H2 Topic 16 – Electromagnetism

Magnetically levitated (maglev) trains exploit the physics of electromagnetism to (i) lift a train off the tracks to minimize friction and (ii) accelerate or perform regenerative braking of the train.

Content

- Concept of a magnetic field
- Magnetic fields due to currents
- Force on a current-carrying conductor
- Force between current-carrying conductors
- Force on a moving charge

Learning Objectives:

Candidates should be able to:

(a) show an understanding that a magnetic field is an example of a field of force produced either by current-carrying conductors or by permanent magnets

(b) sketch flux patterns due to currents in a long straight wire, a flat circular coil and a long solenoid

(c) use $B = \dfrac{\mu_0 I}{2\pi d}$, $B = \dfrac{\mu_0 N I}{2r}$ and $B = \mu_0 n I$ for flux densities of the fields due to currents in a long straight wire, a flat circular coil and a long solenoid respectively

(d) show an understanding that the magnetic field due to a solenoid may be influenced by the presence of a ferrous core

(e) show an understanding that a current-carrying conductor placed in a magnetic field might experience a force

(f) recall and solve problems using the equation $F = BIl \sin\theta$, with directions as interpreted by Fleming's left-hand rule

(g) define magnetic flux density

(h) show an understanding of how the force on a current-carrying conductor can be used to measure the flux density of a magnetic field using a current balance

(i) explain the forces between current-carrying conductors and predict the direction of the forces

(j) predict the direction of the force on a charge moving in a magnetic field

(k) recall and solve problems using $F = BQv \sin\theta$

(l) describe and analyse deflections of beams of charged particles by uniform electric and uniform magnetic fields

(m) explain how electric and magnetic fields can be used in velocity selection for charged particles.

16.0 Introduction

Earlier in gravitational fields and electric fields, we described the directions along which bodies will experience forces, and quantitatively found the field strengths. Back at the secondary level we could deduce the direction of force that will act on a current-carrying conductor subject to a magnetic field. At the A-Levels, we will go on further to quantify how strong a magnetic field is.

16.1 Magnetic Field

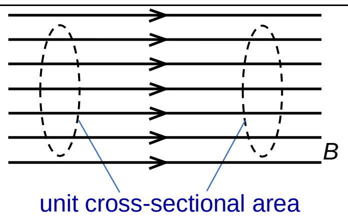
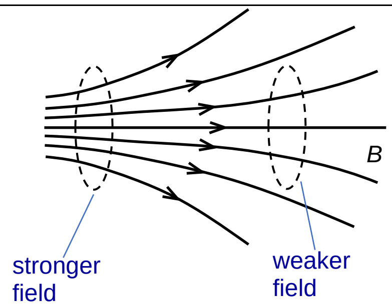
A **magnetic field** is

a region of space in which
a permanent magnet,
a current-carrying conductor or
a moving charge

may experience a force

Curiously the definition reads as "*may* experience a force"; recall that if the direction of current is parallel to the magnetic field lines, there will be no force due to the magnetic field.

Strength of a magnetic field is represented by a vector quantity called the **magnetic flux density *B***.

uniform magnetic field	non-uniform magnetic field
	
Parallel and equally-spaced lines. Number of lines passing through an area (for comparison) is the same.	

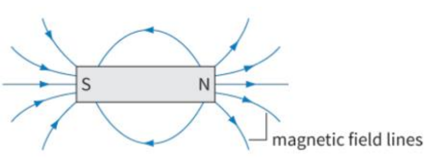
16.2 Magnetic Field Lines

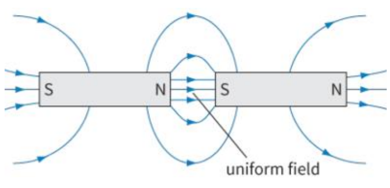
As with gravitational fields and electric fields, we represent magnetic fields using magnetic field lines.

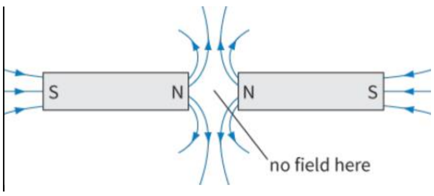
magnetic field lines:

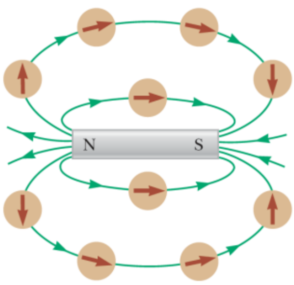
- come out of north poles and go into south poles
- (the tangent) shows the direction of force that a 'free' magnetic north pole will experience at that point
- are closer together where the field is stronger

Field lines do not intersect. When two or more fields interact, the resultant is a vector addition.



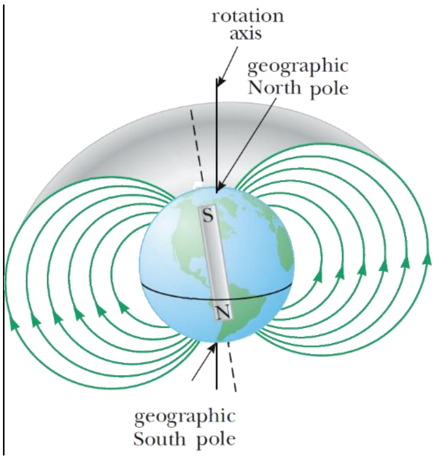




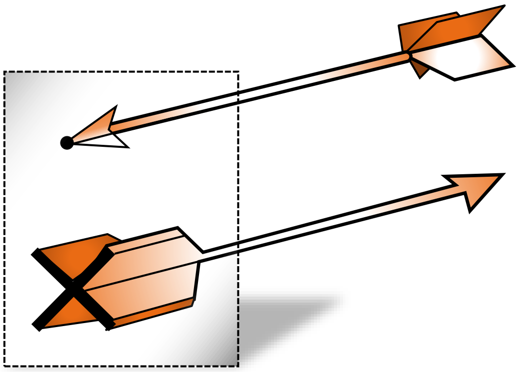
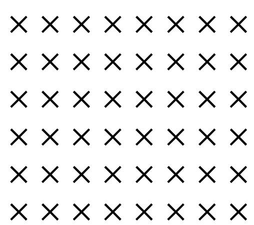
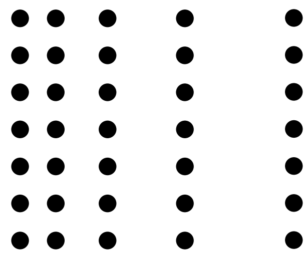


A compass needle will point in the field direction at that point

A compass needle aligns itself with the Earth's magnetic field. Therefore, the geographic North pole is a magnetic South pole; only then can the North-pointing ends of compass needles point in the "correct" direction.

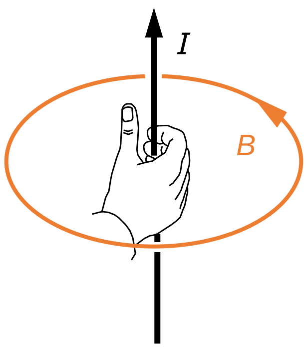
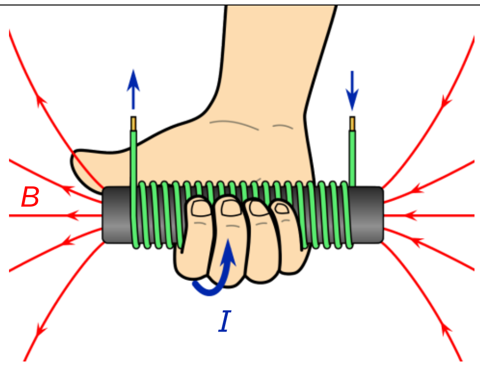


We use dots and crosses to represent magnetic field lines going into and out of the plane of paper.

	uniform magnetic field pointing into plane of paper	non-uniform magnetic field pointing out of plane of paper
 plane of paper		

