Chapter 20 NUCLEAR PHYSICS

Content

- The nucleus
- Isotopes
- Nuclear processes
- Mass defect and nuclear binding energy
- Radioactive decay
- Biological effects of radiation

Learning Outcomes

Candidates should be able to:

(a) infer from the results of the Rutherford α-particle scattering experiment the existence and small size of the atomic nucleus

(b) distinguish between nucleon number (mass number) and proton number (atomic number)

(c) show an understanding that an element can exist in various isotopic forms each with a different number of neutrons in the nucleus

(d) use the usual notation for the representation of nuclides and represent simple nuclear reactions by nuclear equations of the form $^{14}_{7}\text{N} + ^{4}_{2}\text{He} \rightarrow ^{17}_{8}\text{O} + ^{1}_{1}\text{H}$

(e) state and apply to problem solving the concept that nucleon number, charge and mass–energy are all conserved in nuclear processes

(f) show an understanding of the concept of mass defect

(g) recall and apply the equivalence relationship between energy and mass as represented by $E = mc^2$ to solve problems

(h) show an understanding of the concept of nuclear binding energy and its relation to mass defect

(i) sketch the variation of binding energy per nucleon with nucleon number

(j) explain the relevance of binding energy per nucleon to nuclear fusion and to nuclear fission

(k) show an understanding of the spontaneous and random nature of nuclear decay

(l) infer the random nature of radioactive decay from the fluctuations in count rate

(m) show an understanding of the origin and significance of background radiation

(n) show an understanding of the nature of α, β and γ radiations (knowledge of positron emission is not required)

(o) *(not for H1)* show an understanding of how the conservation laws for energy and momentum in β decay were used to predict the existence of the neutrino (knowledge of antineutrino and antiparticles is not required) (*cont'd next page*)

(p) *(not for H1)* define the terms activity and decay constant and recall and solve problems using the equation $A = \lambda N$

(q) *(not for H1)* infer and sketch the exponential nature of radioactive decay and solve problems using the relationship $x = x_0 \exp(-\lambda t)$ where x could represent activity, number of undecayed particles and received count rate

(r) define half-life

(for H1 students: define half-life and use the term to solve problems which might involve information in tables or decay curves)

(s) *(not for H1)* solve problems using the relation $\lambda = \dfrac{\ln 2}{t_{1/2}}$

(t) discuss qualitatively the effects, both direct and indirect, of ionizing radiation on living tissues and cells.

20.1 THE RUTHERFORD α -PARTICLE SCATTERING EXPERIMENT

STRUCTURE OF THE ATOM

Atoms, the basic building blocks of matter, were once thought to be the smallest indivisible particle. However, with his discovery of the **electrons** in 1897, J.J. Thomson concluded that electrons are part of an atom. He further postulated that these very light and negatively charged electrons were distributed throughout a uniform sea of positive charges in an atom, much like the way that plums are evenly distributed in a pudding. He thus named this model as the 'plum-pudding' model.

In 1909, under the direction of Ernest Rutherford, Hans Geiger and Ernest Marsden investigated the structure of an atom. As shown in Fig. 20.1, a beam of α-particles (helium nucleus) having an energy of 7.7 MeV emitted by the decay of radium is directed towards a thin gold foil. The deflected α-particles were detected as flashes on the fluorescent screen. To minimise the scattering of the α-particles by air molecules, their experimental set up was enclosed in vacuum.

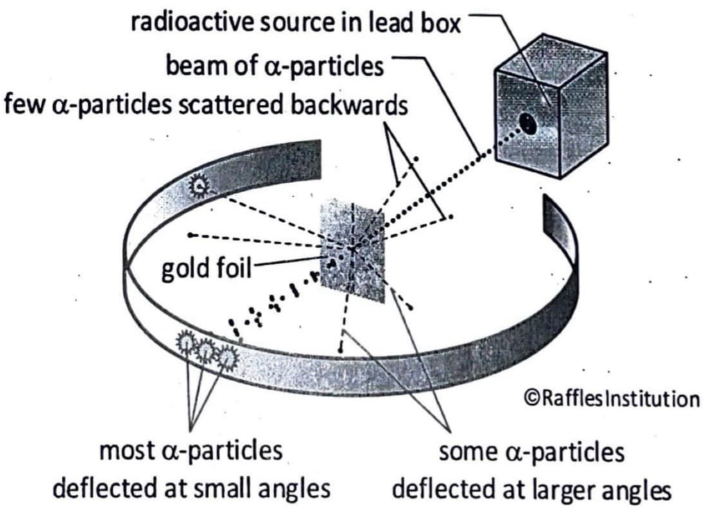


Fig. 20.1

Models of Atom

Pre-1897

Indivisible Atom
An atom was thought to be the smallest component of matter and could not be further broken down into smaller constituents.

1897

Plum Pudding Model
Atoms compose of the negatively charged electrons evenly distributed within a cloud of positive charges.

1911

Planetary Model
Atoms consists of negatively charged electrons orbiting around a very small, dense, positively charged nucleus.

1913

Bohr Model
Similar to Rutherford's model but electrons are in stationary states and hence do not radiate EM energy.

Based on the 'plum-pudding' model of the atom, they had expected the α-particles to be deflected by only a very small angle ($\sim 1°$ at most). Much to their surprise, a **small number** of α-particles (1 in about 8000) were deflected at large angles ($> 90°$) and a few were even scattered backwards ($\sim 180°$).

Current

Quantum Model
Electrons are distributed in region described by the electron density cloud around a positively charged nucleus. The greatest probability of locating the electron is at the densest region.

To explain the large deflections of some of the α-particles, Rutherford theorized in 1911 that all the **positive charge** of the atom (and practically all its mass) was concentrated in a tiny space inside the atom called the **nucleus** and the electrons orbit around the nucleus (similar to planets revolving around the sun).

So, when an α-particle (having a charge of +2e) were to come close to a gold nucleus (having a charge of +79e), it will be strongly repelled and thus suffers a large deflection.

It is therefore easy to deduce that the amount of deflection depends on how close the path of the incoming α-particle is to the gold nucleus. This distance is called the **impact parameter,** *b*, and is indicated in Fig. 20.2 below.

The impact parameter *b* is the perpendicular distance between the path of the projectile and the centre of the field (the target).

If the impart parameter is large, (for instance, α-particles 4 & 5) then the repulsion between the α-particle and the gold nucleus is small and the **deflection** is therefore small. For small *b* (particles 3, 2 & 1), the repulsion gets larger and similarly, the **deflection,** as shown in Fig. 20.2.

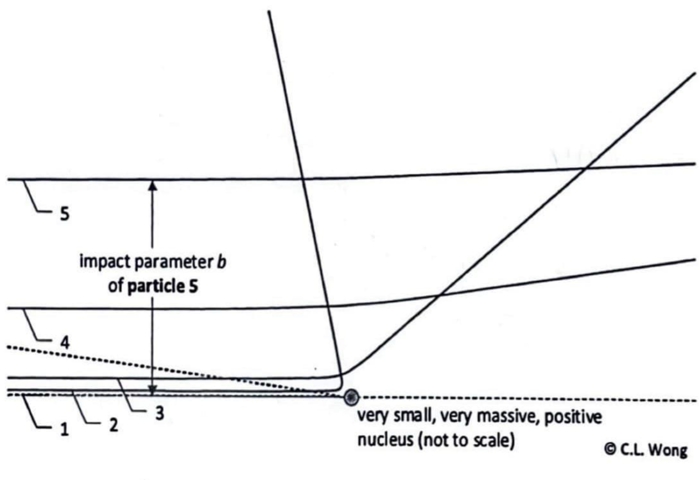


Fig. 20.2

In the α-particle scattering experiment initiated by Rutherford, it was found that most of the α-particles passed through the foil with very little deflection. This showed that most α-particles were too far away from the gold nuclei to be significantly deflected by the Coulomb repulsive force. Only a small number of α-particles showed large deflections. This led Rutherford to conclude that the positive nucleus of the atom is very tiny compared to the overall size of the atom. In other words, most of the atom is empty space!

The gold foil used in Rutherford's experiment has a thickness of 8.6×10^{-6} cm . For comparison, our hair is 1000 to 10000 times thicker.

The size of gold atom is 2.88 Å. So the gold foil is about 300 atoms thick.

Fig. 20.3 below illustrates the paths of α-particles passing through a very thin gold foil.

Rutherford estimated the size of the nucleus to be of the order of a few femto-metre[1] (see Example 1).

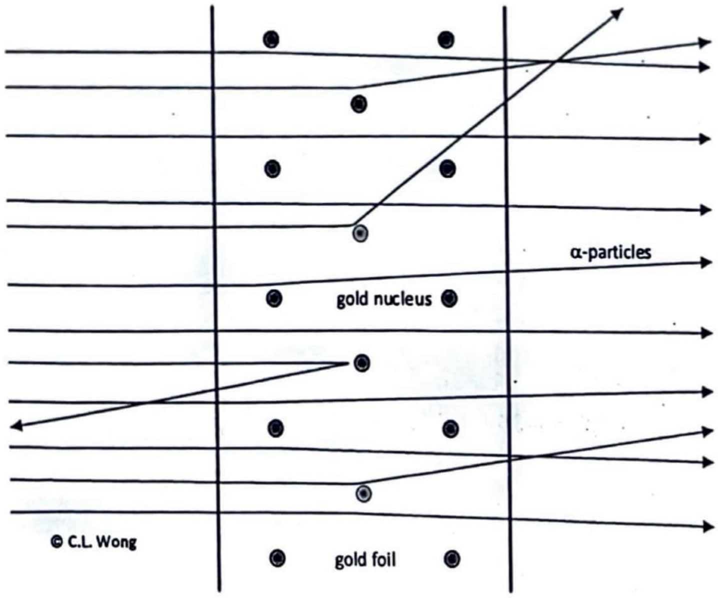


Fig. 20.3

[1] 1 fm = 1×10^{-15} m .

Example 1 **Distance of Closest Approach**

An α-particle (+2*e* charge) of energy 7.7 MeV is directed head-on (zero impact parameter) towards a gold nucleus (+79*e* charge). Calculate the distance of closest approach for the α-particle.

Solution:

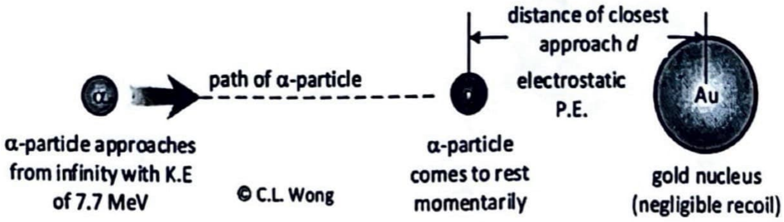


Fig. 20.3

Since the gold nucleus is much more massive than the α-particle, the recoil of the gold nucleus is negligible.

As the α-particle approaches the nucleus, its kinetic energy is gradually converted to electrostatic potential energy. At distance of closest approach *d*, the α-particle is momentarily at rest. By the principle of conservation of energy,

Loss in K.E. of α-particle = Gain in electrostatic P.E. of system

$$7.7 \times 1.60 \times 10^{-13} = \frac{79 \times 2 \times (1.60 \times 10^{-19})^2}{4\pi\varepsilon_0 d}$$

$$d = 2.95 \times 10^{-14}\ \text{m}$$

The size of the nucleus ranges from 1.8 fm to 15 fm while the size of an atom ranges from 30 to 300 pm.

An atom is approximately 20 000 times larger than its nucleus. If the nucleus is the size of a marble (1 cm), then the atom will be the size of two football fields (200 m).

THE NUCLEUS

After the discovery of the nucleus of an atom, Rutherford turned his attention to the scattering of α-particles on lighter nuclei. By 1917, Rutherford had succeeded in producing the hydrogen nuclei by bombarding hydrogen gas with α-particles. When he repeated the experiment with pure nitrogen gas, hydrogen nuclei were also detected. He concluded that the hydrogen nucleus (hydrogen being the simplest and lightest element), is the building block of the nucleus. Rutherford named this particle the **proton**.

However, the disparity between the atomic number of an atom and its atomic mass led Rutherford to hypothesize the existence of a neutral particle in the nucleus. In 1931, physicists detected an extremely penetrating radiation produced when very energetic α-particles bombarded the nuclei of beryllium, boron and lithium. This radiation was initially thought to be gamma radiation.

The *nucleus* is a very dense region at the centre of an atom, containing protons and neutrons.

Nuclei is the plural of nucleus.

Nuclide is the atomic species with a specific mass number *A* and proton number *Z*.

In 1932, James Chadwick, a former student of Rutherford, performed a series of experiments that conclusively dismissed the gamma radiation hypothesis. Instead, he found that the new radiation consisted of neutral particles which had approximately the same mass as the protons. Chadwick named this particle the **neutron**.

It is now an established fact that the nucleus of an atom consists of protons and neutrons, collectively known as **nucleons**.

https://en.wikipedia.org/wiki/Discovery_of_the_neutron

PROPERTIES OF PROTON, NEUTRON AND ELECTRON

Properties	proton	neutron	electron
charge/ C	1.602177×10^{-19}	0	$-1.602177 \times 10^{-19}$
rest mass/ kg	1.672622×10^{-27}	1.674927×10^{-27}	9.109383×10^{-31}
rest energy/ MeV	938.2720	939.5654	0.5109989

In nuclear physics, the masses of subatomic particles are typically given to higher precision.

NUCLEAR FORCE

How is it possible to pack protons into such a tiny space as the nucleus when they repel each other so strongly when they are near? Surely there must be an even stronger **attractive force** required to bind the protons and neutrons together in the nucleus. Indeed, it is this force, called the **nuclear force,** that binds all the protons and neutrons together in the nucleus. Note that the nuclear force is:

- **the same** between a proton and a proton, a proton and a neutron and a neutron and another neutron.
- **extremely short range,** and acts over 1 to 3 fm, or, not beyond the closest nucleon.

The nuclear force is a residual effect of the strong interaction. The strong interaction is one of the four fundamental interactions of nature, the others being electromagnetism, the weak interaction and gravitation.

Example 2

Repulsive Force between Protons

Calculate the Coulomb repulsive force between two protons separated by a distance of 1 fm.

Solution:

$$F = \frac{(1.60 \times 10^{-19})^2}{4\pi\varepsilon_0 \times (1.80 \times 10^{-15})^2}$$

$$= 230 \text{ N}$$

Example 3 **Density of Nucleus**

Studies have shown that the nucleus of an atom is approximately spherical with constant density. The nuclear radius is given by

$$R = R_0 A^{1/3}$$

where R_0 has a value ranging from 1.2 to 1.3 fm. Estimate the density of the nucleus.

