PEICAI SECONDARY SCHOOL

SECONDARY 5 NORMAL ACADEMIC
PRELIMINARY EXAMINATION 2021

CANDIDATE NAME	SOLUTIONS

CLASS | | | | REGISTER NUMBER | | |

ADDITIONAL MATHEMATICS 4047/02

Paper 2

27 August 2021

2 hours 30 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your register number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

This document consists of **23** printed pages and **1** blank page.

Setter: Mrs Ho Thuk Lan

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

1 Solve the equation $2\cos 2x = 3 - 5\sin x$ for $0 \le x \le$. [5]

$2\left(1-2\sin^2 x\right) = 3 - 5\sin x$

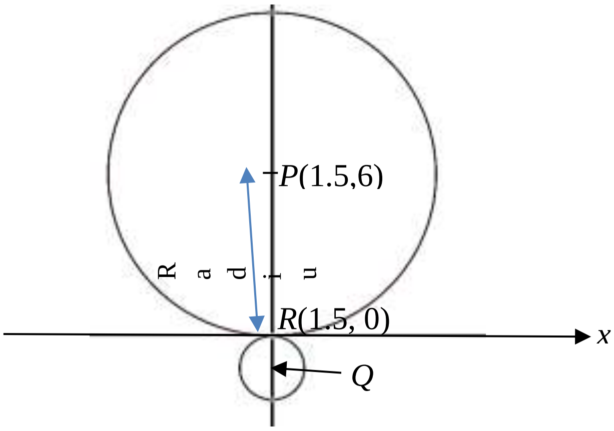
$4\sin^2 x - 5\sin x + 1 = 0$

$(\sin x - 1)(4\sin x - 1) = 0$

$\sin x = 1$ $x = \frac{\pi}{2} = 1.57$	$\sin x = \frac{1}{4}$ basic angle, $\alpha = \sin^{-1}\frac{1}{4}$ $x = 0.253,\ 2.89$

x = 0.253, 1.57, 2.89

2



The diagram shows two circles C_1 and C_2 with centres P and Q respectively. Both circles C_1 and C_2 is tangent to the x axis at R (1.5, 0). The radius of C_1 is 6 units.

(i) Find the coordinates of the centre, P of circle C_1. [1]

$P = (1.5,\ 6)$

(ii) Find the equation of the circle C_1. [1]

$(x-1.5)^2 + (y-6)^2 = 36$

(iii) Given that the area of C_1 is 18 times the area of C_2, find the coordinates of the centre, Q of C_2. [3]

$\frac{Area_{C_2}}{Area_{C_1}} = \left(\frac{RQ}{PR}\right)^2$ $\frac{Area_{C_2}}{18 Area_{C_2}} = \left(\frac{RQ}{6}\right)^2$ $\left(\frac{RQ}{6}\right)^2 = \frac{1}{18}$ $RQ = \sqrt{2}$	$Area_{C_1} = \pi(6)^2$ $Area_{C_1} = 36\pi$ $Area_{C_2} = \frac{36\pi}{18} = 2\pi$ $\pi(r_2)^2 = 2\pi$ $r_2 = \sqrt{2}$

$Q = (1.5,\ \sqrt{2})$

3 **(i)** Using $\sin 3x = \sin(2x + x)$, show that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

$\sin 3x = \sin(2x + x)$

$\sin 3x = \sin 2x \cos x + \cos 2x \sin x$

$\sin 3x = (2\sin x \cos x)\cos x + (1 - 2\sin^2 x)\sin x$

$\sin 3x = 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$\sin 3x = 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$

$\sin 3x = 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$\sin 3x = 3\sin x - 4\sin^3 x$ (shown)

(ii) State the amplitude and period of $y = 6\sin x - 8\sin^3 x$. [2]