16.3 Magnetic Fields due to Current

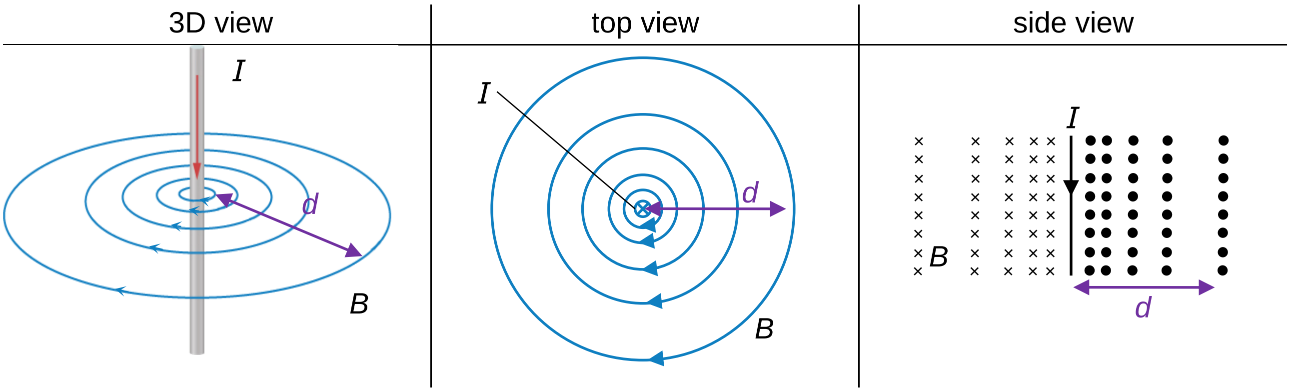
When using the right-hand grip rule, the 4 fingers can be either the flow of current or the direction of the magnetic field lines:

four fingers give direction of magnetic field lines	four fingers aligned to direction of current flow
	

There are three geometries you need to be familiar with:

1. Magnetic flux density due to current in a long straight wire
2. Magnetic flux density due to a flat circular coil
3. Magnetic flux density due to a long solenoid

16.3.1 Magnetic Flux Density due to Current in a Long Straight Wire



The magnetic field lines are concentric circles centred on the wire. The radius of each larger circle is increasingly larger because the magnetic field is weaker further away from the wire.

The magnetic flux density (*B*) due to a long straight wire at a perpendicular distance *d* away from the wire carrying current I is given in the list of data and formulae. You have to remember what the individual quantities refer to.

$$\boxed{B = \frac{\mu_0 I}{2\pi d}}$$

μ_0 : permeability of free space (H m^{-1})

I : current in the conductor (A)

d : distance from the conductor (m)

Example 1

A vertical wire X carries a current of 5.0 A downwards.

(a) Find the magnetic flux density due to the current at point P that is 10 cm east of X.

(b) The horizontal magnetic flux density due to Earth's magnetic field is 40 μT. Find the resultant magnetic flux density at P.

(c) Determine the location relative to X where the resultant magnetic flux density is zero.

(d) State if the resultant magnetic flux density 10 cm North of X is greater than at P.

Solution

(a) $$B_{\text{wire}} = \frac{\mu_0 I}{2\pi d} = \frac{(4\pi \times 10^{-7})(5.0)}{2\pi(10 \times 10^{-2})}$$

$$= 1.0 \times 10^{-5}\ \text{T (towards South)}$$

(b) $$B_{\text{net}} = B_{\text{Earth}} - B_{\text{wire}}$$

$$= 40 \times 10^{-6} - 1.0 \times 10^{-5}$$

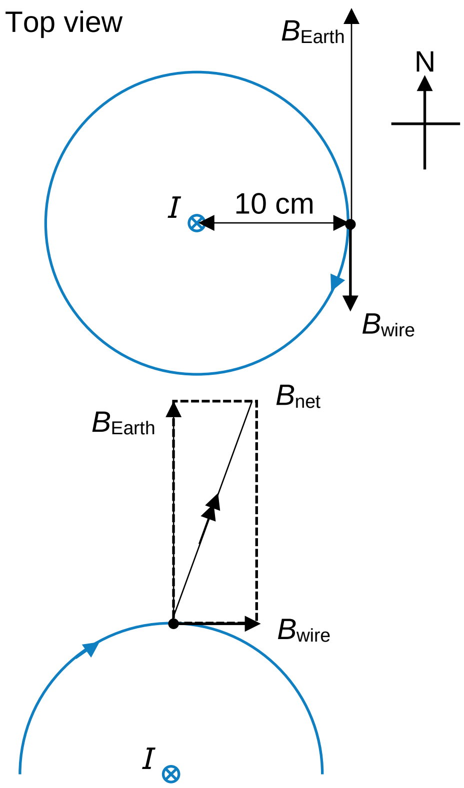
$$= 3.0 \times 10^{-5}\ \text{T (towards North)}$$

(c) $$B_{\text{net}} = B_{\text{Earth}} - B_{\text{wire}} = 0$$

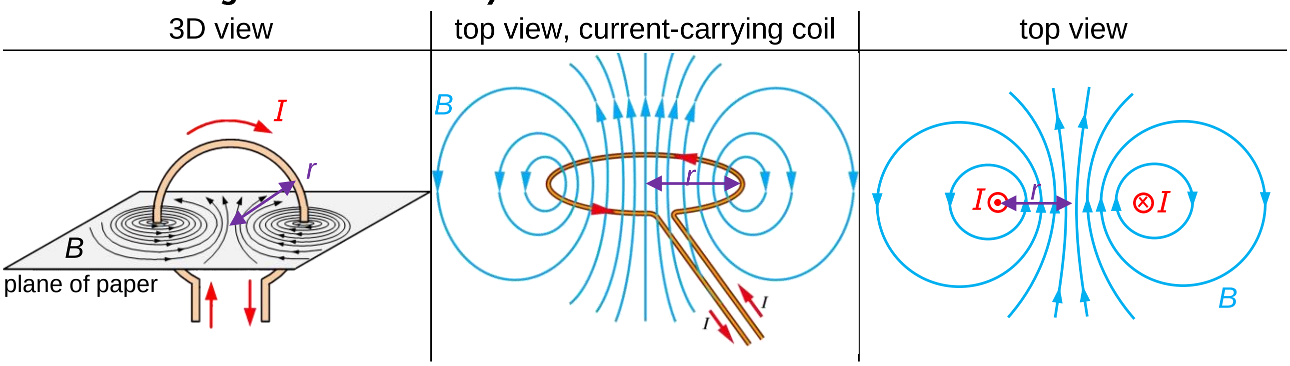
$$\frac{\mu_0 I}{2\pi d_{\text{new}}} = 40 \times 10^{-6}$$

$$d_{\text{new}} = 0.025\ \text{m} \quad \text{East of X}$$

(d) larger in magnitude.
magnetic flux density of Earth vectorially added to magnetic flux density due to current carrying wire.



16.3.2 Magnetic Flux Density due to a Flat Circular Coil



The magnetic flux density at the centre of a flat circular coil with radius r, number of turns N, carrying current I is given in the list of data and formulae. We need to know what the individual quantities refer to.

$$\boxed{B = \frac{\mu_0 N I}{2r}}$$

μ_0 : permeability of free space (H m^{-1})

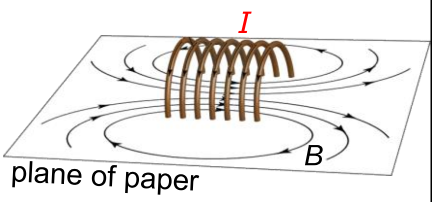
N : number of turns

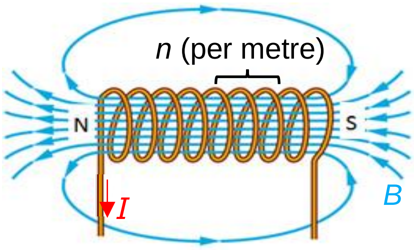
I : current in the conductor (A)

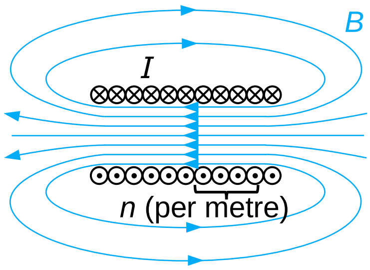
r : radius of coil (m)

16.3.3 Magnetic Flux Density due to a Long Solenoid

3D view	top view, current-carrying coil	top view







A *long* solenoid is one where its length is significantly larger than its (cylindrical) radius. The magnetic field lines are mostly parallel, straight and equally-spaced in the centre region of the long solenoid - the field is more uniform towards the centre of the coil. The field lines begin to diverge nearer the ends of the solenoid.

$$\boxed{B = \mu_0 n I}$$

The magnetic flux density along the axis of a long solenoid that has *n* number of turns of wire per unit length is given in the list of data and formulae. We need to know what the individual quantities refer to.

μ_0 : permeability of free space (H m^{-1})
n : number of turns per unit length
I : current in the conductor (A)

Note that μ_0 is the permeability of free space, a constant that describes how magnetic fields behave in free space (or vacuum), similar to how permittivity of free space ε_0 describes the behaviour of electric fields. The equation is **not** valid if a ferrous (e.g. soft iron) core is inside the solenoid.

