Name: Register Number: Class:

For Marker's Use
90

南僑中學

NAN CHIAU HIGH SCHOOL

Parents' signature:

PRELIMINARY EXAMINATION 2023
SECONDARY FOUR EXPRESS

ADDITIONAL MATHEMATICS
Paper 1

4049/01
22 August 2023, Tuesday

Candidates answer on the Question Paper.

2 hours 15 minutes

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{r}a^{n-r}b^r + \cdots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 (a) Each side of an equilateral triangle measures $\left(\frac{1}{2+\sqrt{6}}\right)$ cm. Find, without using a calculator, the area of the equilateral triangle, in cm^2, in the form $(a\sqrt{3}+b\sqrt{2})$, where a and b are rational numbers. [4]

(b) Solve the equation $\sqrt{x+3} = \frac{1}{\sqrt{x+3}} - \frac{5}{6}$. [3]

2 (a) Differentiate $x^5 \ln x^2$ with respect to x, where $x > 0$. [2]

(b) Hence, find $\int x^4 \ln x \, dx$. [3]

3 (a) Express $-9-2x^2-4x$ in the form $a(x+b)^2+c$ and hence state the maximum value of $-9-2x^2-4x$. [3]

(b) Hence, or otherwise, determine the range of values of k for which $k-9-2x^2-4x=-2$ has two real and distinct roots. [2]

4 Express $\dfrac{3x^3+2}{(x+1)(x^2-1)}$ in partial fractions. [6]

5 (a) Solve the equation $\log_7 x + \log_{49} x^2 = 6$. [3]

(b) Given $y = \lg(x^2 + 8x + 15) - \lg(x + 4)$, state the range of values of x for which y exists. [2]

6 Given that $\sin\theta = c$ and $90° < \theta < 180°$,

(a) express $\sin 2\theta$ in terms of c, [2]

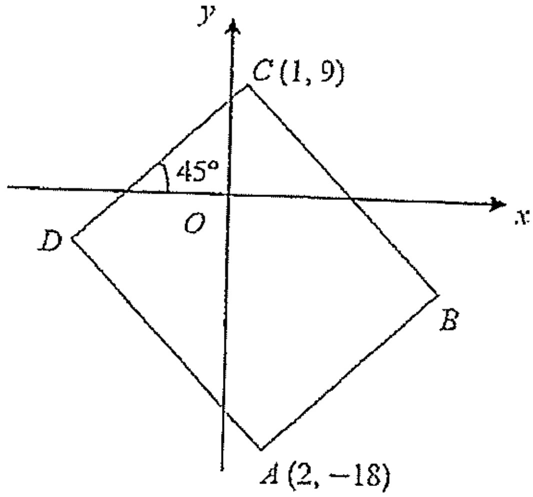
(b) express $\cos(\theta + 30°)$ in terms of c, [3]

(c) find the principal value of $\sin^{-1}(-c)$ in terms of θ. [1]

7 (a) Prove the identity $\frac{4\sin\theta+4\sin^2\theta}{\sec\theta+\tan\theta} = 2\sin 2\theta$. [4]

(b) Hence, solve the equation $\frac{4\sin\theta+4\sin^2\theta}{\sec\theta+\tan\theta} = 0.7$ for $-\pi \leq \theta \leq \pi$. [3]

8 The diagram shows a rectangle $ABCD$ with A (2, −18) and C (1, 9). DC makes an angle of 45° with the positive direction of the x-axis.



(a) Show that the coordinates of B is (15, −5). [5]

(b) Given that E is a point on DB produced such that $DB = BE$, find the area of triangle ACE. [4]

9 A solid of height x cm has a volume of V cm^3. It has a uniform cross-sectional area given by $(px^2 + qx)$ cm^2, where p and q are constants.

Corresponding values of x and V are shown in the table below.

x	1	2	3	4
V	4	28	90	208

(a) Using suitable variables, draw, on the grid below, a straight line graph and hence estimate the value of p and of q. [5]

Continued working space for (a)

(b) Using your values of p and q obtained in (a), calculate the value of x for which the solid is a cylinder with height half the length of its base radius. [2]

(c) Explain how another straight line drawn on the same grid as (a) can lead to an estimated value of x for which the solid is a cylinder with height half its base radius.

Draw this line and hence verify your value of x found in part (b). [3]

10 Jean trains physically by running along a straight track. At time t seconds after leaving a fixed point O, its velocity v m/s is given by $v = 2.5\sin\left(\frac{1}{2}t\right)$. She makes the first turn back to O when she reaches point A.

(a) Find the acceleration when $t = \frac{\pi}{2}$. [2]

(b) Find the distance OA. [4]

When Jean returns to fixed point O for the fourth time from A, her subsequent velocity towards point A, v m/s is given by $v = 0.25t - 4\pi$ for $t > k$, where k is a constant.