Solution:

Consider a nucleus of mass number A. The mass of this nucleus is approximately equal to $m_p A$. Thus. the density of nucleus is

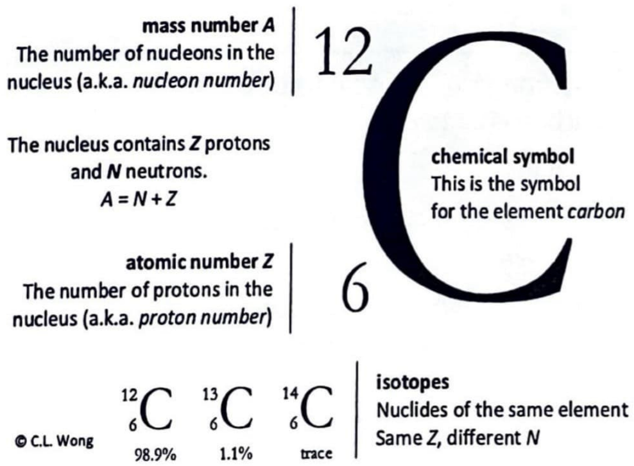
$$\text{Density } \rho = \frac{m_p A}{\text{Volume } V} = \frac{m_p A}{\frac{4}{3}\pi R^3} = \frac{m_p A}{\frac{4}{3}\pi R_0^3 A}$$

$$= \frac{1.672622 \times 10^{-27}}{\frac{4}{3}\pi \times (1.25 \times 10^{-15})^3}$$

$$= 2.04 \times 10^{17} \text{ kg m}^{-3}$$

20.2 NOMENCLATURE

THE STANDARD NUCLEAR NOTATION



Mass number A is not the same as atomic mass. Mass number is an integer and has no unit. It is simply the number of nucleons in the nucleus. Atomic mass has unit of kg or u.

Nuclide vs. Isotopes
A *nuclide* is an atomic species characterised by a unique value of Z and N. These two terms are generally used interchangeably.

NUCLIDES AND ISOTOPES

Definition:

A nuclide is the atomic species with a specific mass number A and proton number Z (e.g. $^{56}_{26}Fe$)

Definition:

Isotopes are nuclides that have the same number of protons but different number of neutrons.

E.g., $^{12}_{6}C$ and $^{14}_{6}C$ are isotopes of carbon, also referred to as carbon-12 and carbon-14. Isotopes of the same element exhibit identical chemical properties and exist in different proportions in nature, e.g. $^{16}_{8}O$ (99.8%), $^{17}_{8}O$ (0.037%) and $^{18}_{8}O$ (0.163%).

In the case of hydrogen, its isotopes are given specific names: hydrogen ($^{1}_{1}H$), deuterium ($^{2}_{1}H$) and tritium ($^{3}_{1}H$).

A **radioisotope** is an isotope that is radioactive, e.g. Iodine-131. Radioisotopes can be naturally occurring or produced artificially.

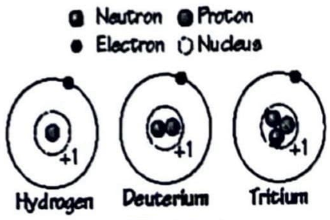


Fig 20.4

UNIFIED ATOMIC MASS CONSTANT (u)

Definition:

One atomic mass unit (1 u) is equal to one-twelfth of the mass of a carbon–12 atom.

The mass of carbon-12 atom is exactly $12u$ while the mass of its nucleus is $11.996706u$.

The atomic mass unit u is the standard unit used to represent mass on an atomic scale.

By definition, 1 mole of carbon-12 contains 6.022×10^{23} atoms with a mass of 0.012 kg. So, in terms of kg, the unified atomic mass unit u is

$$1\,u = \frac{1}{12}\times\frac{0.012}{6.022\times10^{23}} = 1.660539\times10^{-27}\ \text{kg}$$

The value of u 1.66×10^{-27} kg is given in the data page during examinations.

Example 4 **"Missing Mass"**

Calculate the total mass of 6 protons, 6 neutrons and 6 electrons in terms of *u*.

Solution:

Mass of proton: $m_p = \dfrac{1.672622 \times 10^{-27}}{1.660539 \times 10^{-27}} = 1.007277\, u$

Mass of neutron: $m_n = \dfrac{1.674927 \times 10^{-27}}{1.660539 \times 10^{-27}} = 1.008665\, u$

Mass of electron: $m_e = \dfrac{9.109383 \times 10^{-31}}{1.660539 \times 10^{-27}} = 0.000549\, u$

Total mass of 6 protons, 6 neutrons and 6 electrons: $m_{total} = 6(1.007277 + 1.008665 + 0.000549)u$

$= 12.09894\, u$

Why is the mass of the carbon-12 atom (which is 12 u) less than the total mass of its constituents?

20.3 MASS DEFECT & BINDING ENERGY

EQUIVALENCE OF MASS AND ENERGY

In 1905, Einstein established that **energy** and **mass** are **equivalent** quantities related by

$$E = m c^2 \qquad \textbf{(20.1)}$$

A particle with rest mass m_0 is said to have *rest mass energy* $E_0 = m_0 c^2$.

When an object emits energy (example, when it emits light), its mass decreases. The decrease in mass Δm would be equal to the amount of energy emitted ΔE divided by the square of the speed of light, i.e.,

$$\Delta m = \Delta E / c^2$$

Even an object at rest has an enormous amount of energy by virtue of its mass alone. This energy is known as **rest mass energy.** The rest mass energy of a body is in addition to any kinetic energy or potential energy it may have.

Probably the clearest experimental proof of the equivalence of mass and energy occurs in nuclear interactions, where large amounts of energy are released and the energy released is accompanied by a decrease in mass.

Mass *disappears* when energy is **lost** and **mass *increases*** when energy is **gained.**

In view of the equivalence of mass and energy, the classical principle of conservation of energy must be regarded as a *special case* of the principle of conservation of mass-energy. However, this more general principle needs to be invoked only when dealing with certain nuclear phenomena or when speeds comparable to the speed of light are involved.

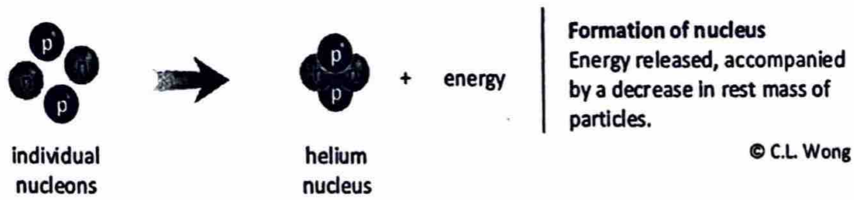


Fig. 20.5

MASS DEFECT

Definition:

The mass defect of a nucleus is the difference between the total mass of the separate nucleons and the combined mass of the nucleus.

Formula

The **mass defect of the nucleus** is

$$\Delta m_{\text{nucleus}} = Zm_p + (A-Z)m_n - m_{\text{nucleus}} \quad \textbf{(20.2)}$$

$\Delta m_{\text{atom}} \approx \Delta m_{\text{nucleus}}$

NUCLEAR BINDING ENERGY

Definition:

Nuclear binding energy is the energy equivalent of the mass defect of a nucleus. It is the energy required to separate to infinity all the nucleons of a nucleus.

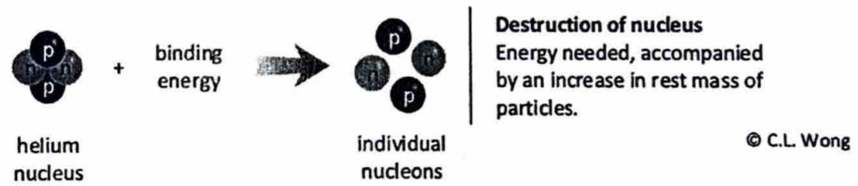


Fig. 20.6

This process is accompanied by an **increase** in the rest mass of the system. Thus, the nuclear binding energy (B.E.) and the mass defect Δm of the nucleus are related by the equation

$$\text{B.E.} = \Delta m_{\text{nucleus}} c^2 \quad \textbf{(20.3)}$$

Atomic binding energy is the energy required to break up an atom into free electrons and individual nucleons.

Electronic binding energy is the energy required to completely remove all electrons from an atom. Electronic binding energies are typically much smaller than nuclear binding energies so they may be neglected.

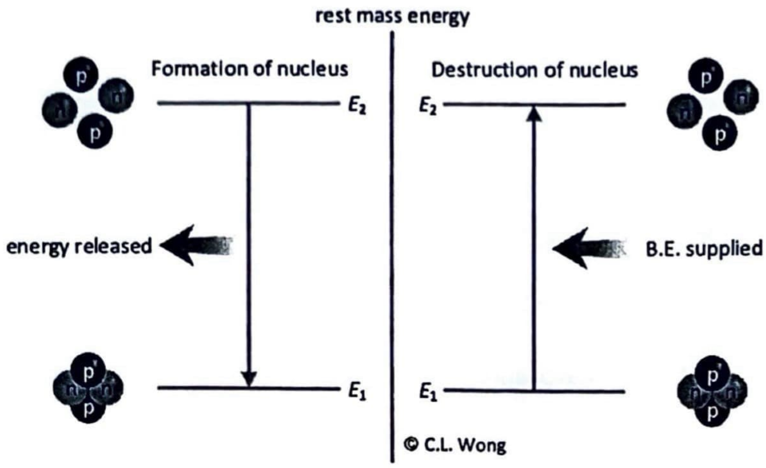


Fig. 20.7

The energy released during the formation of a nucleus is numerically equal to the binding energy of the nucleus – however, it is not the definition of nuclear binding energy.

Example 5 **Pair-Production**

A gamma ray photon of energy *E* can interact with a nucleus that is at rest to produce an electron and its anti-particle, a positron. This process is known as pair-production. Calculate the minimum energy of this gamma ray photon.

Solution:

$$\gamma \rightarrow e^- + e^+$$

By the principle of mass-energy equivalence, the energy of the gamma ray photon must at least be equal to the total rest mass energy of both electron and positron.

A positron is the anti-particle of an electron. It has the same mass as an electron but of the opposite charge.

At this minimum energy, the electron and positron are formed at rest. However, the total momentum of the system must be non-zero. Hence pair production is possible only when the gamma ray photon interacts with a massive nucleus. The momentum of the photon is transferred to the nucleus without appreciable increase in energy of the nucleus.

Gamma ray photon has momentum.

Pair production can also be achieved with two gamma ray photons travelling in opposite directions.

Example 6 **Nuclear Binding Energy**

Calculate the binding energy in MeV of the deuteron $^{2}_{1}H$, $^{56}_{26}Fe$ and $^{236}_{92}U$ given that

mass of deuteron $m_D = 3.343583 \times 10^{-27}$ kg

mass of iron 56 $m_{Fe} = 92.861315 \times 10^{-27}$ kg

mass of uranium 236 $m_U = 391.889065 \times 10^{-27}$ kg

Quote your answer to the level of accuracy justified by these values, explaining why you claim this precision.

Solution:

Mass defect for $^{2}_{1}H$

$$\Delta m = m_p + m_n - m_{nucleus}$$
$$= (1.672622 + 1.674927 - 3.343583) \times 10^{-27}$$
$$= \mathbf{0.003966 \times 10^{-27}\ kg}$$

Binding energy of $^{2}_{1}H$

$$B.E. = \Delta m \times c^2 = \frac{0.003966 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}}$$
$$= \mathbf{2.23\ MeV}$$

Mass defect for $^{56}_{26}Fe$

$$\Delta m = Zm_p + (A - Z)m_n - m_{nucleus}$$
$$= (26 \times 1.672622 + 30 \times 1.674927 - 92.861315) \times 10^{-27}$$
$$= \mathbf{0.874667 \times 10^{-27}\ kg}$$

Binding energy of $^{56}_{26}Fe$

$$B.E. = \Delta m \times c^2 = \frac{0.874667 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}}$$
$$= \mathbf{492\ MeV}$$

Mass defect for $^{236}_{92}U$

$$\Delta m = Zm_p + (A - Z)m_n - m_{nucleus}$$
$$= (92 \times 1.672622 + 144 \times 1.674927 - 391.889065) \times 10^{-27}$$
$$= \mathbf{3.181647 \times 10^{-27}\ kg}$$

Binding energy of $^{236}_{92}U$

$$B.E. = \Delta m \times c^2 = \frac{3.181647 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}}$$
$$= \mathbf{1790\ MeV}$$

BINDING ENERGY PER NUCLEON & NUCLEAR STABILITY

Since nuclear binding energy is the energy needed to separate the nucleus into its constituent nucleons, this energy should therefore be an indication of the **stability** of the nucleus. It is natural that the nuclear binding energy of a larger or heavier nucleus is greater than that of a lighter nucleus simply because a heavier nucleus has more nucleons. But this does not mean that a heavier nucleus is more stable. To judge more accurately, the **binding energy per nucleon** ought to be calculated. This is the energy required to remove a nucleon from the nucleus and is given by the equation:

Formula

$$\text{binding energy per nucleon} = \frac{\text{B.E.}}{A} \quad \textbf{(20.4)}$$

The **higher** its value, the **more stable** is the nucleus.

Definition:

Binding energy per nucleon is the total energy needed to completely separate all the nucleons in a nucleus to infinity divided by the number of nucleons in the nucleus.

Graph of Binding Energy Per Nucleon

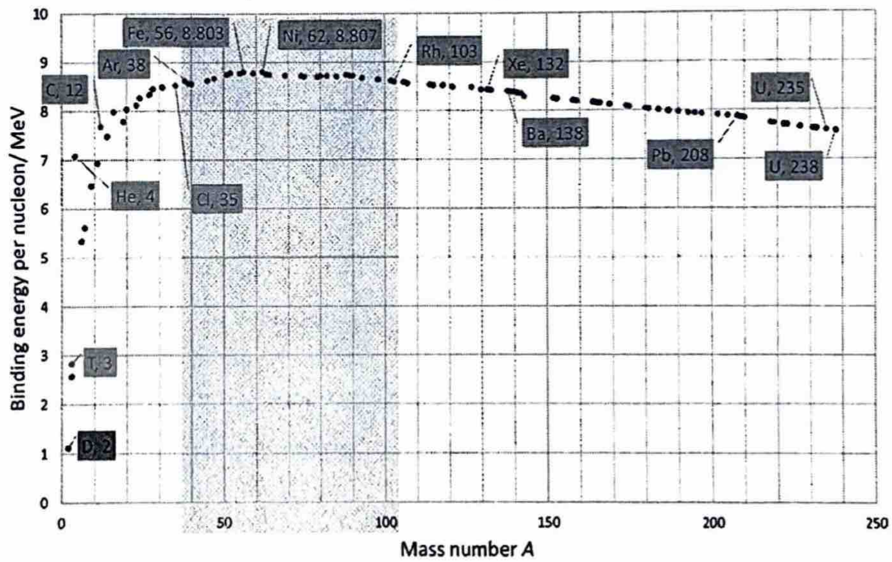
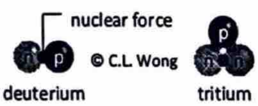


Fig. 20.8

We can interpret *binding energy per nucleon* as the *average energy* require to remove a nucleon from the nucleus until it is completely disintegrated.