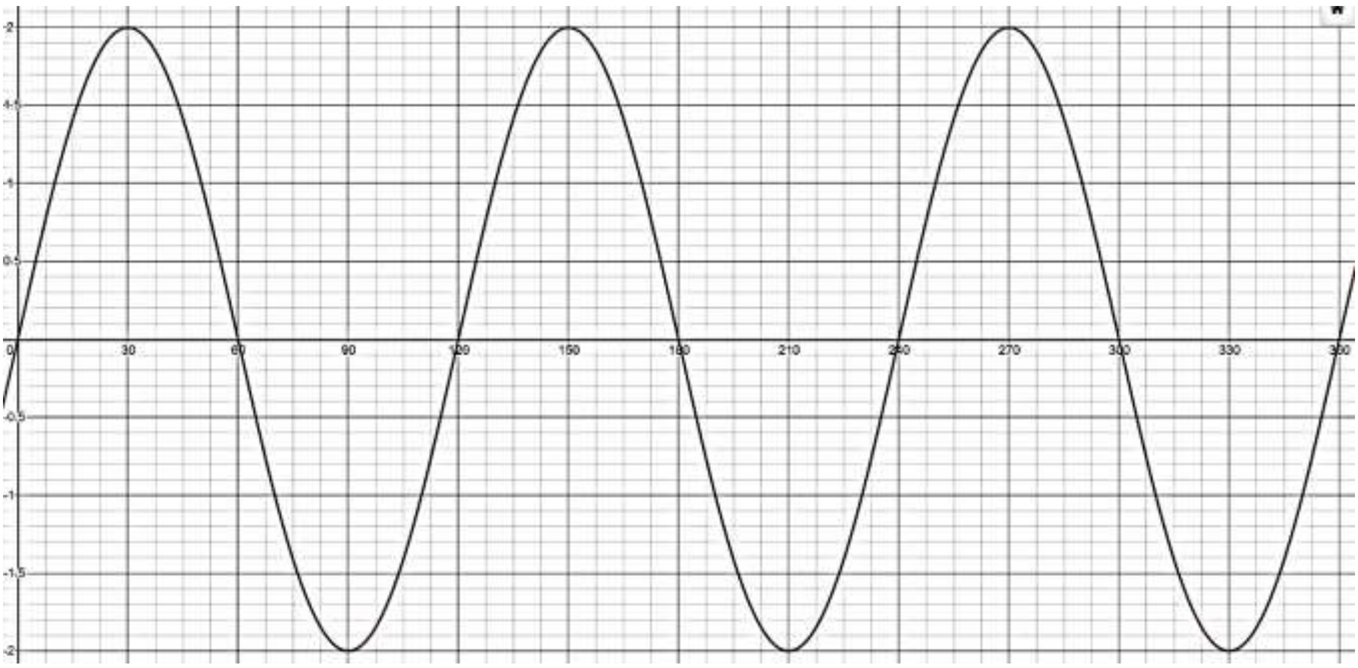
$y = 6\sin x - 8\sin^3 x$

$y = 2(3\sin x - 4\sin^3 x)$

$y = 2\sin 3x$

Amplitude = 2
Period = 120°

(iii) Sketch the graph of $y = 6\sin x - 8\sin^3 x$ for $0° \le x \le 360°$. [3]



4 A glass of hot water was left to cool in the fridge. The temperature, T °C, of the water decreases with time, t minutes. The table shows the measured values of T and t.