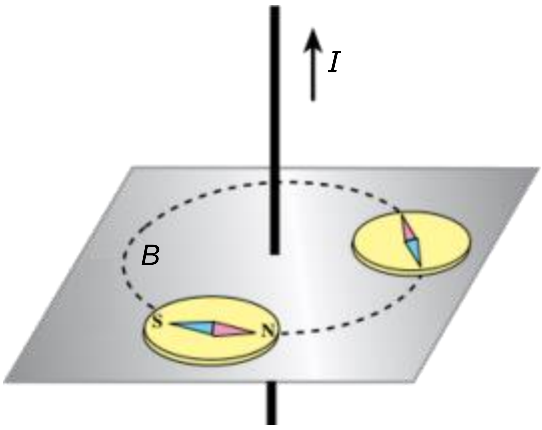
Question: Explain the effects of inserting an iron core into a current carrying solenoid.

A soft iron core is one that is easily magnetized and de-magnetised.
When a soft iron core is inserted in a solenoid,

the core is magnetized
in the same direction as the magnetic flux density set up by the solenoid,
it adds to the magnetic flux density and strengthens the net magnetic field

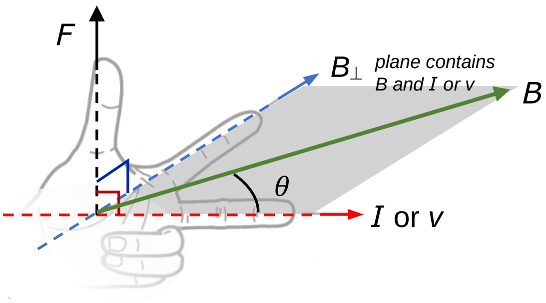
16.4 Magnetic Force

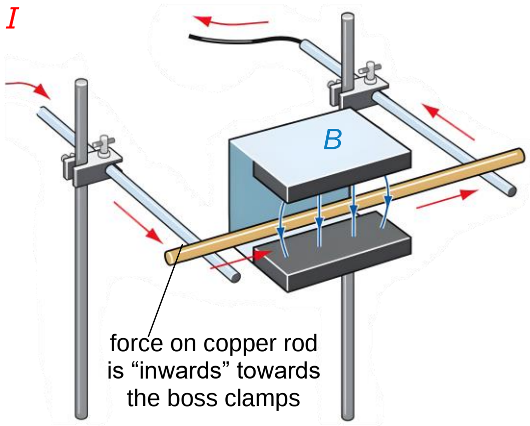
A current-carrying wire causes the magnetised needles of nearby compasses to deflect until the needles are aligned to the magnetic flux generated. The ends of the magnetised needles each experience a magnetic force during the realignment. By Newton's Third Law, the needles must exert a force equal in magnitude, opposite in direction and of the same type, back on the current-carrying wire during realignment.



Each of the needles are permanent magnets that set up their own individual magnetic fields; which can cause forces to be exerted (back) on the current-carrying wire. A current-carrying conductor in a magnetic field _may_ experience a force.

Fleming's left-hand rule predicts the direction of magnetic force on a current-carrying conductor placed in a magnetic field. Note that the magnetic force *F* is always perpendicular to **both** magnetic flux density and direction of current flow. Use the Fleming's left-hand rule to deduce the magnetic force acting on the rod in the figure below.





The magnetic force acting on a straight current carrying conductor in a magnetic field is given by:

$$\boxed{F = BIL \sin\theta}$$

From definition of current:

$$IL = \left(\frac{Q}{t}\right)L = Q\left(\frac{L}{t}\right) = Qv$$

the magnetic force acting on a moving charged particle in a magnetic field is given by:

$$\boxed{F = BQv \sin\theta}$$

F: magnetic force, in newtons (N)
B: magnetic flux density, in tesla (T)
I: current flowing in conductor, in ampere (A)
L: length of current-carrying conductor in magnetic field, in metres (m)
Q: the charge on a moving charged particle, in Coulombs (C)
v: speed of the moving charged particle (m s^{-1})

Note: It is the component of the magnetic flux density normal to the current flow that contributes to the magnetic force, $B_\perp = (B\sin\theta)$.

Magnetic flux density *B* is

the force acting per unit current
per unit length on a wire
carrying a current that is normal to the magnetic field.

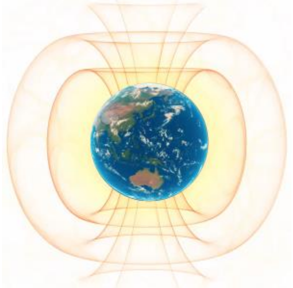
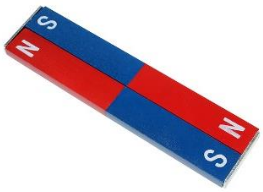
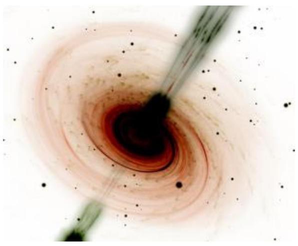
$$B = \frac{F}{IL\left(\sin 90°\right)}$$

Magnetic force is maximum when I and B are normal to each other $\left(\sin 90°\right)$ and is zero when the conductor is parallel to the field $\left(\sin 0° \text{ or } \sin 180°\right)$

This is the mechanism for defining magnetic flux density *B*.

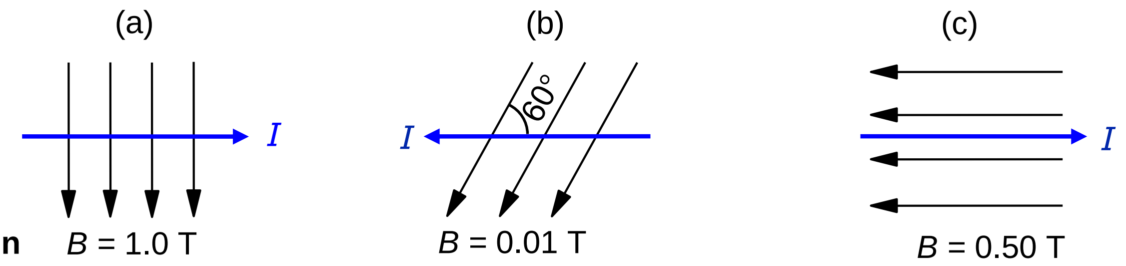
The SI unit of *B* is the tesla (T). *One tesla* is the uniform magnetic flux density which, acting normally to a long straight wire carrying a current of 1 ampere, causes a force per unit length of 1 N m^{-1} to act on the conductor. Here are some approximate magnetic flux densities:

$$1\text{ T} = \frac{1\text{ N}}{(1\text{ A})(1\text{ m})(\sin 90°)}$$

Earth's surface	small bar magnet	large electromagnet	neutron star surface
			
~ 10^{-5} T	~ 10^{-2} T	~ 1.5 T	~ 10^{8} T

Example 2

Each diagram below shows a magnetic field that lines in the plane of paper and a wire of length 0.75 m carrying a current I of 10 A. Find the force for each of the cases.



Solution

(a)

$F = BIL\sin\theta$

$= (1.0)(10)(0.75)(\sin 90°)$

$= 7.5$ N (into page)

(b)

$F = BIL\sin\theta$

$= (0.01)(10)(0.75)(\sin 60°)$

$= 0.065$ N (out of page)

(c)

$F = BIL(\sin 180°)$

$= 0$ N

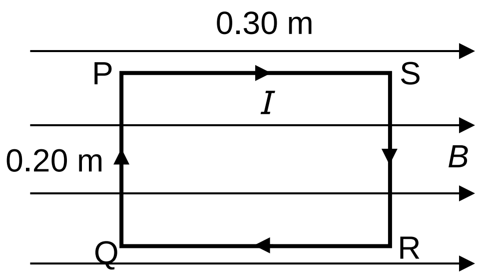
Note: Be careful to select the correct angle for the sin θ, especially in situations like (b). Those familiar with vector cross product may wish to know that $F = \vec{I}L \times \vec{B}$

Example 3

A rectangular coil PRQS of length 0.30 m and breadth 0.20 m carries a current of 0.50 A. It is placed in a magnetic field of flux density 50 mT acting to the right. Find the torque.