The following observations can be made from the graph:

- Light elements from deuterium ($^{2}_{1}H$) to chlorine ($^{35}_{17}Cl$) have **increasing** binding energy per nucleon. In this region, the increase in attractive nuclear forces per nucleon (as more nucleons are added) outweighs the increase in Coulomb repulsion between protons.
- $^{4}_{2}He$ (7.084 MeV) is unusually stable amongst light nuclei.



Deuterium has 0.5 nuclear force pairing per nucleon while tritium has 1 nuclear force pairing per nucleon.

- Elements with intermediate mass number ($^{38}_{18}Ar$ to $^{103}_{45}Rh$) are relatively stable (high binding energy per nucleon).

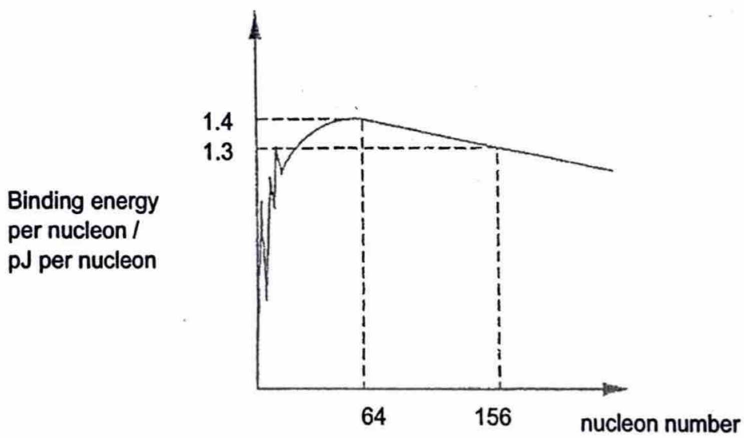
Binding energy of these nuclei are typically greater than 8.6 MeV.

- $^{62}_{28}Ni$ (8.795 MeV), $^{58}_{26}Fe$ (8.792 MeV) and $^{56}_{26}Fe$ (8.790 MeV) are the three most stable nuclei.

$^{56}_{26}Fe$ is more abundant than $^{62}_{28}Ni$ and $^{58}_{26}Fe$ in the universe.

- Heavier nuclei exhibit **decreasing** binding energy per nucleon. In this region, the Coulomb repulsion between the protons becomes more prominent.

Example 7 The sketch graph shows how the binding energy per nucleon varies with the nucleon number for naturally occurring nuclides.



Calculate the total binding energy of the nuclide $^{156}_{64}Gd$.

[J98/I/29]

Solution:

Total binding energy

$$\text{B.E.} = (\text{B.E. per nucleon}) \times \text{number of nucleons}$$
$$= 1.3 \times 10^{-12} \times 156$$
$$= \mathbf{200 \times 10^{-12}\ J}$$

pJ = pico-joules = 10^{-12}

20.4 NUCLEAR REACTIONS

A **nuclear reaction** is a process during which two nuclei, or a nucleus and a subatomic particle (proton, neutron or energetic electron), **collide** with each other to produce one or more nuclides that is/are different from the original nuclides. Hence, a nuclear reaction must cause **nuclear transformation** of at least one nuclide to another. Nuclear reactions may either be natural or artificial. Examples of nuclear reactions are presented in the table below.

Natural	Artificial
Fusion of light elements, such as hydrogen, powers the star and is responsible for the creation of virtually all elements.	Fusion of deuterium and tritium or fission of $^{235}_{92}U$ / $^{239}_{94}Pu$ in thermonuclear weapons. These are uncontrolled nuclear chain reactions.
Collisions between solar wind (mainly plasma of protons) and our upper atmosphere produce pions and kaons.	Fission of $^{235}_{92}U$ in nuclear power plants generates tremendous amount of energy. These are controlled nuclear chain reactions.

Pions and kaons are subatomic particles known as mesons which are made up of a quark and an antiquark.

BALANCING NUCLEAR EQUATIONS & CALCULATING THE ENERGY RELEASED

All nuclear reactions may be presented in the form of an equation in the standard form similar to chemical reactions. In a nuclear equation, the mass number and charge of the particles must be conserved.

Momentum and *mass-energy* are also conserved in a nuclear reaction.

The notations of common particles involved in nuclear reactions are given in the table below.

Name	Symbol
proton	$^{1}_{1}H$ or $^{1}_{1}p$
neutron	$^{1}_{0}n$
electron	$^{0}_{-1}e$
positron	$^{0}_{1}e$
α-particle	$^{4}_{2}He$

Example 8 **Discovery of the Proton**

The discovery of the proton took place after the first induced nuclear reaction was carried in a laboratory by Rutherford. The equation for this reaction may be written as $^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$

Calculate the energy released in the above reaction given that the rest masses of the four atoms are:

$m_{\text{N-atom}} = 14.003074\,u$; $m_{\text{He-atom}} = 4.002603\,u$;

$m_{\text{O-atom}} = 16.999131\,u$; $m_{\text{H-atom}} = 1.007825\,u$

Atomic masses may be used in the calculation for energy release as there is an equal number of electrons on both sides of the equation. The electron binding energies of the atoms may be neglected.

Solution:

The decrease in mass in the reaction is

$$\text{Mass defect } \Delta m = \begin{pmatrix} 14.003074\,u \\ +\,4.002603\,u \end{pmatrix} - \begin{pmatrix} 16.999131\,u \\ +\,1.007825\,u \end{pmatrix}$$

$$= -0.001279\,u$$

The **negative** sign means that the total mass of the products is greater than that of the reactants. In other words, there is an increase in mass.

The energy released for this reaction is

$$\text{Energy released } \Delta E = \frac{-0.001279 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}}$$

$$= -1.15\text{ MeV}$$

This energy release is also known as the *Q-value*, the reaction energy. *Q* is positive for *exothermic* reaction and negative for *endothermic* reactions.

From the energy point of view, this reaction is not likely to happen due to the energy shortfall. So how did this reaction occur for Rutherford to discover the proton? The answer lies with the α-particle. The energy shortfall in the reaction was provided by the kinetic energy of the α-particle.

Nuclear reactions that are not energetically favourable may proceed if the parent nuclei have sufficient kinetic energy.

NUCLEAR FISSION

Definition: **Nuclear fission is the splitting of a heavy nucleus into two lighter nuclei of approximately the same mass.**

The fission of a heavy nucleus is typically initiated by a slow (or thermal) neutron and the fission products include two or three slow neutrons and gamma ray photons. An enormous amount of energy is released during nuclear fission of a very heavy nucleus.

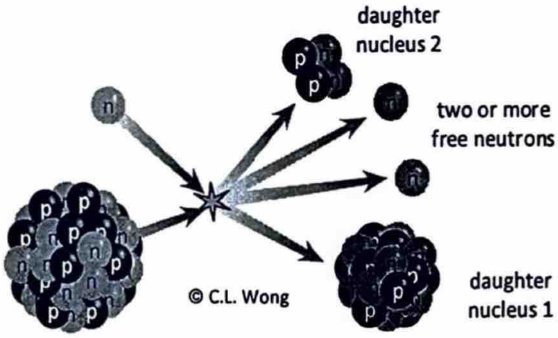


Fig. 20.9

Since the slow neutrons produced in one fission reaction can each trigger another fission reaction, a chain reaction is very quickly set up. This can easily get out of control (this is the case in an exploding atomic bomb where a huge amount of energy is released) unless steps are taken to control the rate of the reactions (this is done in a nuclear reactor).

The production or emission of energy via nuclear fission of heavy nuclei can be explained by the concept of **binding energy per nucleon**.

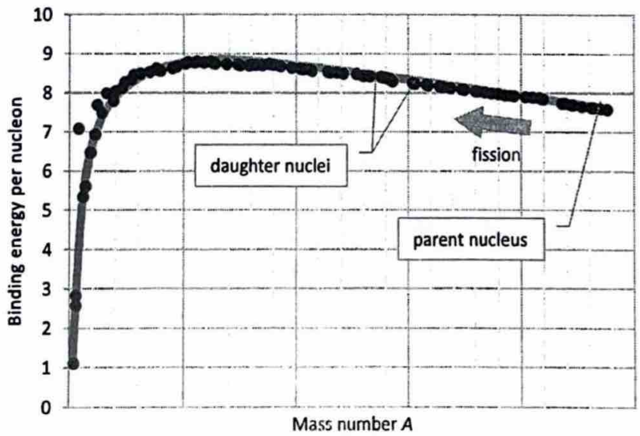


Fig. 20.10

Fig. 20.10 shows that the lighter "daughter" nuclei have **higher** binding energy per nucleon than the "parent" nucleus. This means that the formation of the daughter nuclei releases more energy than that required to completely disintegrate the parent nucleus. The energy E released in a fission reaction can be shown to be equal to the difference in B.E.'s of the reactants and that of the products. That is,

$E = \text{B.E.}_{\text{products}} - \text{B.E.}_{\text{reactants}}$ **(20.5)***

We can visualise fission as a two-step process: the disintegration of the parent nucleus and the formation of the daughter nuclei.

***See Appendix** for derivation of equation (20.5).

NUCLEAR CHAIN REACTION

In 1933, Leó Szilárd hypothesized that neutron-driven fission of heavy nuclei could create a **nuclear chain reaction** (Fig. 20.11). Assuming that the number of secondary neutrons produced by fission is greater than one, then each reaction can initiate additional reactions and an exponentially increasing number of reactions can be achieved through a chain reaction. His hypothesis led to the possibility of yielding large amounts of energy for civilian (nuclear power plant) and military use (atomic bomb).

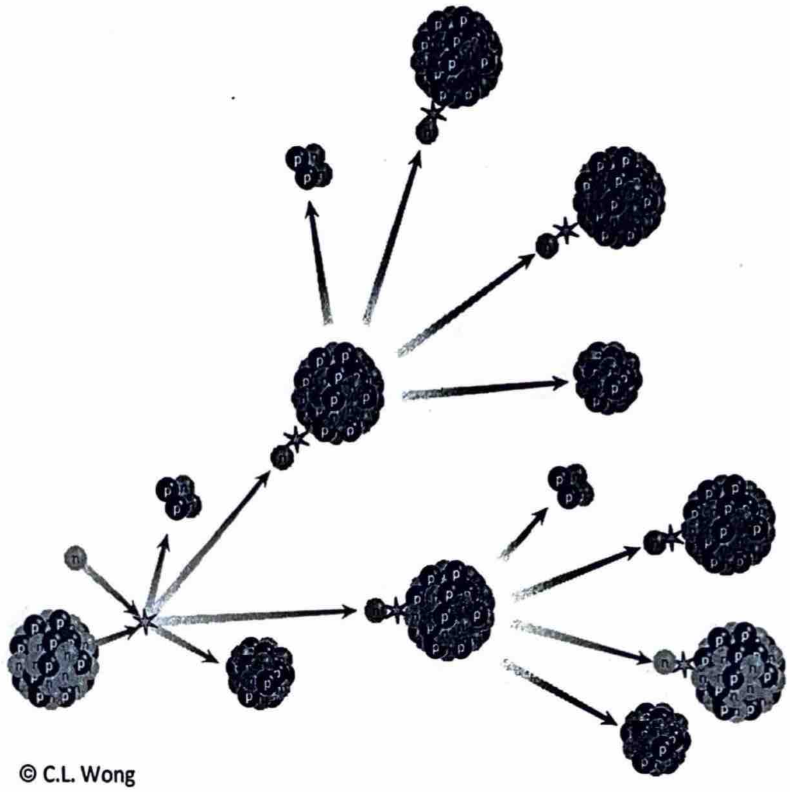


Fig. 20.11

NUCLEAR FUSION

Definition:

Nuclear fusion occurs when two light nuclei combine to form a nucleus of greater mass.

Energy is released during nuclear fusion of very light nuclei.

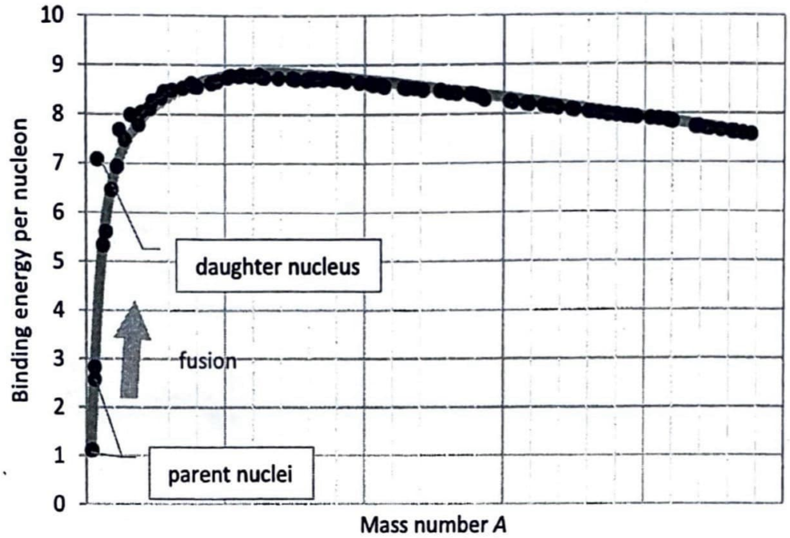


Fig. 20.12

Fig. 20.12 shows that the highest possible energy released per nucleon is by fusing the nuclei of the lightest known element, hydrogen.

Hydrogen nucleus has zero binding energy (not shown).

The Sun derives its power from the fusion of hydrogen nuclei (actually protons) to form a helium nucleus through a **proton-proton cycle**. The cycle consists of three distinct nuclear reactions and begins with the fusion of protons to form deuterium with the release of 0.42 MeV of energy:

Protons must have very high energy to overcome mutual Coulomb repulsion. This would require a temperature in excess of millions of Kelvin.

$$ {}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + {}^{0}_{1}e + \nu \qquad \textbf{(20.6)} $$

A positron ${}^{0}_{1}e$ and a neutrino ν are released at the same time.

The deuterium nucleus then fuses with a proton to form a helium-3 nucleus with the release of 5.49 MeV of energy:

$$ {}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma \qquad \textbf{(20.7)} $$

γ is a gamma ray photon.