t (min)	10	20	30	40	50
T (°C)	60	37	23	14	9

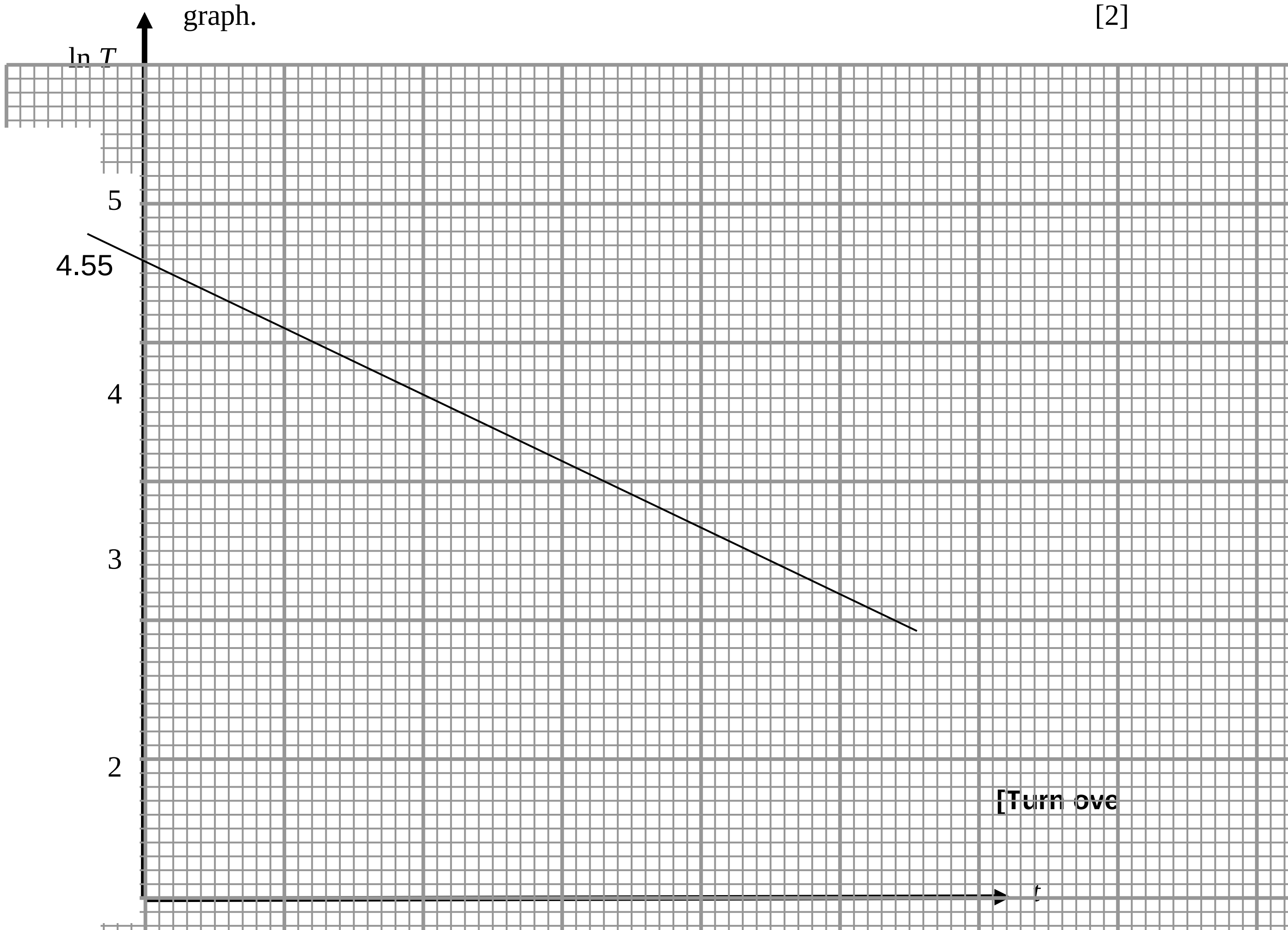
It is known that t and T are related by the equation $T = ae^{-kt}$ where a and k are constants.

(i) Explain clearly how a and k can be calculated when a graph of ln T against t is drawn. [2]

$T = ae^{-kt}$ $\ln T = \ln ae^{-kt}$ $\ln T = \ln a + \ln e^{-kt}$ $\ln T = -kt + \ln a$	$k =$ gradient ln a is the ln T – intercept, c ln a = c $a = e^{c}$

t (min)	10	20	30	40	50
ln T (°C)	4.09	3.61	3.14	2.64	2.20

(ii) On the grid below, plot ln T against t for the given data and draw a straight line graph. [2]



Use your graph to estimate

(iii) the value of *a* and of *k*. [3]

10 20 30 40 50

ln *a* is the ln *T* – intercept, *c*

$a = e^{c}$

$\ln a = 4.55$

$a = e^{4.55}$

$a = 94.6$ (3SF)

(iv) the temperature of the water at the beginning of the experiment. [1]

Temperature = 94.6°C

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5 A bus travelling on a straight road passes a traffic light at junction *P* with speed 11 m/s and, 2 minutes later, passes another traffic light at junction *Q* with speed 21 m/s. During the journey from *P* to *Q*, the acceleration, $a = kt - 2$ m/s², where *k* is a constant and *t* seconds is the time after passing *P*.