Solution

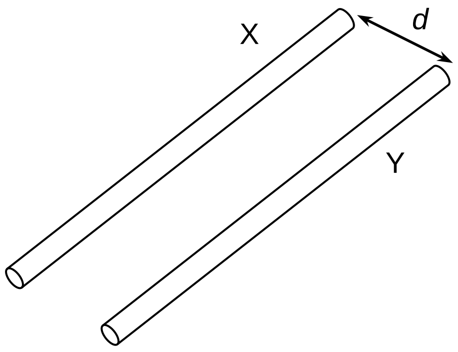
No force along PS and QR because I//B.
Force on PQ is equal in magnitude and opposite in direction as to force on SR.



torque $\tau = FL_{PS} = \left[BIL_{PQ}(\sin 90°)\right]L_{PS} = \left[(50\times10^{-3})(0.50)(0.20)\right](0.30) = 1.5\times10^{-3}$ N

16.4.1 Force between two parallel current-carrying conductors

Two long wires X and Y run parallel to, and near each other:

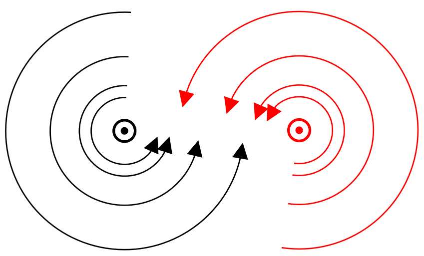
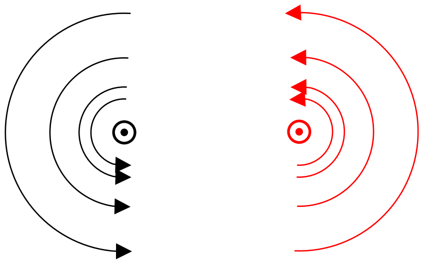
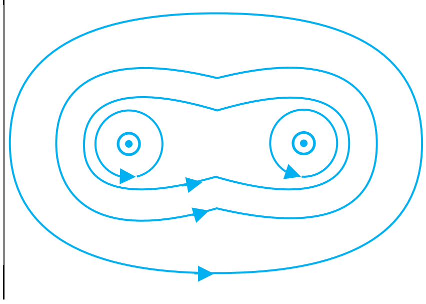


Use the right-hand grip rule to help visualise the fields due to the individual currents.

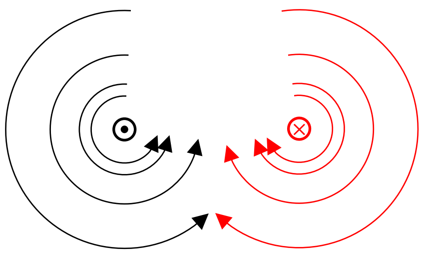
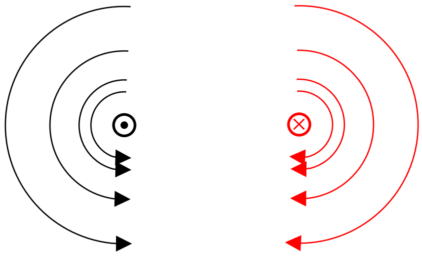
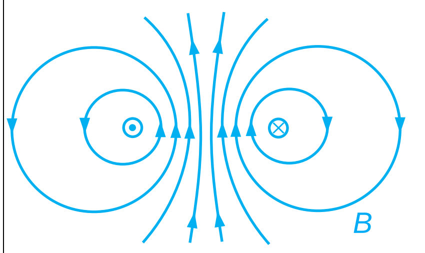
current in same direction, wires attract

current in opposite direction, wires repel

For wires carrying current in the same direction:

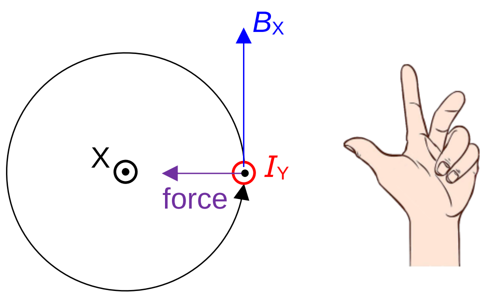
individual *B*-fields cancel out in region between wires	individual *B*-fields reinforce in region outside wire pair	greater density of flux lines outside of wire pair, resembles single wire when very far away
		

For wires carrying current in the opposite direction:

individual *B*-fields reinforce in region between wires	individual *B*-fields cancel out in region outside wire pair	greater density of flux lines in between wire pair
		

The above visualization suggests that the regions of greater magnetic flux density "push" the wires towards region of lower magnetic flux density. However, it is not possible to quantify the magnitude of force using the method.

Another way of accounting for the force is to treat one wire as purely carrying current and therefore experiencing a magnetic force; in the magnetic field that is generated by the other wire.

<table>
<tr><th>current along same direction</th><th>current in opposite direction</th></tr>
<tr><td>


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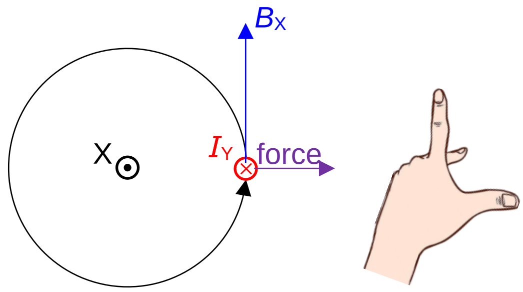

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<tr><td>Current in X sets up anti-clockwise magnetic field lines around wire X.
Direction of magnetic flux lines (due to wire X) at wire Y is vertically upwards.
By Fleming's Left Hand Rule, wire Y experiences a force to the left.
By Newton's 3rd Law, wire X experiences a force of the same type, equal in magnitude and opposite in direction therefore to the right.</td><td>Current in X sets up anti-clockwise magnetic field lines around wire X.
Direction of magnetic flux lines (due to wire X) at wire Y is vertically upwards.
By Fleming's Left Hand Rule, wire Y experiences a force to the right.
By Newton's 3rd Law, wire X experiences a force of the same type, equal in magnitude and opposite in direction therefore to the left.</td></tr>
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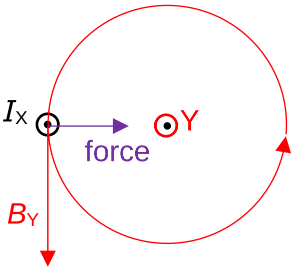

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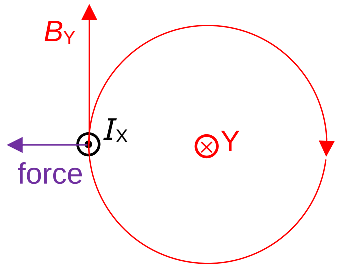

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Example 4

2 long parallel wires X and Y separated by a distance d carry currents I_X and I_Y respectively. Show that the magnetic force per unit length of wire is directly proportional to the product of the two current and inversely proportional to the separation distance between the wires.

Solution

magnetic flux density due to a long straight wire X:

$$B_X = \frac{\mu_0 I_X}{2\pi d}$$

magnetic force:

$$F = BIL\sin\theta$$

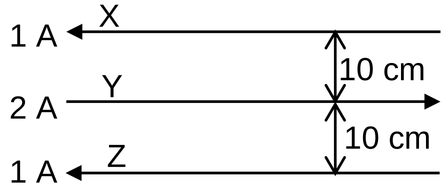
$$F_{\text{on Y}} = B_X I_Y L_Y (\sin 90°)$$

$$\frac{F_{\text{on Y}}}{L_Y} = B_X I_Y$$

$$= \left(\frac{\mu_0}{2\pi}\right)\frac{I_X I_Y}{d}$$

Example 5

3 long parallel wires X, Y and Z are as shown.
When the current in Z is switched off, the force per unit length acting on Y is 4.0×10^{-6} N m^{-1}.
Current in Z is then restored.
Find the force per unit length on Z.



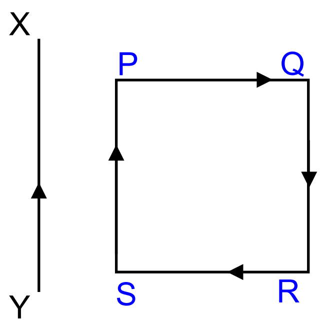
Solution

force per unit length is directly proportional to product of currents and inversely proportional to distance of separation:

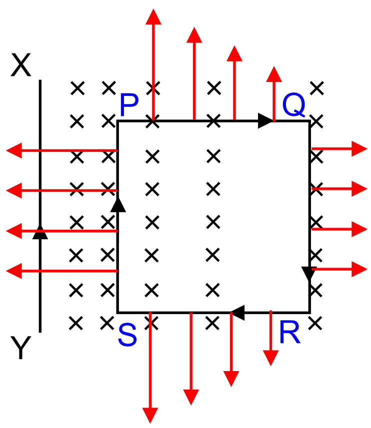
Example 6

A long straight wire XY lies in the same plane as a square loop of wire PQRS which is free to move. The sides PS and QR are initially parallel to XY. The wire and loop carry steady currents as shown in the diagram. Which of the following best describes the subsequent motion of PQRS?

A) It will move away from the long wire.
B) It will move towards the long wire.
C) It will rotate about the axis parallel to XY.
D) It will be unaffected.
E) It will contract.