The helium-3 nuclei from two such reactions then combine to give a helium-4 nucleus and a pair of protons. 12.86 MeV of energy is liberated in this reaction.

$$ {}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + 2{}^{1}_{1}H \qquad \textbf{(20.8)} $$

Lastly, the positron (from 20.6a) will annihilate a single electron to produce two gamma ray photons of 1.022 MeV in total:

$$ {}^{0}_{1}e + {}^{0}_{-1}e \rightarrow 2\gamma \qquad \textbf{(20.9)} $$

This is the opposite of pair production.

In one proton-proton cycle, reactions (20.6), (20.7) and (20.9) occur twice while reaction (20.8) once. The net effect of the above reactions (20.6) to (20.9) is to convert four protons and two electrons into a single helium-4 nucleus:

$$ 4{}^{1}_{1}H + 2{}^{0}_{-1}e \rightarrow {}^{4}_{2}He $$

with the release of 26.7 MeV of energy.

Formula:

Energy released in nuclear reaction = $(\text{mass}_{reactants} - \text{mass}_{products})\, c^2$
OR
Energy released in nuclear reaction = $\text{B.E.}_{products} - \text{B.E.}_{reactants}$

Example 9 **Nuclear Fission**

Write down the nuclear equation for the neutron-induced fission of plutonium-239 (${}^{239}_{94}Pu$) that results in barium-143 (${}^{143}_{56}Ba$) and strontium-95 (${}^{95}_{38}Sr$), and identify other particles that may be produced in this reaction. Calculate the energy released.

$m_{\text{Pu-atom}} = 239.052165\ u$, $m_{\text{Ba-atom}} = 142.920625\ u$,
$m_{\text{Sr-atom}} = 94.919351\ u$

Solution:

This is a neutron-induced fission reaction:

$$ {}^{239}_{94}\text{Pu} + {}^{1}_{0}\text{n} \rightarrow {}^{143}_{56}\text{Ba} + {}^{95}_{38}\text{Sr} + 2{}^{1}_{0}\text{n} $$

Mass difference of the reaction is

$$ \begin{aligned} \Delta m &= \text{mass of original nuclei} - \text{mass of final nuclei} \\ &= (239.52165 - 142.920625 - 94.919351 - 1.008665)u \\ &= 0.203524u \end{aligned} $$

Energy released is $E = \Delta m \times c^2$

$$ = \frac{0.203524 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}} $$

$$ = \mathbf{190\ MeV} $$

Example 10 **Nuclear Fusion**

Although ordinary hydrogen is abundant in the universe, the fusion between two hydrogen nuclei does not occur readily. Fusion research has therefore focused on reactions involving the heavier hydrogen isotopes. The deuterium-deuterium (D-D) reaction is one such possible reaction:

$${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + {}^{1}_{0}\text{n}$$

Calculate the energy released in this reaction given the binding energy per nucleon of deuterium and helium-3 are 1.114 MeV and 2.576 MeV, respectively.

Solution:

To calculate the energy released, we need to compute the binding energies of ${}^{2}_{1}\text{H}$ and of ${}^{3}_{2}\text{He}$.

Free neutrons have zero binding energy.

$$\text{B.E. of } {}^{2}_{1}\text{H} = 2 \times 1.114 = 2.228 \text{ MeV}$$

$$\text{B.E. of } {}^{3}_{2}\text{He} = 3 \times 2.576 = 7.728 \text{ MeV}$$

Hence the energy released is

$$E = \text{B.E. of final nuclei} - \text{B.E. of original nuclei}$$

$$= 7.728 - 2 \times 2.228 = 3.272 \text{ MeV}$$

20.5 RADIOACTIVITY

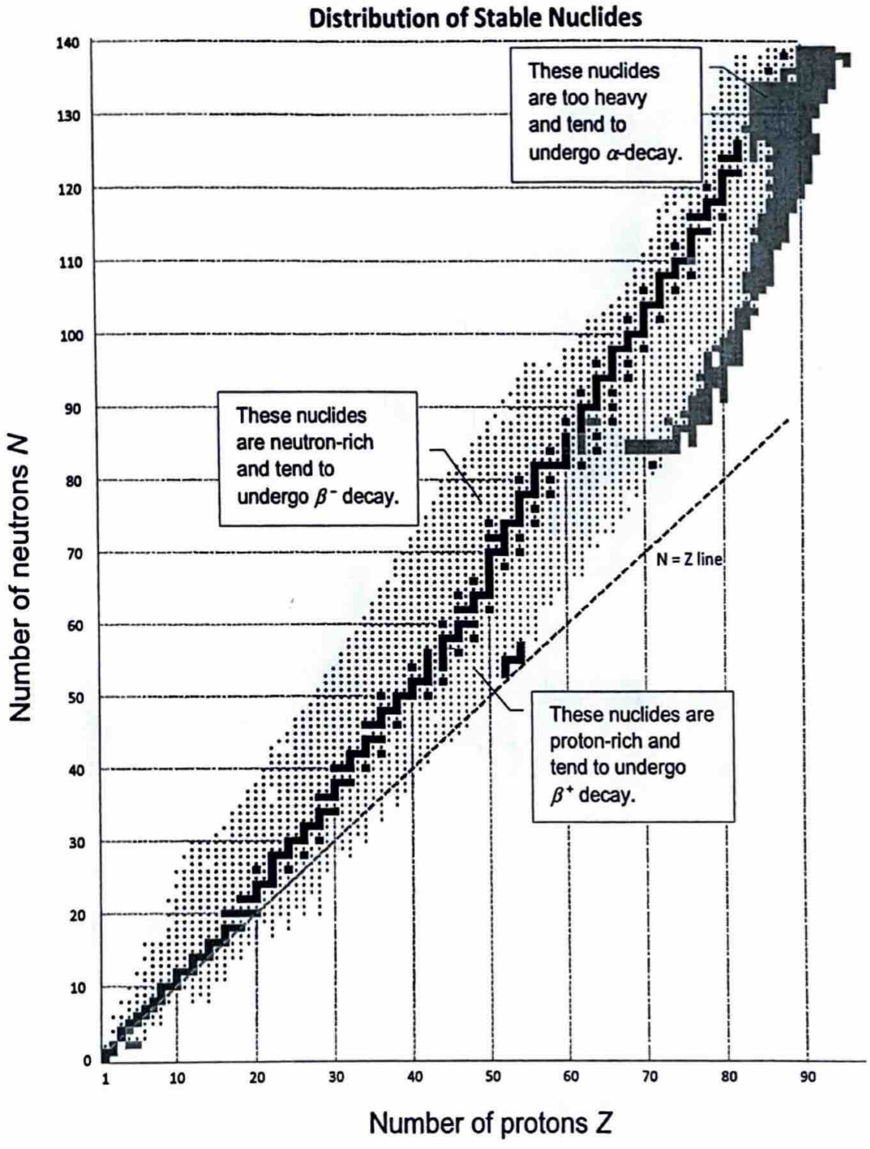
RADIOACTIVITY & STABILITY OF NUCLEI

There are at least 3300 known nuclides (both natural and artificial), but only 254 of them are stable (they are indicated as black dots in the chart below). An element with an unstable nucleus is known as a **radioactive nuclide (or radionuclide)**.

Nuclide is the atomic species with a specific mass number A and proton number Z.

Lead ($Z = 82$) is the heaviest element to have stable isotopes.

Nuclides with even Z and even N are most stable while nuclides with odd Z and odd N are least stable due to pairing effects.



■ These nuclides are very heavy (have many nucleons) and tend to undergo α-decay.

• These nuclides are neutron-rich and tend to undergo β^- decay.

♦ These nuclides are proton-rich and tend to undergo β^+ decay.

Fig. 20.13

A radioactive nuclide will spontaneously decay to a more stable nuclide by emitting some forms of ionizing radiation. This decay process is known as **radioactivity** and the atom is said to have undergone **radioactive decay**.

Radioactive decay is both **spontaneous** and **random. It is spontaneous because** the probability of decay is unaffected by any external factors such as temperature, pressure or chemical composition. It is random because it is impossible to predict which nucleus will decay next. This random nature of radioactive decay results in the **fluctuations** in the count rate measured by a detector.

However, the probability that a given atom or nucleus will decay is constant over time. Hence a mathematical description of the decay rate of a large number of identical radioactive atoms can be formulated.

Definitions:

Radioactivity or **radioactive decay** is the process whereby a radionuclide is transformed into a more stable nuclide by emitting some forms of ionizing radiation.

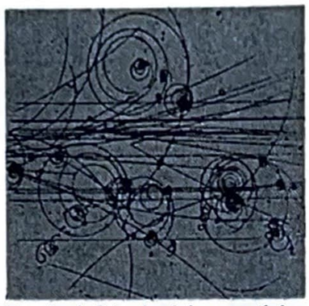
Radioactive decay is **spontaneous** because the probability of decay of a nucleus is unaffected by any external factors such as temperature, pressure or chemical composition.

Radioactive decay is random as it is impossible to predict which nucleus will decay next even though the probability of decay per unit time is identical for all nuclei. This results in fluctuations in the count rate measured by a detector.

TYPES OF RADIATIONS

Early researchers detected **three** types of **ionizing radiation** in the study of radioactive emission. These radiations are given the names **alpha, beta** and **gamma** radiations (in increasing order of their **penetrating power**). In a cloud chamber (see Appendix), these radiations leave behind distinguishable tracks that can be photographed and studied.

- Alpha radiations produce tracks that are **short** and **thick (strong** ionizing power) and of **similar length** (same kinetic energies)
- Beta radiations produce tracks that are **long** and **thin (moderate** ionizing power) and of **varying lengths** (a range of kinetic energies).
- Gamma radiations (**low** ionizing power) leave behind a cloud of wiggly, short and thin tracks due to ionized electrons.



Tracks left by ionising particles in a cloud chamber.

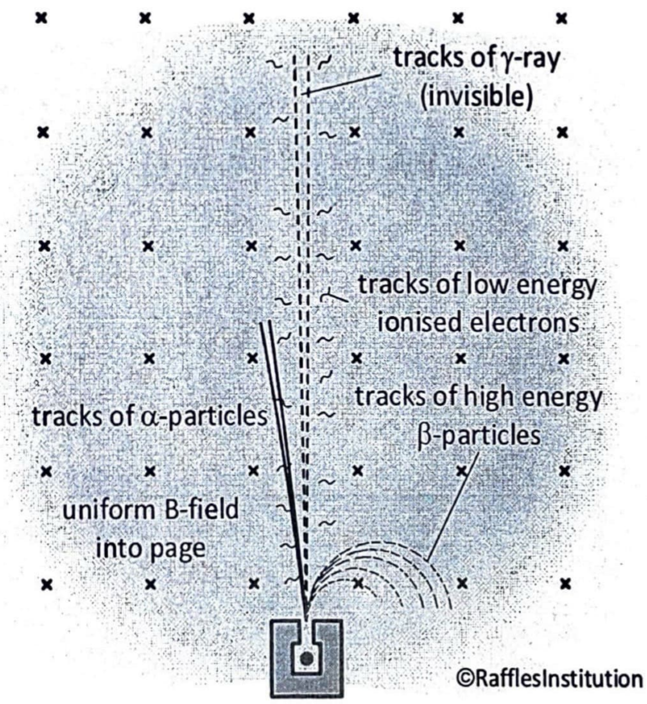


Fig. 20.14 Deflection of α, β and γ radiations in a magnetic field.

When a magnetic field is applied, alpha radiations and beta radiations are deflected in opposite directions. Based on the direction of the magnetic field and the deflection of the radiations, we can deduce that alpha radiations are positive while beta radiations are negative. On the other hand, gamma radiations are not deflected so it must be neutral.

By measuring the specific charge (q/m) of alpha and beta radiations, it was determined that alpha radiations are helium nuclei ($^{4}_{2}He$) and beta radiations are electrons ($^{0}_{-1}e$). Further investigations revealed that gamma radiations are electromagnetic (EM) radiations.

In 1914, Rutherford observed the reflection of gamma radiations from crystal surfaces, proving that gamma radiations are EM radiations.

ALPHA DECAY

Alpha decay refers to the spontaneous emission of an α-particle (helium nucleus) by a radioactive nucleus. The nuclear equation is

$$^{A}_{Z}X \rightarrow ^{A-4}_{Z-2}Y + ^{4}_{2}He \qquad (20.10)$$

where X and Y are the chemical symbols for parent and daughter nuclide respectively.

The typical kinetic energy of the α-particles ranges from 4-9 MeV. However, **all** α-particles emitted by a particular radionuclide have the **same kinetic energy** (see **Example 12**).

As there are only two particles produced after the decay, they will **recoil** from each other if the parent nucleus was at rest initially, as linear momentum is conserved.

Alpha decay is common in nuclei with too many nucleons (mass number $A > 209$). The emission of an α-particle decreases the mass number by 4 and results in a more stable daughter nucleus.

The α-particle eventually acquires two electrons to become a helium atom.

The daughter nucleus may still be radioactive.

BETA DECAY

Beta decay refers to the spontaneous emission of a β-particle {electron $^{0}_{-1}e$ (called β^- decay), or positron $^{0}_{+1}e$ (called β^+ decay)}, by a radioactive nucleus. The nuclear equations are as follows:

$$^{A}_{Z}X \rightarrow ^{A}_{Z+1}Y + ^{0}_{-1}e + ^{0}_{0}\bar{\nu} \quad (\beta^-) \qquad (20.11)$$

$$^{A}_{Z}X \rightarrow ^{A}_{Z-1}Y + ^{0}_{1}e + ^{0}_{0}\nu \quad (\beta^+) \qquad (20.12)$$

Beta particles are created in the nucleus.

$^{0}_{0}\nu$ and $^{0}_{0}\bar{\nu}$ are the neutrino and anti-neutrino, respectively. Neutrinos are elementary particles that interacts weakly with matter. As such they cannot be detected directly.

β^- decay is common in nuclei with **too many neutrons.** Electron emission from the nucleus converts one of the neutrons into a proton:

$$^{1}_{0}n \rightarrow ^{1}_{1}p + ^{0}_{-1}e + ^{0}_{0}\bar{\nu}$$

Assume beta decay to be β^- decay, unless stated otherwise.

β^+ decay is common in nuclei with **too many protons.** Positron emission from the nucleus converts one of the protons into a neutron:

$$^{1}_{1}p \rightarrow ^{1}_{0}n + ^{0}_{1}e + ^{0}_{0}\nu$$

A free proton does not decay into a neutron. Can you explain why?

Because it is lighter than the neutron; so you need to supply energy to transform a proton into a neutron.

Neutrino in beta decay

(not for H1)

Conservation laws for energy and momentum in a β–decay were used to predict the existence of the neutrino.

Conservation of Energy:

(1) The sum of the kinetic energy of the daughter nucleus and the β-particle is less than the total energy released in a given decay process ⇒ this violates the Principle of Conservation of Energy, if these are the only two particles produced in a β-decay. Therefore, another particle, the anti-neutrino, must have been emitted and it carries away the rest of the energy produced.