(i) Show that $k = \frac{5}{144}$. [5]

Speed = 11 m/s 2 min later Speed = 21 m/s

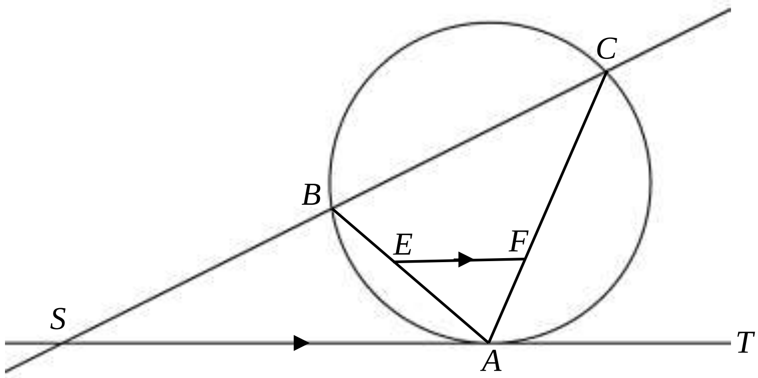
P ———————————— Q

$a = kt - 2$ $v = \frac{kt^2}{2} - 2t + c$ At *P*, *t* = 0, *v* = 11 $11 = \frac{k0^2}{2} - 2(0) + c$ $c = 11$ $v = \frac{kt^2}{2} - 2t + 11$	At *Q*, *t* = 2 min, *v* = 21m/s $21 = \frac{k(120)^2}{2} - 2(120) + 11$ $21 = 7200k - 229$ $7200k = 250$ $k = \frac{5}{144}$

(ii) Find the distance between *P* and *Q*. [4]

velocity, $v = \frac{5t^2}{288} - 2t + 11$ displacement, $s = \frac{5t^3}{288(3)} - t^2 + 11t + c$ $s = \frac{5}{864}t^3 - t^2 + 11t + c$	At P, $t = 0$, $s = 0$ $s = \frac{5}{864}t^3 - t^2 + 11t + c$ $c = 0$ $s = \frac{5}{864}t^3 - t^2 + 11t$ $s = \frac{5}{864}(120)^3 - (120)^2 + 11(120)$ $s = -3080$ Distance = 3080 m

6



The diagram shows a circle passing through the points *A*, *B* and *C*. The point *B* lies on the line *SC*. *ST* is a tangent to the circle at *A*. The points *E* and *F* lie on *AB* and *AC* respectively. Given that *EF* is parallel to *ST*, show that *BCFE* is a cyclic quadrilateral. [5]

BAS = *ACB* (Alternate segment Theorem)
BAS = *AEF* (Alternate angles, *EF*//*ST*)
BEF = 180° *AEF* (Adjacent angles on a straight line)
BEF = 180° *BAS* = 180° *ACB*
ACB + *BEF* = *ACB* + 180° *ACB*
ACB + *BEF* =180°

CAT = *ABC* (Alternate segment Theorem)
CAT = *EFA* (Alternate angles, *EF*//*ST*)
EFC = 180° *EFA* (Adjacent angles on a straight line)
EFC = 180° *CAT* = 180° *ABC*
ABC + *EFC* = *ABC* + 180° *ABC*
ABC + *EFC* =180°

Since *ACB* + *BEF* =180° (angles in opposite segments)
and *ABC* + *EFC* =180° (angles in opposite segments)
Therfore *BCFE* is a cyclic quadrilateral.

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7 The equation of a polynomial given is by $f(x)=3x^3+5x^2+7x-3$.

(i) Find the remainder when f(x) is divided by x + 1. [1]

$f(-1)=3(-1)^3+5(-1)^2+7(-1)-3$
$f(-1)=-3+5-7-3=-8$
Remainder = 8

(i) Show that 3x 1 is a factor of f(x). [1]

$$f\left(\frac{1}{3}\right)=3\left(\frac{1}{3}\right)^3+5\left(\frac{1}{3}\right)^2+7\left(\frac{1}{3}\right)-3$$
$$f\left(\frac{1}{3}\right)=\left(\frac{1}{9}\right)+\left(\frac{5}{9}\right)+\left(\frac{7}{3}\right)-3$$
$$f\left(\frac{1}{3}\right)=0$$

Since $f\left(\frac{1}{3}\right)=0$, therefore 3$x$ 1 is a factor of f(x).

(iii) Show that the equation f(x) = 0 has only one real root. [4]

$$\begin{array}{r} x^2+2x+3 \\ 3x-1\ \overline{)3x^3+5x^2+7x-3} \\ -3x^3+x^2 \\ \hline 6x^2+7x-3 \\ -6x^2+2x \\ \hline 9x-3 \\ -9x+3 \\ \hline 0 \end{array}$$

$f(x)=3x^3+5x^2+7x-3$

$f(x)=(3x-1)(x^2+2x+3)$

$f(x)=0$

$(3x-1)(x^2+2x+3)=0$

$x=\frac{1}{3}$ or $x=\frac{-2\pm\sqrt{-8}}{2}$

Since $\sqrt{-8}$ is undefined, there are no real roots for $x^2+2x+3=0$.
Hence f(x) = 0 has only one real root.