Solution



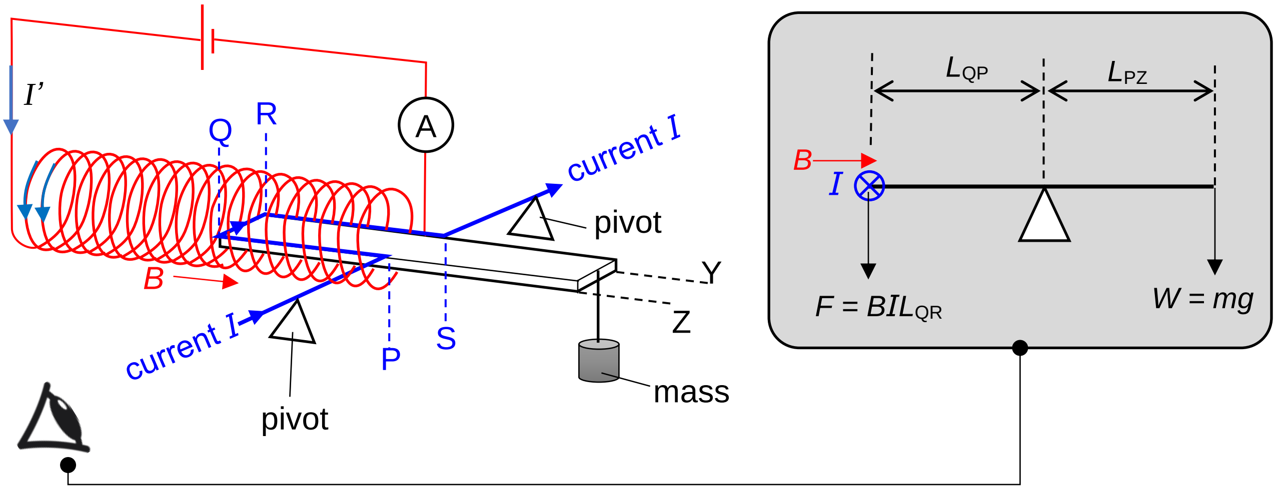
PQ experiences a force upwards
SR experiences a force downwards
The magnitude of the forces are equal,
So no net force vertically.

PS is attracted to XY with greater force than QR is repelled from XY.
Net horizontal force on PRQS towards XY.

Note: magnitude of force on PQ or SR varies with distance from XY and thus is not uniform

16.4.2 Force on a current-carrying conductor to find *B*

A *current balance* usually has a conducting wire frame. One end of the frame is subject to a magnetic flux density *B* and the other end balanced by a mass which provides weight. The frame is pivoted and maintained horizontally. Through principle of moments we can determine the magnitude of *B*.



Example 7

A solenoid coil 15 cm long with 100 turns rests horizontally as shown above. The ammeter reads 20 A when the light rigid frame ZQRY is horizontally pivoted on its mid-point axis PS. Sides QR and YZ are of length 1.5 cm. The counter mass is of mass 1.8×10^{-2} g. Find I.

Solution

$$B_{\text{solenoid}} = \mu_0 nI = \mu_0 \left(\frac{N}{l}\right) I$$

$$= \mu_0 \left(\frac{100}{15 \times 10^{-2}}\right) 20 = \frac{40000\mu_0}{3}$$

By principle of moments, sum of anticlockwise moments equals sum of clockwise moments about pivot,

$$F\left(\cancel{L_{QP}}\right) = W\left(\cancel{L_{PZ}}\right)$$

$$BI_{QR}L_{QR} = mg$$

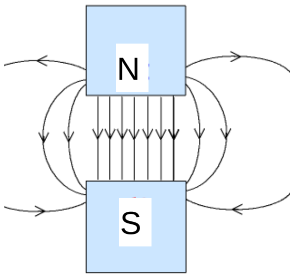
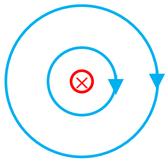
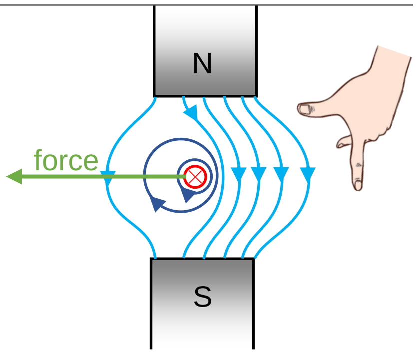
$$I_{QR} = \frac{mg}{BL_{QR}}$$

$$= \frac{\left(1.8 \times 10^{-2} \times 10^{-3}\right)(9.81)}{\left(\frac{40000\mu_0}{3}\right)\left(1.5 \times 10^{-2}\right)} = 0.703 \text{ A}$$

Note: Be sure to distinguish between the currents flowing through the solenoid and the current-carrying conductor. Some questions will show a series connection therefore the current value through these two items will be the same but this is not always the case.

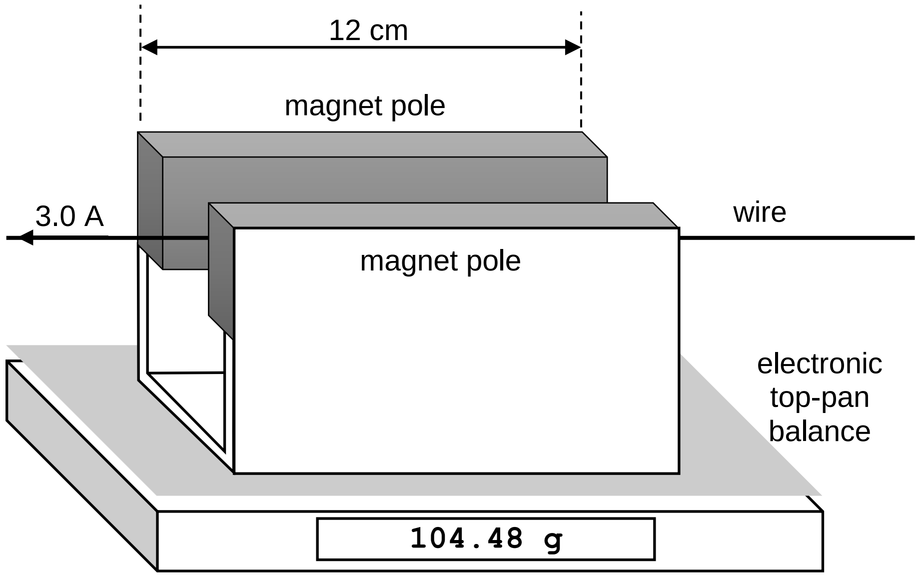
16.4.3 Force on a current-carrying conductor near a permanent magnet

Similar to how we visualise the resultant magnetic field between 2 parallel wires, we can add the individual fields of a current-carrying wire and that between the poles of a permanent magnet:

B-field between poles of magnet(s)	*B*-field around wire carrying current into plane of page	greater density of flux lines in between wire pair
		

Example 8

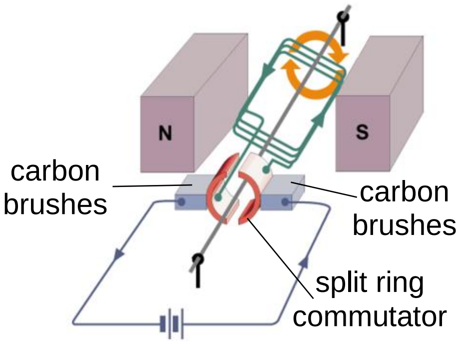
The reading on the top-pan balance changes from 102.48 g to 104.48 g when a current of 3.0 A is switched in the wire. Assuming the magnetic field in-between the magnetic poles is approximately uniform, find the magnetic flux density in between the poles.

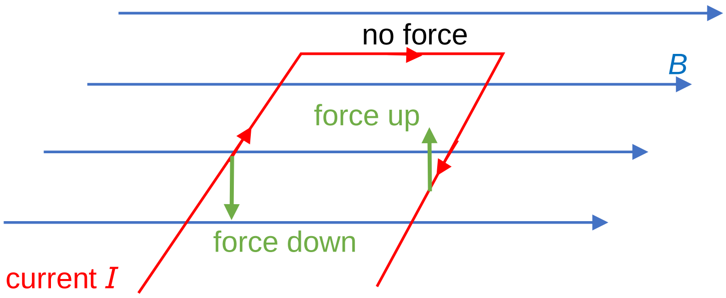


Solution

force on magnet is downwards so by Newton's 3rd Law, force on wire is upwards. by Fleming's left-hand rule, magnetic field lines is directed out of plane of paper.