(2) The kinetic energies of the β-particles emitted in a particular decay reaction are not fixed but range from zero to a maximum value ⇒ the energy is shared between the β-particle and the anti-neutrino.

The anti-neutrino is neutral and nearly undetectable (invisible). It has no biological/chemical effect.

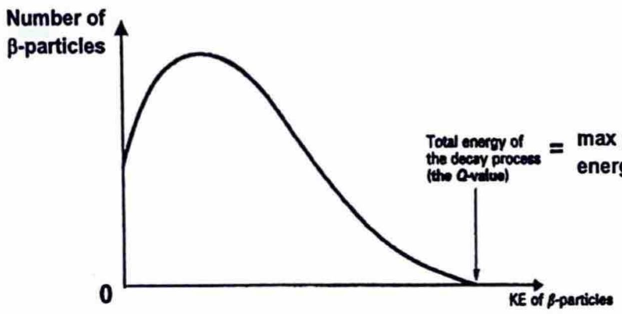


Fig. 20.15

Conservation of Momentum:

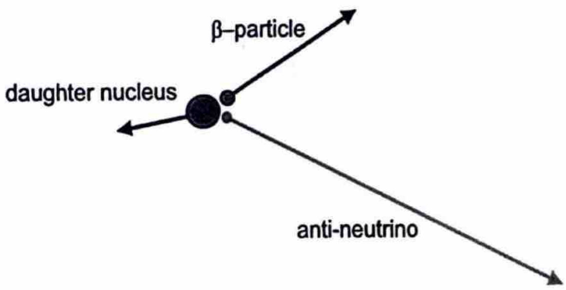


Fig. 20.16

(1) The tracks (formed in a Cloud Chamber) of the daughter nucleus and the β-particle alone do not form a straight line ⇒ momentum is not conserved, if these are the only two particles formed after the decay. Hence this violates the Principle of Conservation of Momentum.

(2) Only by including the momentum of the invisible anti-neutrino can the Principle of Conservation of Momentum be satisfied.

In a cloud chamber, ionizing radiations form visible tracks.

The anti-neutrino is neutral therefore it does not produce ionization to create a track in a Cloud Chamber.

GAMMA DECAY

When a nucleus has just undergone an alpha or a beta decay, the daughter nucleus may be left in an excited state (this means that it possesses too much energy and will emit the excess energy subsequently). As with an excited atom that de-excites back to its ground state by emitting a photon, an excited nucleus will also de-excite to its ground state by emitting a photon. However, because the emitted photon in this case is much more energetic, it is actually a γ-ray photon whereas in the case of excited atoms, the photons emitted are less energetic and are not γ-ray photons. They can be x-ray photons, uv light photons or just visible light photons.

$$^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z}X + \gamma \qquad \textbf{(20.13)}$$

where * indicates that a nucleus is in an excited state.

Example 11 **Radioactive Decay Series**

Uranium-238 ($^{238}_{92}U$) undergoes a series of decays to a final stable nuclide. The particles emitted in successive decays are

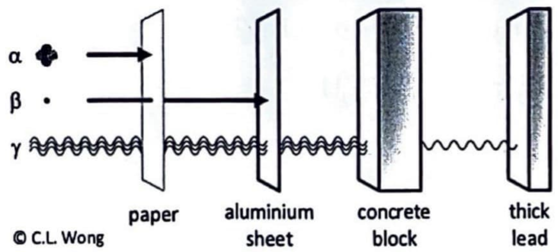
α β β α α

Which nuclide is not produced during this series of decay?

A $^{228}_{88}Ra$ **B** $^{230}_{90}Th$ **C** $^{234}_{91}Pa$ **D** $^{234}_{92}U$

Solution: A

Properties of Alpha, Beta & Gamma Radiations

Properties	alpha	beta (β^-)	gamma
Nature	helium nucleus	electron	γ-ray photon
Charge	$+2e$	$-e$	0
Rest Mass/ kg	6.644657×10^{-27}	9.109383×10^{-31}	0
Energy	 well-defined 4 – 9 MeV	 range from 0.01 to 5 MeV	 discrete energies (keV – MeV)
Speed	$0.05c$	up to $0.9c$	c
Ionizing power	strong	moderate	very weak
Range in air	a few centimetres	a few metres	very long
Penetrating power	low	medium	high
Stopped by			

Ionizing power is the ability of the radiation to knock out electrons from atoms.

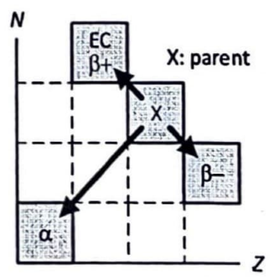
Penetrating power is the ability of the radiation to pass through a material before being completely absorbed. A radiation with high ionizing power has low penetration power.

When particles are absorbed by a material, their energies are converted into **internal energy** of the material.

Summary of Alpha, Beta & Gamma Decays

Decay	Transformation	Example	Reason
α-decay	${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He$	${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$	Nucleus too large
β^- decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}e + \bar{\nu}$	${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e + \bar{\nu}$	Nucleus has too many neutrons
β^+ decay	${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}e + \nu$	${}^{64}_{29}Cu \rightarrow {}^{64}_{28}Ni + {}^{0}_{1}e + \nu$	Nucleus has too many protons
γ-decay	${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + \gamma$	${}^{87}_{38}Sr^* \rightarrow {}^{87}_{38}Sr + \gamma$	Nucleus has excess energy after a decay
Electron capture	${}^{A}_{Z}X + {}^{0}_{-1}e \rightarrow {}^{A}_{Z-1}Y + \nu$	${}^{64}_{29}Cu + {}^{0}_{-1}e \rightarrow {}^{64}_{28}Ni + \nu$	Nucleus has too many protons

Transition diagram



Electron capture (EC) refers to the absorption of orbital electron (usually from K-shell) by a radioactive nucleus. The captured electron converts a proton to a neutron and a neutrino. EC is detected by observing emitted x-ray of the daughter atom.

Example 12 **Alpha Decay**

A stationary nucleus of $^{238}_{92}U$ decays into a Thorium nucleus by the emission of an α-particle.

(a) Calculate the energy released.

(b) Calculate the kinetic energy of the alpha particle.

Masses of nuclei:

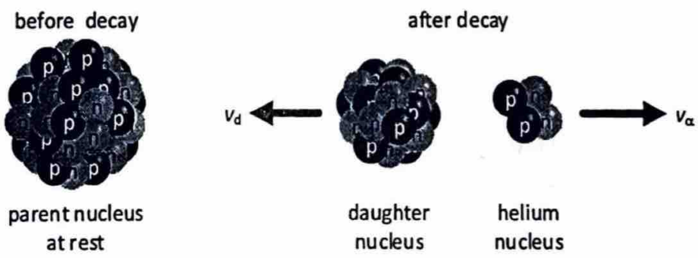
$m_{U\text{-}238} = 238.000281u$; $m_{Th\text{-}234} = 233.994192\ u$; $m_\alpha = 4.001505\ u$

Solution:

$$\Delta E = \frac{0.004584 \times 1.66 \times 10^{-27} \times (3.00 \times 10^8)^2}{1.60 \times 10^{-13}}$$

$$= \mathbf{4.28\ MeV}$$

(b) Since the total momentum of the system is conserved, and since there are only two particles involved in the decay (daughter nucleus and α-particle), these two particles will move in opposite directions after the decay. The two particles are said to recoil from each other.



$$0 = m_\alpha \times v_\alpha + m_d \times -v_d$$

$$\frac{v_\alpha}{v_d} = \frac{m_d}{m_\alpha}$$

$$\frac{\frac{1}{2}m_\alpha v_\alpha^2}{\frac{1}{2}m_d v_d^2} = \frac{m_\alpha \times m_d^2}{m_d \times m_\alpha^2} = \frac{m_d}{m_\alpha}$$

$$\text{K.E.}_\alpha = \frac{m_d}{m_\alpha + m_d} \times \Delta E$$

$$= \frac{233.994192\ u}{4.001505\ u + 233.994192\ u} \times 4.28$$

The daughter nucleus should carry away the rest of the (4.28 – 4.21) = 0.07 MeV of energy.

Example 13 **Beta Decay**

Calculate the energy released in the β decay of helium-6.

Masses of nuclei:

$m_{He\text{-}6} = 6.017788\ u$; $m_{Li\text{-}6} = 6.013475\ u$; $m_e = 0.000549\ u$

Solution:

$${}^{6}_{2}\text{He} \rightarrow {}^{6}_{3}\text{Li} + {}^{0}_{-1}\text{e} + {}^{0}_{0}\bar{\nu}$$

decrease in mass Δm = Mass of He-6 − Mass of Li-6 & electron

= 6.017788 u − (6.013475 u + 0.000549 u)

= 0.003764 u

Energy released

$$\Delta E = \frac{0.003764 \times 1.66 \times 10^{-27} \times (3.00 \times 10^{8})^2}{1.60 \times 10^{-13}}$$

$$= 3.51\ \text{MeV}$$

BACKGROUND RADIATION (needed for H1)

Background radiation is the ionizing radiation in our environment that humans are constantly exposed to. The sources of background radiation can be natural and artificial. A list of sources is given in the table below.

Natural sources	Artificial sources
Air, water & food	Medical imaging
Soil & rock (terrestrial)	Consumer products/materials
Cosmic rays (from outer space)	Nuclear accidents
Internal	

Table 20.3 Sources of background radiation

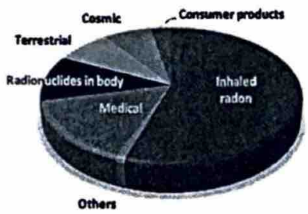


Fig. 20.17 Contributions to background radiation from various sources.

The *Earth* is a **natural** source of background radiation. Radionuclides such as thorium, uranium and radium exist naturally in soil and rock. The decay of these radionuclides produces *radon* – a radioactive gas – which readily seeps out from the ground into underground spaces, buildings and the atmosphere. Radon is by far the biggest source of natural background radiation. Other radioactive sources include underground drinking water, banana, nuts and nearly every natural material.

These nuclides have extremely long half-life (billions of years) and the activity has largely remained constant in the history of mankind.

Food that are radioactive mainly contains potassium and radium.

Cosmic rays from outer space consist of very energetic positively charged ions ranging from protons to iron and even heavier nuclei originating from outside our solar system.

Radioactive nuclides such as potassium-40 and carbon-14 can also be found within *our body* so we are constantly being exposed to ionizing radiations from within. This is known as internal radiation.

Our exposure to the various natural sources of ionizing radiations depends on our location, altitude and even the time of the day.

The most significant source of **artificial** background radiation comes from *medical imaging* – x-rays, computed tomography (CT), positron emission tomography (PET) and single-photon emission computed tomography (SPECT). Radioactive nuclides such as fluorine-18, iodine-123 and technetium-99m are often used as a *tracer*.

Consumer products such as tobacco are radioactive. Heavy smokers are constantly dosed with significant level of radiations to localized spots in the lungs. Construction materials (concrete, stones and granite), petroleum and fuels are also radioactive.

Polonium-210 is found in tobacco products.

The two most infamous *nuclear accidents* occurred in Fukushima, Japan (2011) and Chernobyl, Ukraine (1986) which resulted in significant loss of life and substantial contamination to the immediate vicinity and into the atmosphere. In the Fukushima disaster, millions of litres of contaminated radioactive water were discharged into the Pacific Ocean although the radioactivity level was still deemed to be safe.

Japan is not the only country dumping radioactive waste into the oceans. Many countries with nuclear capabilities have been dumping tons of waste since WW2.

20.6 LAW OF RADIOACTIVE DECAY

LAW OF RADIOACTIVE DECAY

The probabilistic nature of radioactive decay makes it all but impossible to predict which nucleus will decay at a given time. However, since all nuclei of a radioactive nuclide has an equal probability of decaying, it is possible to determine the proportion of the nuclei that will decay in any time interval.

We define the **decay constant** λ as the **probability per unit time** of the decay of a nucleus. According to probability theory, for a sample containing N radioactive nuclei, the **number of radioactive decays per unit time** will be λN. So, the variation with time of the number of undecayed radioactive nuclei dN/dt is given by

Formula

$$\frac{dN}{dt} = -\lambda N \qquad \textbf{(20.14)}$$

Here, the negative sign indicates that dN/dt is negative since N, the number of radioactive nuclei, decreases with time.

We can rewrite **Eqn. (20.14)** in the form

$$\frac{dN}{N} = -\lambda dt$$

which, upon integration, gives

Formula

$$N = N_0 e^{-\lambda t} \qquad \textbf{(20.15)}$$

where N_0 represents the number of undecayed radioactive nuclei at time $t = 0$. This is the **law of radioactive decay**.

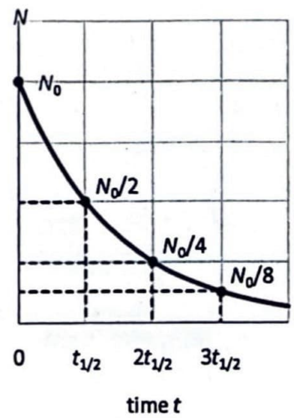


Fig. 20.18 Exponential decrease in the number of radioactive nuclei.

ACTIVITY (not for H1)

The **activity** A of a radioactive source is the **number of radioactive decays per unit time** or **rate of radioactive decay** in the source.

$$A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} = A_0 e^{-\lambda t} \qquad \textbf{(20.16)}$$

where $A_0 = \lambda N_0$ is the activity at time $t = 0$.

Formula

$$A = \lambda N$$

$$A = A_0 e^{-\lambda t}$$

The SI unit of activity is the becquerel (Bq) where

Definition: 1 becquerel = 1 decay s^{-1}

Both N and A decay exponentially with time.

Activity of a radioactive sample is high when its nuclei has a high decay constant λ (high probability of decay) or the number of radioactive nuclei N is large.

COROLLARIES

If a source contains N number of a radioactive nuclei, then the amount of radioactive substance is

$$n = \frac{N}{N_A} \qquad \textbf{(20.17)}$$

and the mass of this radioactive substance is

$$m = N \times m_{atom} \qquad \textbf{(20.18a)}$$

$$= n \times M_{molar} \qquad \textbf{(20.18b)}$$

Molar mass M_{molar} is the mass of 1 mole of the radioactive nuclide.

Therefore **Eqn. (20.15)** can be written in the following forms:

$$n = n_0 e^{-\lambda t} \qquad \textbf{(20.19a)}$$

$$m = m_0 e^{-\lambda t} \qquad \textbf{(20.19b)}$$

where n_0 and m_0 are the initial amount of radioactive nuclei and mass of radioactive nuclide, respectively.

[According to learning outcome (q), we can use the equation

Formula $x = x_0 e^{-\lambda t}$

where x can represent activity A, number of undecayed atoms N, count rate C, number of moles n of radioactive substance or mass m of undecayed atoms (undecayed mass)].