(iv) Use your answers to parts **(ii)** and **(iii)** to solve the equation $\frac{3}{2}\left(2^{3y+1}\right)+5(2^{2y})+7(2^y)=3$. [4]

$3\left(2^{-1}\right)\left(2^{3y}\right)\left(2^{1}\right)+5(2^y)^2+7(2^y)=3$

$3\left(2^{3y}\right)+5(2^y)^2+7(2^y)=3$

$3\left(2^{-1}\right)\left(2^{3y}\right)\left(2^{1}\right)+5(2^y)^2+7(2^y)=3$

$3\left(2^{3y}\right)+5(2^y)^2+7(2^y)=3$

Let $x=2^y$

$3x^3+5x^2+7x-3=0$

$(3x-1)(x^2+2x+3)=0$

$x=\frac{1}{3}$

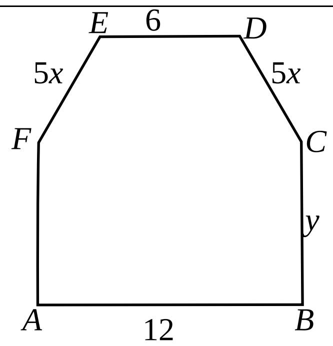
$2^y=\frac{1}{3}$

$\lg 2^y=\lg\frac{1}{3}$

$y=\frac{\lg\frac{1}{3}}{\lg 2}$

$y =$ 1.58 (3SF)

8



A contractor was given 240m of fencing for a playground, He designed the playground *ABCDEF* consisting of a rectangle *ABCF* and an isosceles trapezium *CDEF*. Given that *AB* = 12 m, *DE* = 6 m, *CD* = *EF* = 5*x* m and *BC* = *y* m.

(i) The contractor used 240 m of fencing to enclose the playground. Express *y* in terms of *x*. [1]

$2y+12+6+10x=240$

$y=111-5x$

(ii) Show that the enclosed area, A m^2, of the playground is given by $A=1332-60x+9\sqrt{25x^2-9}$. [2]

Height of trapezium = $\sqrt{(5x)^2-(3)^2}=\sqrt{25x^2-9}$

Area = $12y+\frac{1}{2}(12+6)\sqrt{25x^2-9}$

Area = $12y+9\sqrt{25x^2-9}$

$A=12(111-5x)+9\sqrt{25x^2-9}$

$A=1332-60x+9\sqrt{25x^2-9}$

(iii) Given that *x* can vary, find the stationary value of *A*. [4]

$A = 1332 - 60x + 9\sqrt{25x^2 - 9}$

$A = 1332 - 60x + 9\left(25x^2 - 9\right)^{\frac{1}{2}}$

$\frac{dA}{dx} = -60 + \frac{9}{2}\left(25x^2 - 9\right)^{-\frac{1}{2}}(50x)$

$\frac{dA}{dx} = -60 + 225x\left(25x^2 - 9\right)^{-\frac{1}{2}}$

$\frac{dA}{dx} = -60 + \frac{225x}{\sqrt{25x^2 - 9}}$

For stationary point, $\frac{dA}{dx} = 0$

$-60 + \frac{225x}{\sqrt{25x^2 - 9}} = 0$

$\frac{225x}{\sqrt{25x^2 - 9}} = 60$

$\sqrt{25x^2 - 9} = \frac{225x}{60}$

$\sqrt{25x^2 - 9} = \frac{15x}{4}$

$\sqrt{25x^2 - 9} = \frac{15x}{4}$

$25x^2 - 9 = \frac{225}{16}x^2$

$$\frac{175}{16}x^2 = 9$$

$$x = \sqrt{\frac{144}{175}}$$

$$A = 1332 - 60\left(\sqrt{\frac{144}{175}}\right) + 9\sqrt{25\left(\sqrt{\frac{144}{175}}\right)^2 - 9}$$

$$A = 1332 - 60\left(\frac{12}{5\sqrt{7}}\right) + 9\sqrt{25\left(\frac{144}{175}\right) - 9}$$

$$A = 1332 - \frac{144}{\sqrt{7}} + 9\sqrt{\frac{81}{7}}$$

$$A = 1308.188238 \text{ m}^2$$

$$A = 1310 \text{ m}^2 \text{ (3SF)}$$

Stationary value of $A = 1310 \text{ m}^2$ (3SF)