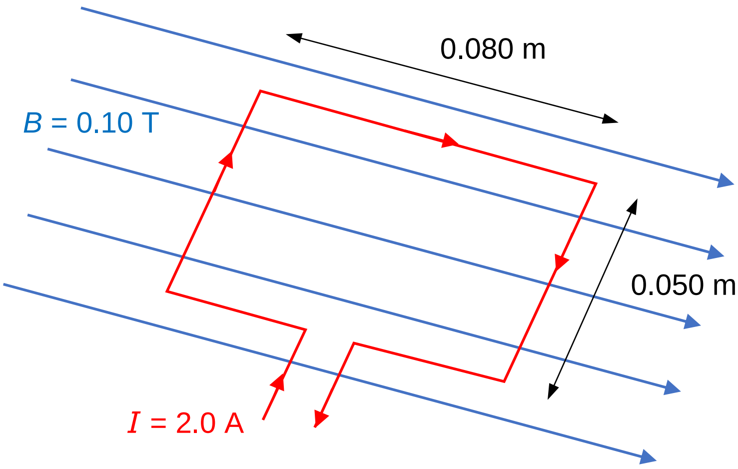
This effect produces torque and is the basic working principle of an electric motor.





Example 9

An electric motor has 100 windings of wire around a rectangular loop as shown. The magnetic flux density is a uniform 0.10 T. The wire carries a current of 2.0 A. Find the torque.



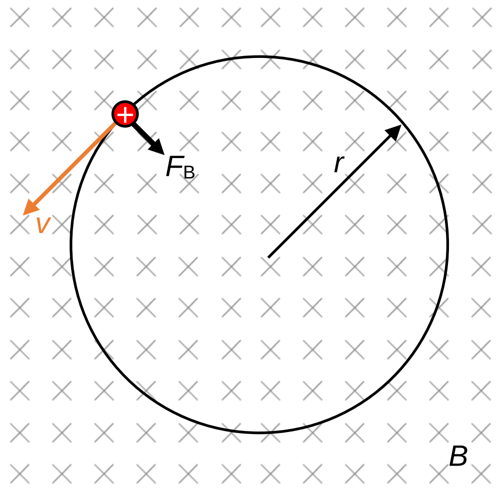
Solution

$$F_{\text{1 wire}} = BIL(\sin\theta) = BIL(\sin 90°) = BIL$$
$$F_{\text{on loop}} = NBIL$$
$$\tau = Fd_{\perp} = (NBIL)d_{\perp}$$
$$= (100)(0.10)(2.0)(0.050)(0.080)$$
$$= 0.080 \text{ N m} \quad (\text{anti-clockwise viewed from commutator})$$

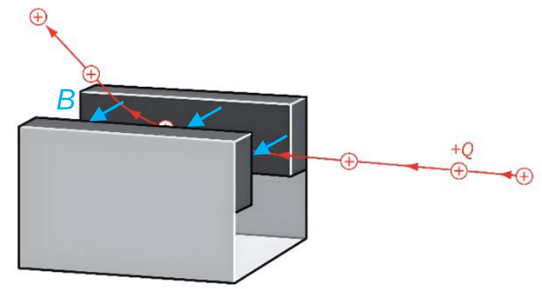
16.5 Force on a moving charge in a magnetic field: circular motion

A current-carrying conductor experiences a force in a magnetic field because “microscopically” moving charges experience magnetic forces. Since current is the rate of flow of charges, the effect manifests “macroscopically” as force on a wire. The magnetic force is always perpendicular to the velocity (as given by Fleming’s left-hand rule), so the motion tends to be circular or part of an arc:

magnetic field wider than circular path



path of charged particle along circular arc



Find expression for radius r:

magnetic force provides centripetal force

$$F_B = F_c$$
$$Bqv(\sin 90°) = \frac{mv^2}{r}$$
$$r = \frac{mv}{Bq}$$

Find expression for period T:

magnetic force provides centripetal force

$$F_B = F_c$$
$$Bq(v)(\sin 90°) = mr\omega^2$$
$$Bq(\cancel{r}\cancel{\omega}) = m\cancel{r}\omega^{\cancel{2}}$$
$$Bq = m\left(\frac{2\pi}{T}\right)$$
$$T = \frac{2\pi m}{Bq}$$

If particle has *additional* component of velocity parrallel to B-field: helical motion

Describing motion

charged particle has velocity normal to magnetic field

experiences a magnetic force perpendicular to both velocity and magnetic field as given by Fleming’s left-hand rule

magnetic force provides centripetal force inside magnetic field

no resultant force acts on particle outside field

by Newton’s 1st Law, particel continues in its state of constant velocity in a straight line

Why magnetic force does no work

magnetic force always directed perpendicular to velocity of charged particle

no displacement in the direction of magnetic force so no work done

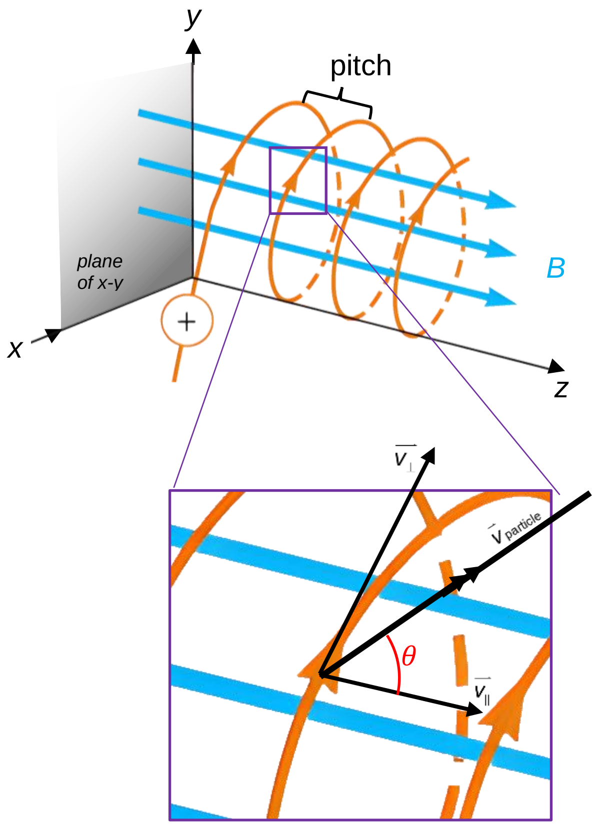
A helical path looks like screw threads.

The circular radius involves the component of velocity parallel to the *x*-*y* plane. We typically denote it $\vec{v}_{\perp}$ as it is perpendicular to *B*.

$v_{\parallel}$ is then the component of velocity that is parallel to *B*. The charged particle's velocity is the vector addition of both components

$$\vec{v}_{\text{particle}} = \vec{v}_{\parallel} + \vec{v}_{\perp}$$

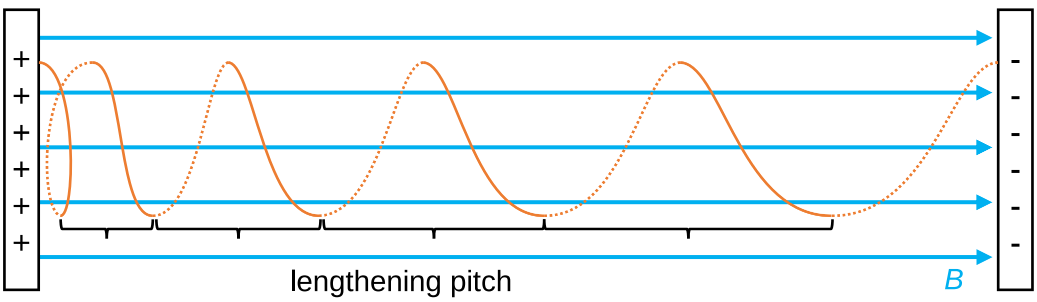
$$\vec{v}_{\perp} = \vec{v}_{\text{particle}} \sin\theta$$
$$\vec{v}_{\parallel} = \vec{v}_{\text{particle}} \cos\theta$$



Pitch is the distance travelled along the magnetic field in one period of revolution so

$$L_{\text{pitch}} = v_{\parallel} T$$

We can create a helical path of increasing or decreasing pitch by accelerating the charged particle along the magnetic field using an electric potential difference:



16.6 Charges in a magnetic field vs electric field

magnetic force $F_B = BQv(\sin\theta)$	**electric field force** $F_E = qE$
acts only on <u>moving</u> charged particle	acts on <u>both</u> stationary and moving charged particles
<u>perpendicular</u> to both magnetic field and direction of motion of charged particle	<u>parallel</u> to the direction of the electric field
Depends on <u>speed and direction</u> of motion of charged particle	<u>independent</u> of <u>speed and direction</u> of motion of the charged particle
results in circular motion	results in parabolic motion

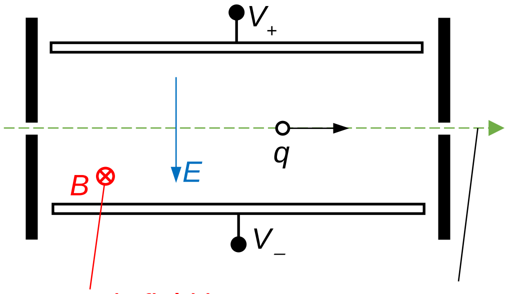
16.7 Crossed-field Velocity Selector

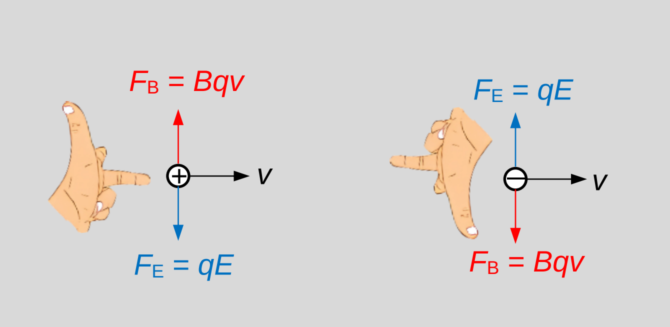
Many experiments require charged particles of well-defined speeds.