HALF-LIFE

The **half-life** $t_{1/2}$ of a radioactive nuclide is the time taken for the **number of undecayed nuclei** to be **reduced to half its original number**. Suppose the initial number of radioactive nuclei is N_0, then after one half-life, only half of the original nuclei remains. So

$$\tfrac{1}{2} N_0 = N_0 e^{-\lambda t_{1/2}}$$

Formula ⇒ $$t_{1/2} = \frac{\ln 2}{\lambda} \qquad \textbf{(20.20)}$$

Decay constant λ and half-life $t_{1/2}$ are inversely related. So, activity of a radioactive sample is high when its half-life is short.

So the activity of a radioactive sample can be written as

$$A_0 = \lambda N_0 = \frac{\ln 2}{t_{1/2}} \times N_0 \qquad \textbf{(20.21)}$$

If $t_{1/2}$ is the time for half the number of radioactive nuclei to decay, does it imply that all radioactive nuclei would decay in two half-lives?

The answer is NO! In the first half-life, one-half of the original nuclei will decay and $\tfrac{1}{2} N_0$ undecayed radioactive nuclei are left. After another half-life, half of those remaining will decay and $\frac{1}{4}N_0$ are left. So, after n half-lives, the number of radioactive nuclei left is

Formula $N = N_0\left(\frac{1}{2}\right)^n = N_0\left(\frac{1}{2}\right)^{t/t_{1/2}}$ **(20.22)**

where n can be an integer or non-integer and t is the time elapsed.

Definitions:

The **activity** of a radioactive source is the number of radioactive decays per unit time in the source.

The **decay constant** is the probability per unit time of the decay of a nucleus.

Half-life $t_{1/2}$ of a radioactive nuclide is the time taken for the number of undecayed nuclei to be reduced to half its original number.

Example 14 Rate of Energy Released (not for H1)

The decay constant of uranium-238 ($^{238}_{92}U$) is 1.55×10^{-10} yr^{-1}. Using your answer to **Example 12** (pg. 29), determine the rate at which energy is released in watts when 1.00 g of $^{238}_{92}U$ decays.

The activity of $^{238}_{92}U$ remains constant for an appreciable amount of time due to its very long half-life.

The energy released per alpha decay of $^{238}_{92}U$ is 4.28 MeV.

- No. of U nuclei N in 1.00 g $= \dfrac{\text{mass of sample}}{\text{mass of U atom}}$

  $= \dfrac{1.00 \times 10^{-3}}{238 \times 1.66 \times 10^{-27}} = 2.53 \times 10^{21}$

- Activity $A = \lambda \times N$

  $= \dfrac{1.55 \times 10^{-10}}{365 \times 86400} \times 2.53 \times 10^{21}$

  * 1 day = 86400 s

  $= 12400$ Bq

- Power $P = 12400 \times 4.28$

  $= 53200$ MeV s^{-1}

  $= 8.51 \times 10^{-9}$ W

Use $m_{\text{U-238 atom}} \approx 238\,u$ when no data is given.

$1\text{ MeV} = 1.60\times10^{-13}$ J

Example 15 Half-life (not for H1)

At time $t = 0$, a radioactive sample contains 3.50 μg of pure carbon-11 ($^{11}_{6}C$), which has a half-life of 20.4 min. Determine

(a) N_0, the initial number of nuclei in the sample,

(b) the activity A of the sample after 8.00 h.

$$\text{Initial no. of C nuclei } N_0 = \frac{\text{mass of sample}}{\text{mass of C atom}} = \frac{3.50 \times 10^{-9}}{11 \times 1.66 \times 10^{-27}}$$

$3.50\ \mu g = 3.50 \times 10^{-9}$ kg.

$m_{C\text{-}11\ atom} \approx 11\ u$

$$= 1.92 \times 10^{17}$$

$$\text{Initial activity } A_0 = \frac{ln2}{t_{1/2} \text{ of C}} \times N_0$$

$$= \frac{ln2}{20.4 \times 60} \times 1.92 \times 10^{17}$$

$$= 1.08 \times 10^{14} \text{ Bq}$$

$$A = A_0 \times e^{-\lambda t}$$

$$= 1.08 \times 10^{14} \times e^{-(ln2/20.4) \times 8.00 \times 60}$$

$$= 8.96 \times 10^{6} \text{ Bq}$$

CARBON DATING

Carbon dating is a technique commonly used to determine the age of organic archeological sample. Nuclear reactions due to cosmic radiations in the upper atmosphere create radioactive carbon-14 ($^{14}_{6}C$) isotope. The ratio of carbon-14 to carbon-12 in carbon dioxide molecules in our atmosphere has a constant value of 1.30×10^{-12}. The carbon atoms in all living organism have this same ratio because all organisms continuously exchange carbon dioxide with their surroundings. When an organism dies, however, it no longer absorbs carbon-14 from the atmosphere and so this ratio decreases as carbon-14 decays with a half-life of 5730 yr.

This means that there is one carbon-14 atom to 7.69×10^{11} carbon-12 atoms.

Using this technique, scientists have been able to identify samples of wood, charcoal, bone and shell between 1000 to 25000 years old. The limit of carbon dating is approximately 60000 years.

Example 16 Carbon Dating (calculation of *Activity* not for H1)

A living specimen contains one atom of carbon-14 ($^{14}_{6}C$; $t_{1/2}$ = 5730 yr) for every 7.69×10^{11} stable carbon-12 ($^{12}_{6}C$). An archaeological sample of wood contains 21.0 mg of carbon. The activity of the sample is measured to be 120 day^{-1}. Determine the age of the sample of wood.

Solution

$$N_{C12} = \frac{21.0\times10^{-6}}{12\times1.66\times10^{-27}} = 1.05\times10^{21}$$

$$N_{C14} = \frac{1.05\times10^{21}}{7.69\times10^{11}} = 1.37\times10^{9}$$

$m_{C\text{-}12\ atom} = 12\ u$. We use mass of carbon-12 to compute N because ratio of carbon-14 to carbon 12 is extremely minuscule.

We should compare the activity of the same mass of carbon in a living specimen and in an archaeological sample.

$$A_0 = \frac{ln2}{t_{1/2}\text{ of C14}} \times N_0 = \frac{ln2}{5730\times365}\times 1.37\times10^{9} = 454\ \text{day}^{-1}$$

$$t = \frac{ln\left(\frac{454}{120}\right)}{ln2/5730} = 11000\ \text{year}$$

Correcting background count rate from observed count rates

In any experiment on radioactivity, the measured count rates should be corrected for the background count rate since background radiation is always present. The count rate due to the radioactive nuclide alone, also known as the true count rate, is the difference between the observed count rate and the background count rate.

True Count = Observed Count – Background Count

Effects of ionising radiation on living tissues and cells

Ionizing radiation can cause cancer and other health effects by breaking chemical bonds in atoms and molecules. Damage occurring at the cellular or molecular level can disrupt the processes that control the rate at which cells grow and replace themselves and permit the uncontrolled growth of cancer cells. Acute health effects include burns and radiation sickness. Radiation sickness can cause premature aging or even death.	***See Appendix** for greater details on effects.

APPENDIX

THE NUCLEAR REACTOR

The extensive use of fossil fuels (coal, oil or gas) to generate electrical power has resulted in elevated global temperature and caused alarming rise in sea level and unpredictable weather patterns. Furthermore, the rapid depletion of the fossil fuels has made it imperative for mankind to explore alternative sources of power.

Alternative sources of power include solar, wind, geothermal hydroelectric and nuclear.

To satisfy the enormous energy needs of mankind, we have been harnessing nuclear power for decades. With major advances in technology and design, nuclear power plants have been made safer but the Chernobyl accident in 1986 and the Fukushima disaster in 2011 are painful reminders of what can happen when things go wrong.

Typical nuclear power plant generates 3000 MW of power.

Advantages of nuclear power

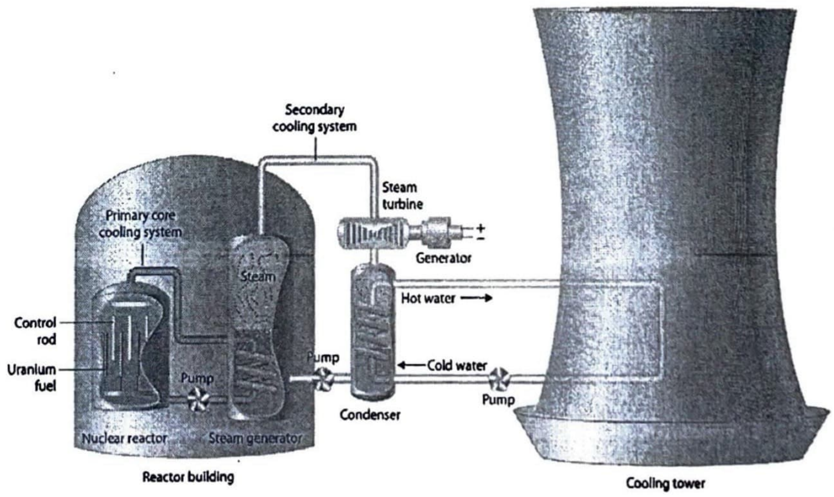
- Significantly lower CO_2 emission than fossil-fuel power plant
- Relatively low fuel consumption (30 tons per year of uranium vs. 9000 tons per year of fossil fuel per power plant)
- Abundant amount of uranium in earth crust
- Spent uranium can be enriched and re-used
- Low operating cost

Disadvantages of nuclear power

- Risk of meltdown and the catastrophic consequences
- Storage of increasing amount of nuclear waste
- Nuclear power plant are expensive to build and decommission
- Proliferation of nuclear weapon from enriched fuel

In this section, we will further explore the inner workings of pressurized-water reactor (PWR) power plant.

The other type being boiling water reactor and supercritical water reactor.



A schematics of a pressurized-water reactor.

In Section 20.4, we learnt that heavy nuclei such as uranium-235 ($^{235}_{92}U$) or plutonium-239 ($^{239}_{94}Pu$) can be made to fission by bombarding with slow thermal neutron. In the process, the neutrons released can cause further fission of the heavy nuclei. This is known as nuclear chain reaction.

Energy of a fast neutron is between 1-2 MeV while that of slow thermal neutron is 0.025 eV.

A typical nuclear reactor may contain as many as 150 to 250 assemblies of 200 to 300 fuel rods. Each of these rods consists of ceramic pallets of enriched uranium dioxide. So each nuclear reactor may contain as much as 80 to 100 tons of uranium.

During fission of the uranium nuclei, a tremendous amount of heat is released and more energetic neutrons are produced. However, these neutrons are too fast to effectively cause further fission so they must be slowed down sufficiently with a moderator (water surrounding the fuel rods served as a moderator and a coolent to prevent a melt down). As fast neutrons pass through the moderator, they experience multiple collisions with the hydrogen atoms in the water. As a result, these neutrons lose energy and slow down.

Most modern nuclear reactors use light water (H_2O) as moderator, but carbon graphite or heavy water (D_2O) may be used instead.

An important parameter describing the operation of a nuclear reactor is the reproduction constant *K*, defined as the average number of neutrons from each fission event that cause another fission event. In a typical uncontrolled fission of uranium-235, an average of 2.5 neutrons are released from each fission event, i.e., $K=2.5$. For sustained and controlled chain reaction, it is important to ensure *K* is close to 1, i.e., the excess neutrons must be removed from the reactor using control rods. The depth at which the control rods are inserted into the nuclear reactor will control the rate of nuclear fission; deeper to reduce the rate of reaction, shallower to increase the rate of reaction.

When $K>1$, the reactor is supercritical and a nuclear meltdown may occur. When $K<1$, the reactor is subcritical and the reaction will eventually dies out.

Now the heat released must be converted to electrical energy. In PWR, water in the reactor is heated to 588 K by the fuel rod. This heated water remains liquid due to the extremely high pressure of 15.5 Mpa in the primary cooling system. It is then pump through a heat exchanger (steam generator) where heat is conducted through the copper piping to turn water in the secondary cooling loop into steam. The steam then drives the steam turbine which in turn drives an electrical generator. It then passes through the condenser where it condenses back to water and before being circulated back to the heat exchanger.

Control rods are usually made of boron, silver, indium and cadmium which are capable of absorbing neutron without undergoing nuclear fission.

One safety feature of PWR is its use of water as a moderator. Neutrons are more effectively "moderated" as the neutrons will experience more frequent collisions with water molecules. However, any increase in temperature of the reactor will cause water to expand, thereby reducing its ability to moderate the neutrons and decreasing the reactivity of the reactor. Therefore, the reactor will stablise itself to a temperature set by the position of the control rods. This makes PWR inherently safer compared to other reactor design.

In the heat exchanger, water in the secondary cooling system are never in contact with the primary cooling system and hence is not contaminated by any radioactive material.

Derivation of Equation (20.5) -- $E = \text{B.E}_{\text{products}} - \text{B.E}_{\text{reactants}}$

We will use the **fission reaction** in **Example 9**:

$$^{239}_{94}\text{Pu} + ^{1}_{0}\text{n} \rightarrow ^{143}_{56}\text{Ba} + ^{95}_{38}\text{Sr} + 2^{1}_{0}\text{n}$$

The **decrease** in mass in the reaction is

$$= (m_{Pu} + m_n) - (m_{Ba} + m_{Sr} + 2m_n) = m_{Pu} - m_{Ba} - m_{Sr} - m_n \quad \text{..... (1)}$$

$$But,\ m_{Pu} = (239-94)m_n + 94m_p - \frac{BE_{Pu}}{c^2} = 145m_n + 94m_p - \frac{BE_{Pu}}{c^2}$$

$$and,\ m_{Ba} = (143-56)m_n + 56m_p - \frac{BE_{Ba}}{c^2} = 87m_n + 56m_p - \frac{BE_{Ba}}{c^2}$$

$$and,\ m_{Sr} = (95-38)m_n + 38m_p - \frac{BE_{Sr}}{c^2} = 57m_n + 38m_p - \frac{BE_{Sr}}{c^2}$$

$$\Rightarrow\ eqn(1) = -\frac{BE_{Pu}}{c^2} + \frac{BE_{Ba}}{c^2} + \frac{BE_{Sr}}{c^2}$$

Therefore, the **energy released** is

$$E = (m_{\text{reactants}} - m_{\text{products}})c^2 = BE_{Ba} + BE_{Sr} - BE_{Pu} = \text{BE}_{\text{products}} - \text{BE}_{\text{reactants}}$$

DANGER OF NEUTRON RADIATION

Neutrons are high-speed subatomic particles that have an exceptional ability to penetrate other materials, including human bodies. Amongst the different types of ionizing radiations (such as α, β and γ radiations), neutrons are the only ones capable of turning a nucleus radioactive. This process, called neutron activation, is used to produce many of the radioactive sources that are used in medical, academic, and industrial applications (including oil exploration).