(iv) Find the nature of this stationary value. Would the contractor be happy or disappointed with the design of his playground? [3]

$$\frac{dA}{dx} = -60 + \frac{225x}{\sqrt{25x^2 - 9}}$$

$$\frac{dA}{dx} = -60 + 225x\left(25x^2 - 9\right)^{-\frac{1}{2}}$$

$$\frac{d^2A}{dx^2} = 225x\left[-\frac{1}{2}\left(25x^2 - 9\right)^{-\frac{3}{2}}(50x)\right] + 225\left(25x^2 - 9\right)^{-\frac{1}{2}}$$

$$\frac{d^2A}{dx^2} = -5625x^2\left[\left(25x^2 - 9\right)^{-\frac{3}{2}}\right] + 225\left(25x^2 - 9\right)^{-\frac{1}{2}}$$

$$\frac{d^2A}{dx^2} = \frac{-5625x^2}{\left(25x^2 - 9\right)\sqrt{25x^2 - 9}} + \frac{225}{\sqrt{25x^2 - 9}}$$

$$\frac{d^2A}{dx^2} = \frac{-5625x^2 + 225(25x^2 - 9)}{\left(25x^2 - 9\right)\sqrt{25x^2 - 9}}$$

$$\frac{d^2A}{dx^2} = \frac{-2025}{\left(25x^2 - 9\right)\sqrt{25x^2 - 9}}$$

When $x = \sqrt{\frac{144}{175}}$

$$\frac{d^2A}{dx^2} = \frac{-2025}{\left(25\left(\sqrt{\frac{144}{175}}\right)^2 - 9\right)\sqrt{25\left(\sqrt{\frac{144}{175}}\right)^2 - 9}}$$

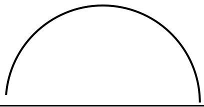
$$\frac{d^2A}{dx^2} = \frac{-2025}{\left(\frac{81}{7}\right)\left(\sqrt{\frac{81}{7}}\right)}$$

$$\frac{d^2A}{dx^2} = -51.44516438 < 0$$

Since $\frac{d^2A}{dx^2} < 0$, the area of the playground is a maximum.

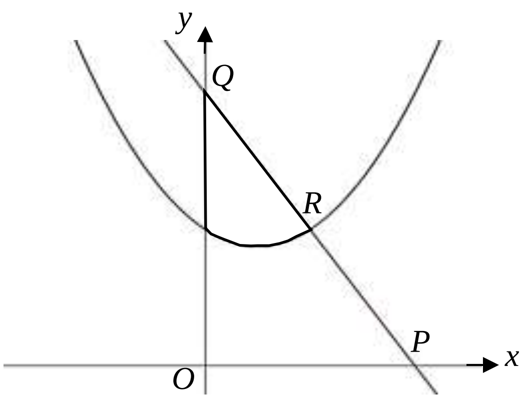
Method 2:

x	0.90	$x = \sqrt{\frac{144}{175}} = 0.907114$	0.92
Sign of $\frac{dy}{dx}$	+	0	
Sketch of tangent			

Sketch of curve	Maximum curve 

As the area of the playground is maximised, the contractor is not disappointed with the design.

9



The diagram shows part of the curve $y = x^2 - x + h$ and the line $y = 4 - 2x$. The line $y = 4 - 2x$ intersects the x axis and the y axis at P and Q respectively. Given that R is the point of intersection of the line $y = 4 - 2x$ and the curve $y = x^2 - x + h$ and that R is the midpoint of PQ, find

(i) the coordinates of R, [3]

At P, $y = 0$ $4 - 2x = 0$ $x = 2$ $P = (2, 0)$	At Q, $x = 0$ $y = 4 - 2(0)$ $y = 4$ $Q = (0, 4)$	$R = \left(\frac{2+0}{2}, \frac{0+4}{4}\right)$ $R = (1,\ 2)$

(i) the value of h, [1]

