix} 250 & 250 & 100 \\ 400 & 200 & 90 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$	
	$AB = \begin{pmatrix} 600 \\ 690 \end{pmatrix}$	B1 – 600 B1 – 690
23aii	*The elements in AB represent the total amount of flour,* *butter and sugar used in making* *a sponge cake and a butter cake respectively.*	B1
23bi	$D = \begin{pmatrix} 20 \\ 48 \\ 15 \end{pmatrix}$	B1
23bii	$E = \begin{pmatrix} 2.5 & 2.5 & 1 \\ 4 & 2 & 0.9 \end{pmatrix}$	B1
24a	$9 - 7x + x^2 = x^2 - 7x + 9$	
	$= (x - 3.5)^2 - 3.25$	B1 - -3.5 B1 - -3.25
	$= -3.25 + (x - 3.5)^2$	

24bi		
		C1 – shape and symmetric curve correct P1 – y-intercept And minimum point
24bii	x = 3.5	B1
25a	0	B1
25b	$Gradient = \dfrac{30}{20}$ $\dfrac{speed}{4} = \dfrac{3}{2}$ $speed = \dfrac{3}{2} \times 4$ $speed = 6m/s$	M1 - $\dfrac{3}{2}$ A1

25c	$1350 = \dfrac{1}{2}(25 + k)30$	M1
	$2700 = (25 + k)30$	
	$\dfrac{2700}{30} = 25 + k$	
	$90 = 25 + k$	
	$90 - 25 = k$	
	$k = 65$	A1
	$\left(\dfrac{1}{2} \times 20 \times 70\right) + (25 \times 30) + \left(\dfrac{1}{2} \times (k - 45) \times 30\right) = 1350$	M1
	$300 + 750 + 15(k - 45) = 1350$	
	$15k = 975$	A1
	$k = 65$	
25d		L1 - Correct label of the distances C1 – correct sketching of graph -1 m if graph is not proportional