Besides its ability to transform our tissues into a radioactive mess, neutron is also highly capable of knocking out protons from the hydrogen atom in water molecules and thereby breaking its chemical bonds. This proton then ionizes any atoms along its path, causing irreparable damage to cells. Damaged cells may die or become abnormal which can result in cancer. Because our body is composed mostly of water, neutrons is up to 10 times more damaging compared to beta radiations or gamma-rays.

Because of their exceptional ability to penetrate other materials, neutrons can travel great distances in air and require very thick hydrogen-containing materials (such as concrete or water) to block them. Fortunately, however, neutron radiation primarily occurs inside a nuclear reactor, where many feet of water provide effective shielding.

Source: https://www.nrc.gov/about-nrc/radiation/rad-health-effects.html

https://en.wikipedia.org/wiki/Neutron_radiation

RADIATION DETECTORS

A radiation detector is a device used to detect, track and/or identify ionizing particles such as those produced by radioactive decay or cosmic radiation. In this section, we will briefly discuss the operation of a few commonly used radiation detectors, namely, the Wilson cloud chamber, the Geiger-Müller tube and the scintillation counter.

THE WILSON CLOUD CHAMBER

Invented by Charles Wilson in 1911, a diffusion cloud chamber is a particle detector that can show the path of a passing ionizing radiation (see **Fig. 20.15** on pg. 4).

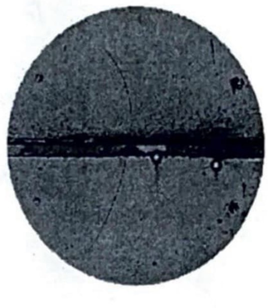


Fig. 20.21 Cloud chamber photo that proved the existence of positron.

A cloud chamber consists of a sealed chamber with a large temperature gradient between the top plate (at room temperature) and cold bottom plate. The bottom plate is cooled to about −78°C by a layer of dry ice beneath it. A felt disc, soaked with isopropyl alcohol, is placed near the top plate. The alcohol then vaporizes and diffuses downwards. Near the bottom plate, the alcohol vapour condenses but just above the bottom plate, a layer of supersaturated alcohol vapour is formed.

A radioactive source, placed inside this chamber, emits ionizing radiation. These ionizing radiations produce ions as they pass through the supersaturated vapour. Since alcohol molecules are polar, the supersaturated vapour is attracted towards the ions and condensation occurs. These condensations produced a trail of visible alcohol droplets which represents the path of the radiations.

Fig. 20.22 A schematic diagram of a cloud chamber.

A video of a modern cloud chamber: www.youtube.com/watch?v=LosCtlh5Flc

Invented by Donald Glaser in 1952, the bubble chamber works on a similar principle as a cloud chamber except that it uses a super-heated and super-saturated liquid. The bubble chamber was the detector of choice for many decades and superseded the cloud chamber as it is capable of tracking more energetic ionizing radiations.

THE GEIGER-MÜLLER TUBE

A Geiger-Müller (GM) tube is a low cost and widely used radiation detector. Invented by Hans Geiger and Walther Müller in 1928, it consists of a metal chamber filled with a low-pressure inert gas such as argon. A potential difference of a few hundred volts is applied between the wall of the tube (cathode) and a thin wire in the centre (anode).

A Geiger-Müller (GM) counter.

When ionizing radiation enters the tube through the mica window, it ionizes the argon atoms to create pairs of positively charged ions and electrons. The strong electric field accelerates the positive ions towards the cathode and electrons towards the anode. Close to the anode, the electrons gain sufficient energies to cause additional ionization of the argon atoms. This creates an electron avalanche and a large output pulse is detected from a single ionizing radiation. The disadvantage of a GM counter is that it is unable to measure the energy of the ionizing radiation or distinguish the type of ionizing radiation.

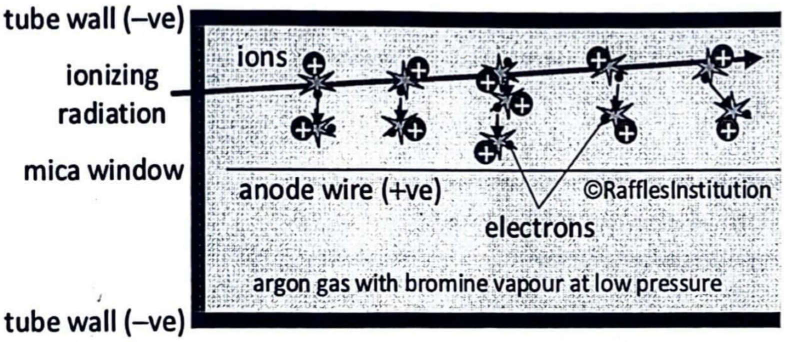


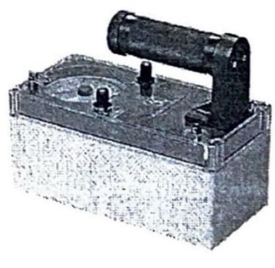
Fig. 20.23 A schematic diagram of a Geiger-Müller tube.

Meanwhile, the positively charged ions acquire electron to become argon atoms upon reaching the cathode. These atoms may subsequently emit photons which can cause further ionization and produce undesirable secondary pulses. Hence, a quenching agent, such as bromine, may be added to minimize secondary pulses. The addition of a quenching agent, however, renders the tube inactive for a short period of time. During this "dead time", the tube is unable to detect the arrival of new ionizing radiation, hence limiting the accuracy and the maximum count rate.

The GM tube is connected to a counter (to measure cumulative count) or a rate-meter (to measure count rate).

THE SCINTILLATION COUNTER

A scintillation counter consists of a scintillator and a photomultiplier. Invented by Sir Samuel Curran in 1944, a scintillator emits photons when ionizing radiation strikes the scintillating crystal. These photons cause the emission of electrons (photoelectric effect) at the photocathode. They are then electrostatically accelerated towards the first dynode of the photomultiplier tube. On impact with the dynode, a single electron releases a number of secondary electrons which are in turn accelerated to strike the second dynode. At each dynode, more electrons are released so that the current is amplified. The resultant output is an electrical pulse which is passed to the processing electronics.



A scintillation counter.

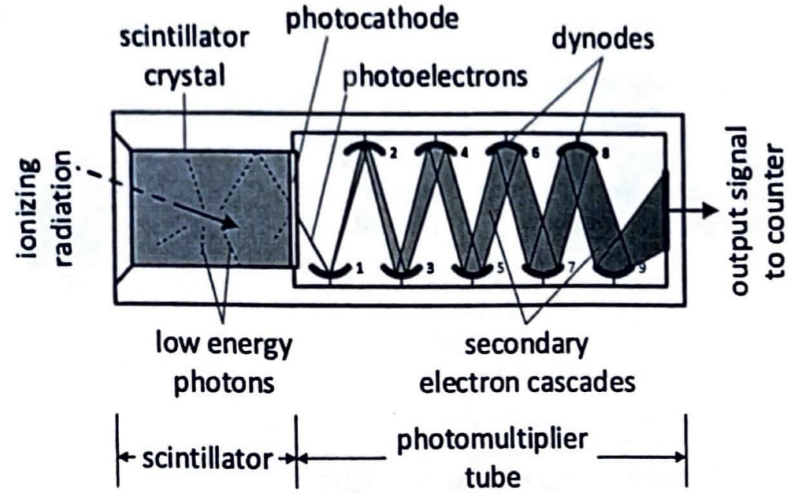


Fig. 20.24 A schematic diagram of a scintillation counter.

Unlike a GM tube, a scintillator can measure both the intensity and the energy of the incident radiation. It can also distinguish between different ionizing radiation through the use of different scintillating crystal – zinc sulphide for α-particles, plastic scintillators for β-particles and sodium iodide for γ-ray photons.

EXPERIMENTAL DETERMINATION OF DECAY CONSTANT

The decay constant or half-life of a radioactive nuclide is usually determined experimentally. The technique described here is suitable for the determination of half-lives which range from a few minutes to a few days.

Alternative technique is required for determination of very short half-lives (< milli-second) or very long half-lives (> months or years).

Before measuring the count rate of a radioactive source, we first measure the count rate C_B due to **background radiation** with a detector, such as Geiger-Müller (GM) tube connected to a rate-meter. This background count rate C_B must be subtracted from the total count rate C_T when a radioactive source is placed at a fixed position in front of the GM tube.

The count rate *C* is a **fraction** of the activity *A* of a radioactive sample as radiations of the decay are emitted in all directions and not all radiations are counted by the detector.

$$C_S = C_T - C_B \qquad \textbf{(20.23)}$$

Once the count rate C_S due to the source is computed, we can then proceed with one of two ways to determine the decay constant.

The variation with time *t* of the count rate C_S due to one such radioactive source is shown in **Fig. (20.19)**.

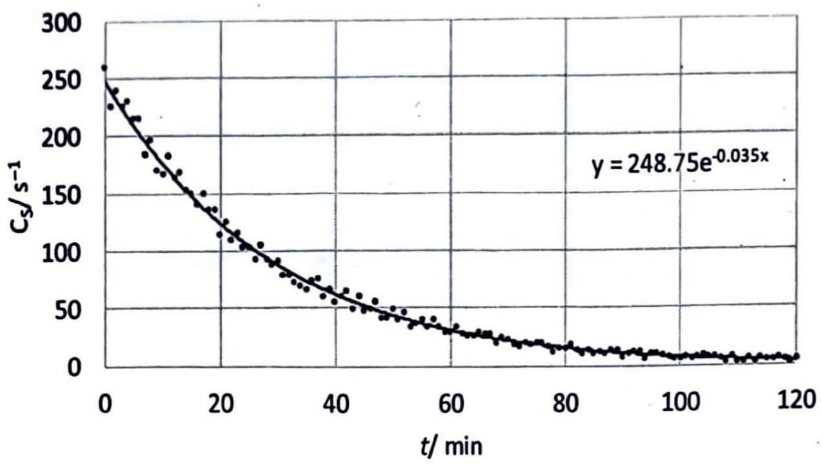


Fig. 20.19 The variation with time *t* of the count rate c_s .

The tabulated values are not shown due to the large number of data points.
How do you tell from the graph that radioactive decay is random in nature ?

In **Fig. 20.19**, an exponential line of best fit is drawn since we know that the count rate C_S varies exponentially with time,

$$C_S = C_{S0} e^{-\lambda t} \qquad \textbf{(20.24)}$$

We read off C_S at two specific times, say, at $t=0$ and $t=60\,\text{s}$,

$$C_{S0} = 250\ \text{s}^{-1};\ C_S = 30\ \text{s}^{-1}$$

Using Eqn. (20.24), we then determine that the decay constant is $-0.035\ \text{min}^{-1}$ and the half-life of radioactive source is 20 min.

Taking natural logarithm on both sides of **Eqn. (20.24)**, we obtain

$$\ln C_S = \ln C_{S0} - \lambda t$$

So a straight line with a gradient of $-\lambda$ would be obtained if $\ln C_S$ is plotted against time *t*.

Fig. (20.20) shows the variation with time *t* of $\ln C_S$ for the same data.

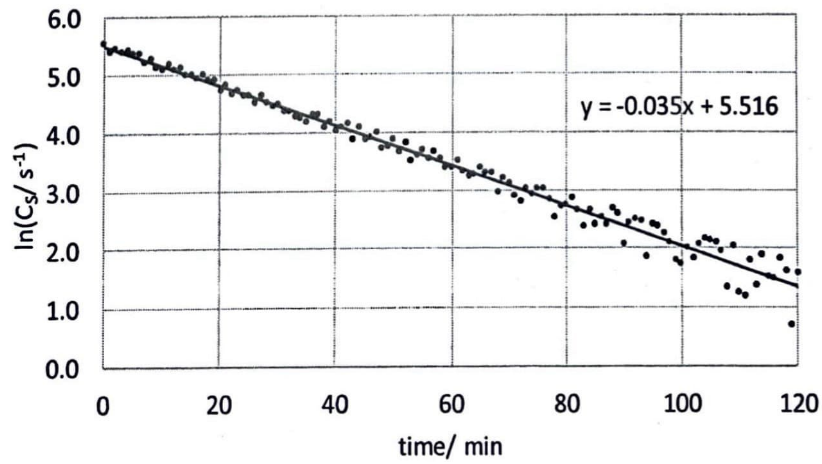


Fig. 20.20 The variation with time *t* of $\ln c_s$.

Do you notice that the scattering of the data points increases with time? This shows that the percentage uncertainty of the count rate increases with decreasing count rate.

STOCHASTIC (RANDOM) HEALTH EFFECTS (needed for H1)

Stochastic effects are associated with long-term, low-level (chronic) exposure to radiation. Increased levels of exposure make these health effects more likely to occur, but do not influence the type or severity of the effect.

Cancer is considered by most people the primary health effect from radiation exposure. Simply put, cancer is the uncontrolled growth of cells. Ordinarily, natural processes control the rate at which cells grow and replace themselves. They also control the body's processes for repairing or replacing damaged tissue. Damage occurring at the cellular or molecular level, can disrupt the control processes, permitting the uncontrolled growth of cancer cells. This is why ionizing radiation's ability to break chemical bonds in atoms and molecules makes it such a potent carcinogen.

Other stochastic effects also occur. Radiation can cause changes in DNA, the "blueprints" that ensure cell repair and replacement which produces a perfect copy of the original cell. Changes in DNA are called mutations.

Sometimes the body fails to repair these mutations or even creates mutations during repair. The mutations can be teratogenic or genetic. Teratogenic mutations are caused by exposure of the fetus in the uterus and affect only the individual who was exposed. Genetic mutations are passed on to offspring.

NON-STOCHASTIC HEALTH EFFECTS

Non-stochastic effects appear in cases of exposure to high levels of radiation, and become more severe as the exposure increases. Short-term, high-level exposure is referred to as 'acute' exposure.

Many non-cancerous health effects of radiation are non-stochastic. Unlike cancer, health effects from 'acute' exposure to radiation usually appear quickly. Acute health effects include burns and radiation sickness. Radiation sickness is also called 'radiation poisoning.' It can cause premature aging or even death. If the dose is fatal, death usually occurs within two months. The symptoms of radiation sickness include: nausea, weakness, hair loss, skin burns or diminished organ function.

Medical patients receiving radiation treatments often experience acute effects, because they are receiving relatively high "bursts" of radiation during treatment.

EFFECTS OF RADIATION TYPE AND EXPOSURE PATHWAY

Both the type of radiation to which the person is exposed and the pathway by which they are exposed influence health effects. Different types of radiation vary in their ability to damage different kinds of tissue. Radiation and radiation emitters (radionuclides) can expose the whole body (direct exposure) or expose tissues inside the body when inhaled or ingested.