$y = x^2 - x + h$

At $R(1,\ 2)$

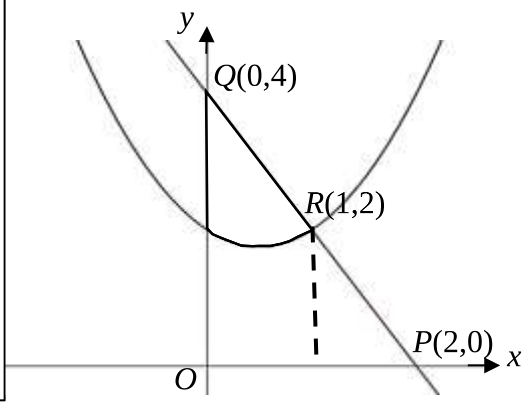
$2 = 1^2 - 1 + h$

$h = 2$

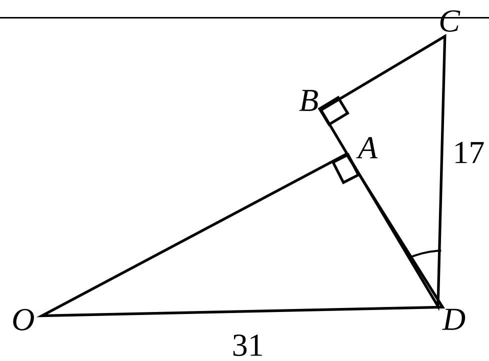
[Turn over

(ii) the area of the shaded region. [6]

Area of trapezium = Area under the curve = Area under the curve = = Shaded area = Area of trapezium – area under curve =



10



The diagram shows three fixed points *O*, *C* and *D* such that *OD* = 31 cm, *CD* = 17 cm and angle *ODC*= 90°. The lines *OA* and *BC* are perpendicular to the line *BD* which makes an angle θ with the line *CD*. The angle θ can vary in such a way that the point *A* lies between the points *B* and *D*.

(i) Show that *OA* + *AB* + *BC* = 48 cos θ 14 sin θ. [3]

In ⊗*BCD*, $\sin\theta = \frac{BC}{17}$ ∴ *BC* = 17 sin θ $\cos\theta = \frac{BD}{17}$ ∴ *BD* = 17 cos θ	In ⊗*OAD*, Since angle *ODC*= 90°, *AOD* = θ $\cos\theta = \frac{OA}{31}$ ∴ *OA* = 31 cos θ $\sin\theta = \frac{AD}{31}$ ∴ *AD* = 31 sin θ *AB* = *BD* *AD* *AB* = 17 cos θ 31 sin θ	*OA* + *AB* + *BC* = 31 cos θ + (17 cos θ 31 sin θ) + 17 sin θ *OA* + *AB* + *BC* = 48 cos θ 14 sin θ. (Shown)

(ii) Express *OA* + *AB* + *BC* in the form $R\cos(\theta + \alpha)$, where *R* > 0 and $0° < \alpha < 90°$. [3]

$R=\sqrt{48^2+14^2}=50$ $\quad\quad$ $\alpha=\tan^{-1}\left(\frac{14}{48}\right)=16.26020471°$

48 cos θ 14 sin θ = 50 cos(θ + 16.3)°

(iii) Find the values of θ for which $OA + AB + BC = 30$. [4]

$50\cos(\theta+16.3)° = 30$

$\cos(\theta+16.3)° = \frac{3}{5}$

Basic angle, $\alpha=\cos^{-1}\frac{3}{5}=53.1301°$

$\theta+16.2602°=53.1301°, 360°-53.1301°$

$\theta=36.9°, 290.6°$ (rejected)

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11 The equation of a curve is $y=x\sin x$.

(i) Find an expression for $\frac{dy}{dx}$. [2]

$y=x\sin x$

$\frac{dy}{dx}=x\cos x+\sin x$

(ii) **Hence** find $\int x\cos x\ dx$. [2]

$\frac{d}{dx}x\sin x=x\cos x+\sin x$

$x\sin x=\int(x\cos x+\sin x)\ dx$

$x\sin x=\int(x\cos x+\sin x)\ dx$

$x\sin x=\int(x\cos x)\ dx+\int\sin x\ dx$

$\int(x\cos x)\ dx=x\sin x-\int\sin x\ dx$

$\int(x\cos x)\ dx=x\sin x-\int\sin x\ dx$

$\int(x\cos x)\ dx=x\sin x+\cos x+c$

(iii) Find an expression for $\frac{d}{dx}\left(x^2\cos x\right)$. [2]