By arranging magnetic fields perpendicularly to electric fields, we can simultaneously subject moving charged particles to magnetic force and electric force of opposing directions and same magnitude.

If the fields are placed correctly, both positive and negative charges can pass through undeflected.

(*The following diagram is deliberately underpopulated to avoid visual overcrowding by field lines*.)





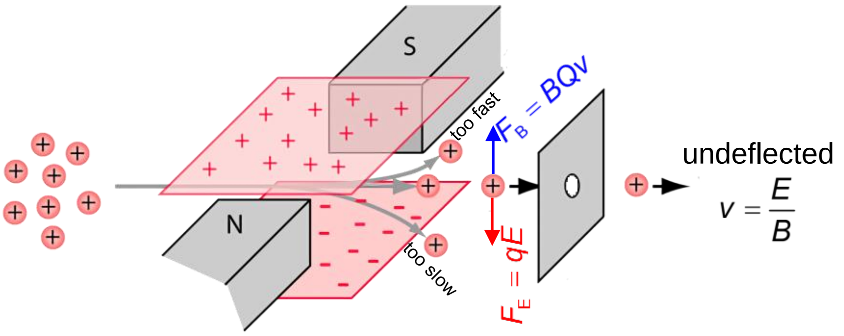
The set-up is known as a ***velocity selector*** as it allows charged particles of a certain velocity to pass through undeflected:

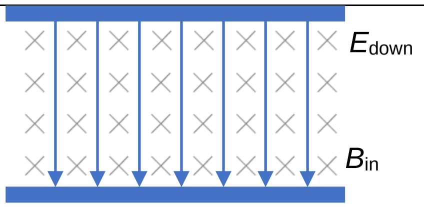
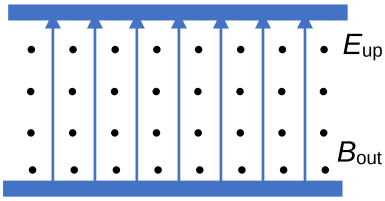
Net force on charged particle is zero for no deflection.

$$F_B - F_E = 0$$
$$F_B = F_E$$
$$B\cancel{q}v = \cancel{q}E$$
$$v = \frac{E}{B}$$

relative magnitudes of F_B and F_E	velocity	deflection in cross-fields
$F_B = F_E$	$v = \frac{E}{B}$	undeflected
$F_B < F_E$	$v < \frac{E}{B}$	direction of electric force
$F_B > F_E$	$v > \frac{E}{B}$	direction of magnetic force

Note: To predict the deflection direction when one of the quantity changes, it may be advisable to use the equation (Bqv = qE) directly. For example, when B increases, the magnetic force will be larger than the electric force. The charged particle will move in the direction of the magnetic force.



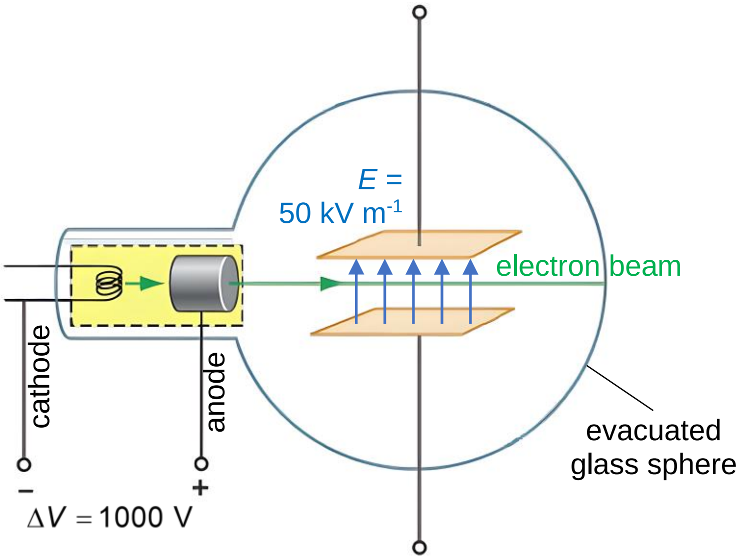
possible cross-field arrangement	alternative cross-field arrangement
	

Example 10

A beam of electrons in a cathode-ray tube is accelerated by a p.d. of 1000 V between the cathode and the anode as shown. The beam is passed through a small hole into a pair of electrostatic deflecting plates which produce an electric field of 50 kV m^{-1} upwards, perpendicular to the initial direction of the beam.

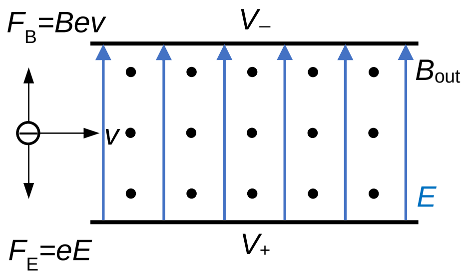
(a) State the direction of magnetic that will allow the electron beam to pass through undeflected.

(b) Find the magnetic flux density required in part (a).



Solution

(a) magnetic field points out of plane of paper

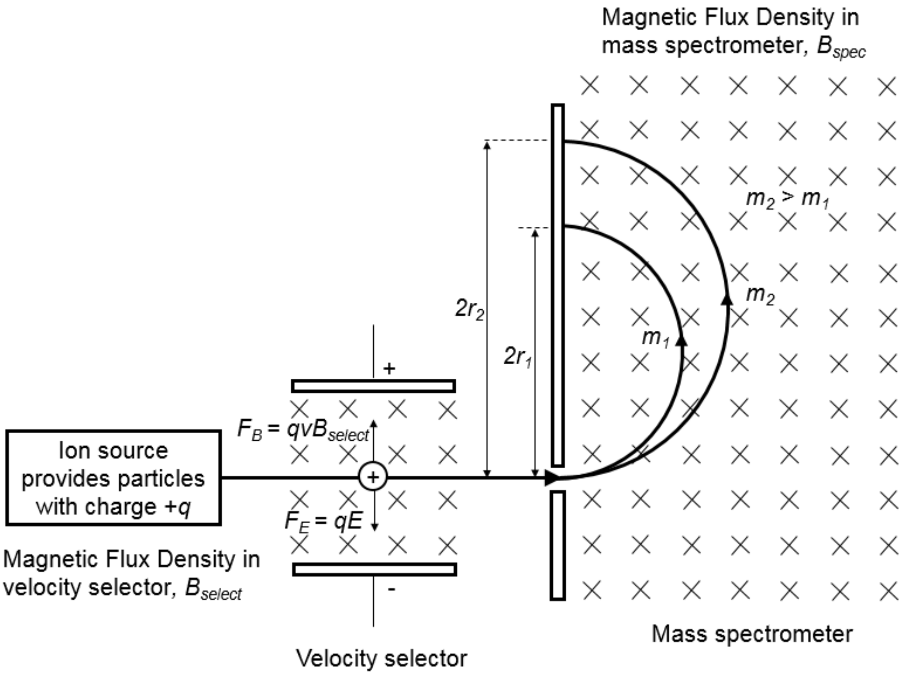


(b) electric potential energy (E_p)of electrons converted into kinetic energy (E_k) between cathode and anode

16.7.1 Mass Spectrometer

The purpose of the mass spectrometer is to separate ions according to their mass-charge ratio $\frac{m}{q}$.

- It consists of a *velocity selector* with magnetic field B_{select} which selects ions with the same velocity, v.
- Then these ions are passed into another magnetic field, B_{spec}. These ions will be collected after completing a semicircle of radius r. (The collector could be a photographic plate to study the ions or a collection of selected specific charge for subsequent experimental process.)