All kinds of ionizing radiation can cause cancer and other health effects. The main difference in the ability of alpha and beta particles and gamma and x-rays to cause health effects is the amount of energy they can deposit in a given space. Their energy determines how far they can penetrate into tissue. It also determines how much energy they are able to transmit directly or indirectly to tissues and the resulting damage.

Although an alpha particle and a gamma ray may have the same amount of energy, inside the body the alpha particle will deposit all of its energy in a very small volume of tissue. The gamma radiation will spread energy over a much larger volume. This occurs because alpha particles have a mass that carries the energy, while gamma rays do not.

Source: http://www.epa.gov/rpdweb00/understand/health_effects.html

ADDITIONAL RESOURCES

Refer to the following links for more in depth discussion:
http://en.wikipedia.org/wiki/Nuclear_reactor
http://en.wikipedia.org/wiki/Nuclear_weapon
ATLAS – The Particles Strike Back
https://www.youtube.com/watch?v=iYRQpcJVQx8

20 NUCLEAR PHYSICS TUTORIAL

H2 PHYSICS 9749

Data

speed of light $c = 3.00 \times 10^{8}\ \text{m s}^{-1}$

elementary charge $e = 1.60 \times 10^{-19}\ \text{C}$

unified atomic mass unit $u = 1.66 \times 10^{-27}\ \text{kg}$

mass of electron $m_e = 0.000549\ u$

mass of proton $m_p = 1.007276\ u$

mass of neutron $m_n = 1.008665\ u$

SELF-CHECK QUESTIONS

Nuclear Reactions

S1 What are the observations and conclusions of Rutherford's alpha-particle scattering experiment?

S2 Distinguish between mass number *A* and atomic number *Z*.

S3 Distinguish between nuclides and isotopes.

S4 What is mass defect and binding energy of a nucleus? How are the two quantities related?

S5 What is binding energy per nucleon and how is binding energy per nucleon related to the stability of nuclei?

S6 What must be conserved in any nuclear reaction?

S7 Explain how fission of heavier nucleus or fusion of lighter nuclei leads to a release of energy.

Radioactivity

S8 What is meant by the *random* and *spontaneous* nature of radioactive decay?

S9 How does the ratio of number of neutrons to protons affect the nuclear stability for (i) light elements, (ii) intermediate elements and (iii) heavy elements?

S10 What are the characteristics of the three main types of radiation?

S11 What is background radiation? How do we deal with background radiation in an experimental determination of half-life of a radioactive nuclide?

S12 State the universal law of radioactive decay.

S13 Define activity and state its relationship with the number of undecayed radioactive nuclei.

S14 Define half-life of a radioactive sample.

S15 State the uses of nuclear reactions and radioactivity. Discuss the possible health hazards of ionizing radiations.

SELF-PRACTICE QUESTIONS

SP1 Which of the following conclusions can be deduced from Rutherford α-particles scattering experiment?

(1) Electrons exist in different orbits.
(2) The α-particles are helium nuclei.
(3) The positive nucleus in an atom is confined in a very small region.

A (1) only

B (1) & (2) only

C (3) only

D (1), (2) and (3)

SP2 Which pair of nuclides has nuclei containing the same number of neutrons?

A $^{107}_{47}Ag$ and $^{104}_{45}Rh$

B $^{109}_{47}Ag$ and $^{109}_{46}Pd$

C $^{108}_{46}Pd$ and $^{109}_{47}Ag$

D $^{105}_{45}Rh$ and $^{106}_{45}Rh$

[N10/1/39]

SP3 The nucleus of the nuclide $^{A}_{Z}X$ has mass M. In terms of the rest mass of the proton m_p and the rest mass of the neutron m_n, what is the binding energy per nucleon of this nucleus?

A $\left(\dfrac{Am_p + Zm_n - M}{Z}\right)c^2$

B $\left(\dfrac{M - Zm_p - Am_p}{A}\right)c^2$

C $\left(\dfrac{M - Zm_p - (A-Z)m_n}{Z}\right)c^2$

D $\left(\dfrac{Zm_p + (A-Z)m_n - M}{A}\right)c^2$

[N98/1/29]

SP4 Calculate the binding energy per nucleon of

(a) deuteron (nucleus of deuterium) if the atomic mass of the deuterium is 2.014102 *u*.

(b) $^{65}_{29}Cu$ nucleus with an atomic mass of 64.927790 *u*.

SP5 High energy alpha-particles can transform nitrogen-14 to oxygen-17:

$$^{14}_{7}N + ^{4}_{2}He \rightarrow ^{17}_{8}O + ^{1}_{1}H$$

The sum of the rest masses of the nitrogen and helium nuclei is 18.006 *u*. The sum of the rest masses of the oxygen and hydrogen nuclei is 18.007 *u*. The energy equivalent of 0.001 *u* is 1 MeV.

What do the data show?

A Mass of 0.001 *u* has been converted into 1 MeV of energy.

B The kinetic energy of the products exceeds the kinetic energy of the reactants by 1 MeV.

C The kinetic energy of the reactants exceeds the kinetic energy of the products by 1 MeV.

D The reactants had only 1 MeV of kinetic energy and all of this was converted into mass.

[N01/1/29]

SP6 Explain how the principle of conservation of energy in β decay were used to predict the existence of the neutrino.

SP7 The following information concerns a sample of a certain radioisotope:

The activity at time zero is A_0; The activity at time *t* is *A*;

The number of undecayed nuclei at time *t* is *N*;

The decay constant is λ; The half-life is $t_{1/2}$.

Which relationship is not correct?

A $A = A_0 \exp(-\lambda t)$ **B** $A = \lambda N$

C $A = t_{1/2} N$ **D** $N = 1.44 t_{1/2} A$

[N98/1/30]

SP8 Which sample of nuclide has the greatest initial activity?

	nuclide	amount/ mole	half-life/ day
A	$^{225}_{89}Ac$	0.003	10
B	$^{228}_{90}Th$	0.1	400
C	$^{228}_{88}Ra$	0.6	2100
D	$^{241}_{94}Pu$	1.0	4800

[N09/1/40]

SP9 The initial activity of a sample of a radioactive isotope containing N_0 nuclei is A_0. What fraction of the original number of nuclei has decayed when the activity has declined to $A_0/3$?

A 1/3 **B** 1/2 **C** 2/3 **D** 1/8

SP10 The decay constant and the half-life are related by the equation $\lambda = \ln 2 / t_{1/2}$. The half-life of $^{60}_{27}Co$ is 5.26 years, calculate

(a) the decay constant of $^{60}_{27}Co$,

(b) the activity of 1.00 g of $^{60}_{27}Co$. Molar mass of $^{60}_{27}Co$ is 60 g mol^{-1}.

[J94/2/7]

SP11 Samples of two radioactive nuclides, **X** and **Y**, each has equal activity A_0 at time $t = 0$. **X** has a half-life of 24 years and **Y** has a half-life of 16 years. The samples are mixed together. What will be the total activity of the mixture after 48 years?

A $\frac{1}{12}A_0$ **B** $\frac{3}{16}A_0$ **C** $\frac{1}{4}A_0$ **D** $\frac{3}{8}A_0$

[N03/1/30]

SP12 A source contains 8.9×10^{16} radioactive nuclei and has an activity of 5.7×10^{7} Bq. Calculate

(a) its decay constant,

(b) its half-life,

(c) the time taken for its activity to fall to 6.0×10^{5} Bq.

DISCUSSION QUESTIONS

Nuclear Reactions

D1 (a) In the α-particle scattering experiment, α-particles, travelling in a vacuum, are incident on a gold foil. Sketch diagrams to illustrate the path of an α-particle if its original path

(i) is directly towards the nucleus of a gold atom,

(ii) passes close to the nucleus of a gold atom,

(iii) passes some distance from the nucleus.

(b) Describe and explain how the α-particle scattering experiment which you have illustrated in part (a) gives evidence for the existence and small size of the nucleus.

D2 During his discovery of the neutron in 1932, James Chadwick, determined the mass of the newly identified particle by firing a beam of fast neutrons towards known targets. As the speed of the neutrons cannot be determined directly, he directed a beam of neutrons (all having the same speed), at two different targets and measuring the maximum recoil speeds of the target nuclei.

(a) Explain why the speed of neutrons could not be determined directly.

(b) Explain the origin of the maximum recoil speed of a target nucleus.

(c) Let the masses of the two target nuclei be m_1 and m_2 and the finals speeds of the target nuclei be v_1 and v_2, respectively.

Show that the mass of the neutron can be calculated from the equation

$$m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(d) Use the following data to calculate the mass of the neutron.

$m_p = 1.00\ u$; $m_N = 14.00\ u$, $v_p = 3.30 \times 10^7$ m s^{-1}, $v_N = 4.70 \times 10^6$ m s^{-1}

Masses of a proton and nitrogen nuclei to be 1.00 *u* and 14.0 *u*, respectively,

D3 When a deuteron (nucleus of deuterium) of mass 2.013553 *u* and negligible kinetic energy is absorbed by a Lithium-6 nucleus of mass 6.013476 *u*, the compound nucleus disintegrates spontaneously into two α-particles, each of mass 4.001505 *u*.

Calculate the energy, in joules, of each α-particle.

D4 When $^{235}_{92}U$ is bombarded by slow neutrons, $^{141}_{56}Ba$ and $^{92}_{36}Kr$ may be produced.

(a) Write the equation for this nuclear reaction.

(b) Name this type of nuclear reaction.

(c) If the binding energy per nucleon of $^{235}_{92}U$ is 7.6 MeV and that of both $^{141}_{56}Ba$ and $^{92}_{36}Kr$ is 8.5 MeV, show that 195 MeV of energy is released in one such reaction.

(d) Suggest two forms of energy which the energy released in (c) is transformed into.

[Modified J97/2/7]

D5 The nuclear fusion of deuterium-2 (^2_1H) to form helium-4 is proposed to produce electrical power. The nuclear reaction is given by

$$^2_1H + ^2_1H \rightarrow ^4_2He + \text{energy}$$

The atomic masses of deuterium-2 and helium-4 are 2.014102 *u* and 4.002603 *u* respectively.

(a) Calculate the mass defect of the reaction.

(b) Calculate the energy released from 1.0 g of deuterium.

(c) What is the mass of deuterium needed each day if the required power output of the fusion power plant is 100 MW and the efficiency of electricity generation is 40%?

Radioactivity

D6 **(a)** The radioactive decay of the isotope cobalt-60 (beta emitter) is both spontaneous and random.

Explain what is meant by

(i) spontaneous decay,

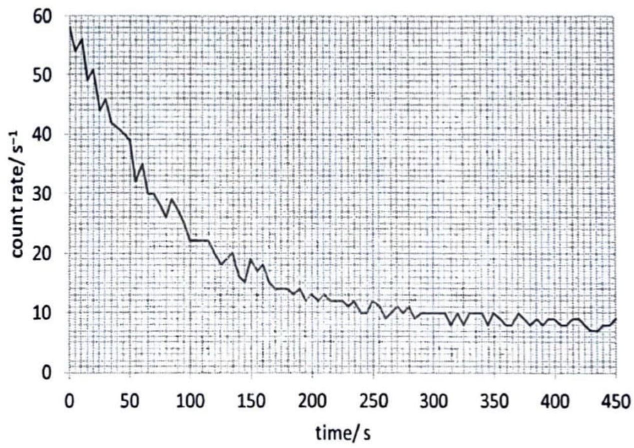
(ii) random decay.

(b) A student has a sample of cobalt-60 – a beta emitter – inside a box. The student defines the half-life of cobalt-60 as the time taken for the number of nuclei inside the box to decay to one half of its initial value.

State and explain one reason why this definition is inappropriate.

[Modified N09/3/4]

D7 An experiment is carried out in which the count rate is measured at a fixed distance from a sample of a certain radioactive material. The figure shows the variation of count rate with time that was obtained.



(a) Explain how you dealt with the problems of

(i) the random nature of the count rate,

(ii) the background radiation.

(b) Use the graph to estimate the half-life of the material.

[Modified N91/2/7]

D8 The hospital treatment of internal tumours requires the use of 2.0 mg of the radionuclide $^{60}_{27}Co$. The half-life of $^{60}_{27}Co$ is 5.3 years and each decay emits an electron of average energy 0.12 MeV followed by γ rays of total energy 2.5 MeV.

(a) **(i)** What is meant by the term "decay constant"?

(ii) Determine the decay constant of $^{60}_{27}Co$.

(b) Calculate

(i) the number of nuclei in 2.0 mg of $^{60}_{27}Co$,

(ii) the initial activity of $^{60}_{27}Co$,

(iii) the initial power output from 2.0 mg of $^{60}_{27}Co$,

(iv) the power output from 2.0 mg of $^{60}_{27}Co$ after 21.2 years.

(c) Explain why the energy absorbed by the internal organs in the body is smaller than the energy emitted by the source.

(d) Nowadays, linear accelerators emitting "megavolt" x-rays are used instead of $^{60}_{27}Co$. Suggest two reasons why linear accelerators may be preferred.

[N88/2/12]

D9 A geologist tries to determine the age of a sample of rock containing $^{40}_{19}K$ which decays to give a stable nuclide $^{40}_{18}Ar$. The activity of the sample is found to be 2.4 Bq, while the original activity of a similar rock having the same mass is 38.4 Bq. The decay constant of $^{40}_{19}K$ is 5.33×10^{-10} yr^{-1}.

(a) **(i)** Define half-life.

(ii) Determine the half-life of $^{40}_{19}K$.

(iii) Determine the age of the sample of rock.

(b) **(i)** Determine the number of $^{40}_{19}K$ nuclei when the activity is 2.4 Bq.

(ii) State whether the half-life of $^{40}_{19}K$ will change if

1. a sample of greater mass of $^{40}_{19}K$ is chosen, and
2. the temperature of the sample of $^{40}_{19}K$ is increased.

ANSWERS

SP1 C

SP2 C

SP3 D

SP4 1.11 MeV, 8.78 MeV

SP5 C

SP7 C

SP8 A

SP9 C

SP10 4.17×10^{-9} s^{-1}, 4.19×10^{13} Bq

SP11 $3A_0/8$

SP12 6.40×10^{-10} s^{-1}, 1.08×10^{9} s or 34.3 yr, 225 yr

D2 1.16 *u*

D3 1.79×10^{-12} J

D4 195 MeV

D5 0.025601*u*, 5.72×10^{11} J, 37.8 g

D7 59.2 s

D8 0.13 yr^{-1} or 4.14×10^{-9} s^{-1}, 2.0×10^{19}, 8.3×10^{10} Bq, 0.035 W, 0.0022 W

D9 1.30×10^{9} yr, 5.20×10^{9} yr, 1.42×10^{17}