$\frac{d}{dx}\left(x^2\cos x\right)=2x\cos x+(-\sin x)x^2$

[Turn over

$$\frac{d}{dx}\left(x^2 \cos x\right) = 2x\cos x - x^2 \sin x$$

(i) Using the result found in part **(ii)** and part **(iii)**, find $\int x^2 \sin x \, dx$. [4]

$$\frac{d}{dx}\left(x^2 \cos x\right) = 2x\cos x + (-\sin x)x^2$$

$$\left(x^2 \cos x\right) = \int \left[2x\cos x + (-\sin x)x^2\right] dx$$

$$x^2 \cos x = \int 2x\cos x \, dx - \int x^2 \sin x \, dx$$

$$\int x^2 \sin x \, dx = 2\int x\cos x \, dx - x^2 \cos x$$

$$\int x^2 \sin x \, dx = 2\left(x\sin x + \cos x + c\right) - x^2 \cos x$$

$$\int x^2 \sin x \, dx = 2x\sin x + 2\cos x - x^2 \cos x + d$$ where $d = 2c$

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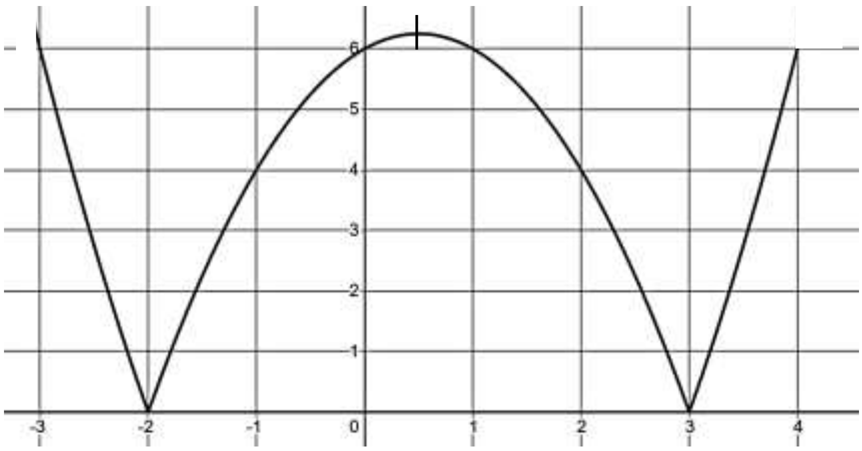
12 **(a)** Sketch the curve of $y = \left|6 + x - x^2\right|$ for $-3 \le x \le 4$, indicating clearly the intercepts and the turning point. [2]



(b) The quadratic equation $2x^2 - x - 3 = 0$ has roots 〈 and ®. Find

(i) the value of $\alpha^3 + \beta^3$. . [5]

Sum of roots, $\alpha + \beta = -\frac{-1}{2} = \frac{1}{2}$

Product of roots, $\alpha\beta = \frac{-3}{2} = -\frac{3}{2}$

$$\alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2)$$

$$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$$
$$(\alpha + \beta)^2 - 2\alpha\beta = \alpha^2 + \beta^2$$

$$(\alpha + \beta)^2 - 2\alpha\beta - \alpha\beta = \alpha^2 + \beta^2 - \alpha\beta$$
$$(\alpha + \beta)^2 - 3\alpha\beta = \alpha^2 + \beta^2 - \alpha\beta$$

$$\alpha^3 + \beta^3 = (\alpha + \beta)\left[(\alpha + \beta)^2 - 3\alpha\beta\right]$$
$$\alpha^3 + \beta^3 = \left(\frac{1}{2}\right)\left[\left(\frac{1}{2}\right)^2 - 3\left(-\frac{3}{2}\right)\right]$$
$$\alpha^3 + \beta^3 = \left(\frac{1}{2}\right)\left[\left(\frac{1}{4}\right) + \left(\frac{9}{2}\right)\right]$$
$$\alpha^3 + \beta^3 = \left(\frac{1}{2}\right)\left[\left(\frac{1+18}{4}\right)\right]$$
$$\alpha^3 + \beta^3 = \frac{19}{8}$$

(ii) a quadratic equation with roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. [3]

Sum of roots, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{\frac{1}{2}}{-\frac{3}{2}} = -\frac{1}{3}$

Product of roots, $\frac{1}{\alpha}\frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{-\frac{3}{2}} = -\frac{2}{3}$

Quadratic equation:

$$x^2 - (\text{sum of roots})x + (\text{product of root}) = 0$$

$$x^2 + \frac{1}{3}x - \frac{2}{3} = 0$$

END OF PAPER