Consider a particle of charge q exiting a velocity selector with speed v.

From page 16 of the notes, we already know that this particle would move in a circular motion when it enters the B_{spec} field as the magnetic force provides the centripetal force.

$$B_{spec}qv = \frac{mv^2}{r}$$

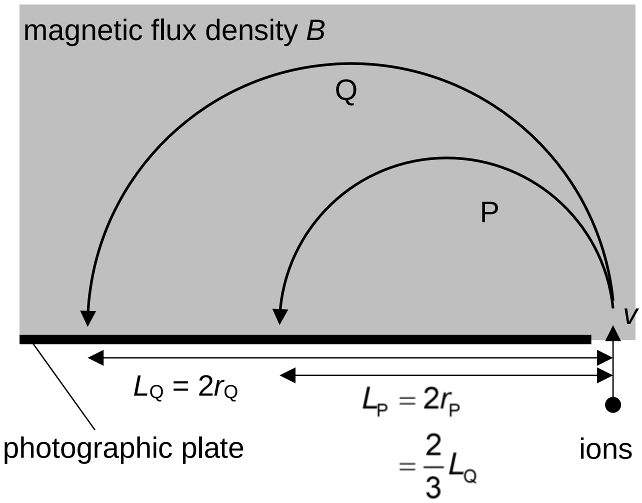
The radius of the circular path is given by $r = \frac{m}{q}\frac{v}{B_{spec}}$

Since v and B_{spec} are kept constant $r \propto \frac{m}{q}$. If q is also kept constant, the larger the m, the larger the r ($r_2 > r_1$ if $m_2 > m_1$).

Hence the particles of different m fall on different parts of the collector and are separated. This is the working principle of the *Mass Spectrometer*. A common term named *specific charge*, $\frac{q}{m}$, is also widely used in physics.

Example 11

Chemists deploy mass spectrometers to separate ions according to their specific charge $\left(\text{ratio of } \dfrac{q}{m}\right)$. Ions P and Q are first passed through a velocity selector and injected at speed *v* into a mass spectrometer which contains a uniform magnetic field. A photographic plate shows the locations where ions P and Q strike as shown.



(a) P and Q arise from atoms which tend to lose electrons when ionised. State the direction of the magnetic field.

(b) Find the ratio of $\dfrac{\text{specific charge of P}}{\text{specific charge of Q}}$.

Solution:

(a) P and Q are positively-charge ions.
by Fleming's left-hand rule, *B* points into plane of paper

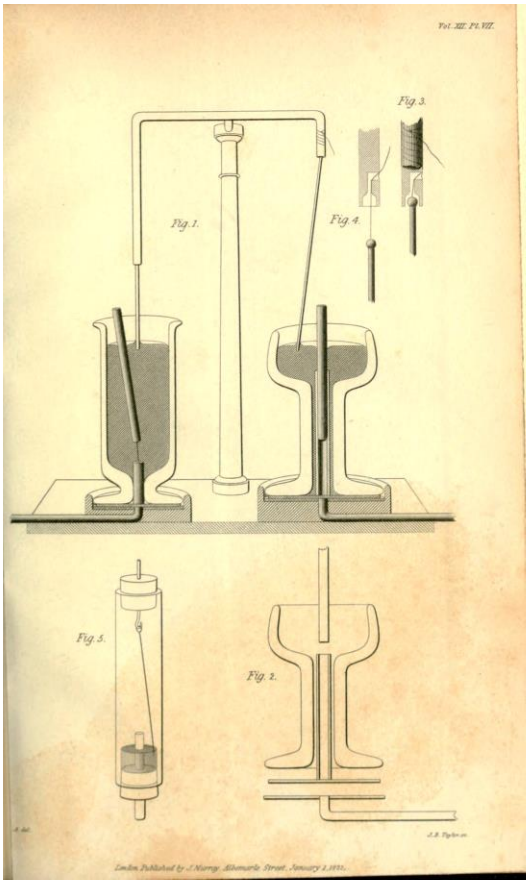
(b) magnetic force provides centripetal force

16.8 Ending Notes

It was on 3 Sep 1821 that Michael Faraday made his big breakthrough. After repeating some of the experiments previously published and devising new ones as his thinking progressed, his notes record that he:

Michael Faraday

> Arranged a magnet needle in a glass tube with mercury about it and by a cork, water, etc. Supported a connecting wire so that the upper end should go into the silver cup and its mercury and the lower move in the channel of mercury round the pole of the needle. The battery arranged with the wire as before. In this way got the revolution of the wire round the pole of the magnet.



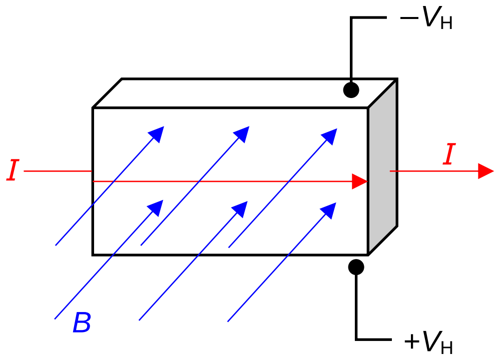
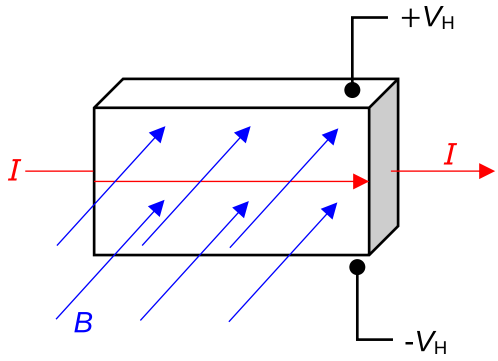
He continued working on the problem the following day, hoping to produce an apparatus that would illustrate what he called ‘electromagnetic rotations’. The result was a device that rotated continuously without human intervention, thus successfully demonstrating the transformation of electrical energy into mechanical energy; in other words, the electric motor.

The electric motor continued to advance even today, in the form of high efficiency engines in the latest electric vehicles.

From small revolutions, it seems, great things would come.

16.9 Appendix - Force on a moving charge in a magnetic field: Hall Effect

A Hall probe measures magnetic flux density using the Hall Effect. When a conductor is carrying a current, a "sidewards" potential difference develops when subject to an external magnetic field. The polarity of this "Hall voltage" V_H depends on the charge of the majority-carrier in that conductor i.e. electrons for metals and maybe positive "holes" in certain type of semiconductors:

negative charge carriers (electrons in metals and some semiconductors)	positive charge carriers (holes in *some other* semiconductors)
	
By Fleming's left-hand rule, the magnetic force on electrons acts upwards. Electrons accumulate at the upper surface of the conductor, forming an electric field that pointing from bottom to top.	By Fleming's left-hand rule, the magnetic force on "holes" (positive charge carriers) acts upwards. The upper surface of the conductor is more positively charged, forming an electric field that pointing from top to bottom.

To find the expression for Hall voltage V_H in a metal

electric field strength due to Hall voltage:

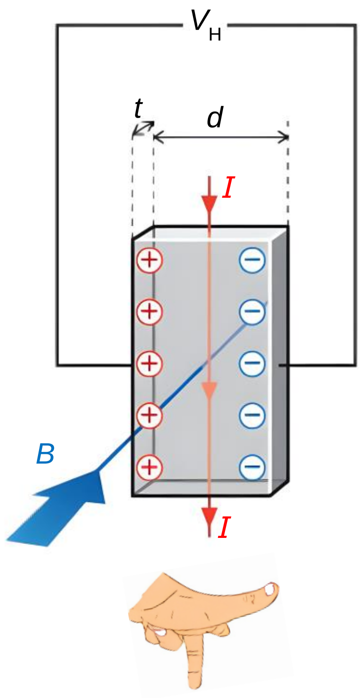
$$E = \frac{V_H}{d}$$

electric force on electron:

$$F_E = qE = q\frac{V_H}{d} \qquad \text{(to the left)}$$

magnetic force on electron of drift velocity:

$$F_B = Bqv \qquad \text{(to the right)}$$



at steady state accounting for current and drift velocity:

$$F_B = F_E$$

$$B\cancel{q}v = \cancel{q}\frac{V_H}{d}$$

$$V_H = Bdv = Bd\left(\frac{I}{nAq}\right) = B\cancel{d}\left(\frac{I}{n\left(t\cancel{d}\right)q}\right) = B\left(\frac{I}{ntq}\right)$$

Observe that for the same operating conditions, the Hall voltage is directly proportional to the magnetic flux density $V_H \propto B$. This makes the instrument easy to read and calibrate.