(c) Show that $k = 16\pi$ and explain why Jean did not return to O subsequently. [2]

(d) Find the total distance covered during the first 60 seconds. [4]

11 The equation of a curve is $y = 3x^2e^{-\frac{1}{2}x}$.

(a) Find the range of values of x for which y is increasing. [4]

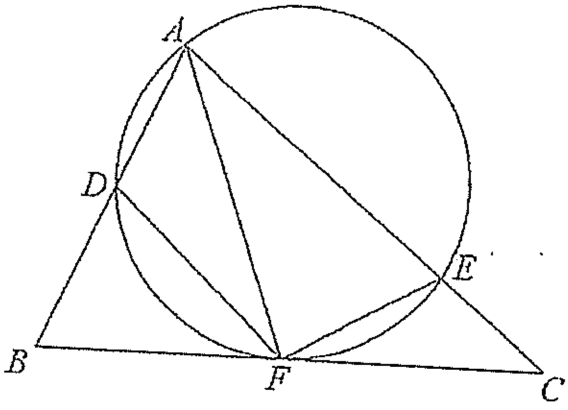
(b) Show that the origin (0, 0) is a minimum point of the curve. [2]

A point, P, lies on the curve such that the normal to the curve at P has a negative gradient.

(c) State the range of values of x-coordinates of P. [1]

(d) Find the equation of the normal at P, given that the x-coordinate of P is 1. [4]

12



In the diagram, the line BC is a tangent to the circle at F. The points A, D, E and F lie on the circumference of the circle. D and F are the midpoints of line AB and BC respectively.

(a) Show that $ADFE$ is a trapezium.

[2]

(b) Show that triangle DFA is similar to triangle EFC.

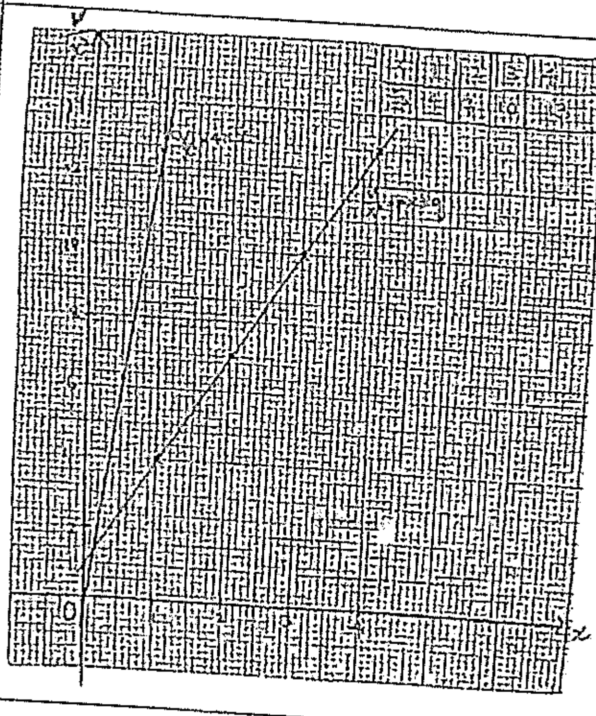
[3]

(c) Prove that $\frac{1}{2} \times AC \times EC = DA \times EF$.

[2]

End of paper

Answer Key:

1a	$\frac{5\sqrt{3}}{8}-\frac{3\sqrt{2}}{4}$
1b	$x=-\frac{23}{9}$
2a	$2x^4+10x^4 lnx$
2b	$\frac{(x^5 lnx)}{5}-\frac{x^5}{25}+c3$
3a	$-2(x+1)^2-7$, $max\ value = -7$
3b	$k>5$
4	$3+\frac{5}{4(x-1)}-\frac{17}{4(x+1)}+\frac{1}{2(x+1)^2}$
5a	343
5b	$x>-3$
6a	$-2c\sqrt{1-c^2}$
6b	$\frac{-1}{2}c-\frac{\sqrt{3-3c^2}}{2}$
6c	θ–180°
7a	As proven
7b	$\theta=0.179, 1.39, -1.75, -2.96$
8a	As shown
8b	546 $units^2$
9a	

	$\frac{y}{x^2} = px + q$ $gradient = p$ $= \frac{10-1}{3-0} = 3$ $q = 1$
9b	0.105
9c	$draw\ \frac{V}{x^2} = 4\pi x$ $From\ graph, x = 0.1$ *This is close to the x value obtained in part (b).*
10a	$\frac{0.884m}{s^2}$ or $\frac{5\sqrt{2}}{8} m/s^2$
10b	$10m$
10c	As shown and any valid argument eg. Proving v>0
10d	$91.8m$
11a	$0 < x < 4$
11b	1st or 2nd derivative test or justifying components of expressions clearly, with proving of y-coordinate
11c	$0 < x < 4$
11d	$y = -\frac{2e^{\frac{1}{2}}}{9}x + \frac{2e^{\frac{1}{2}}}{9} + 3e^{-\frac{1}{2}}$ $or\ y = -0.366x + 2.19$
12a	Midpoint theorem
12b	$\angle EFC = \angle FAE (alt\ segment\ theorem)$ $\angle FAE = \angle AFD (alt\ \angle s, DF \parallel AE)$ $\therefore \angle EFC = \angle AFD$ $180° - \angle ADF = \angle FEA\ (\angle s\ in\ opp\ segment)$ $\angle FEC = 180° - \angle FEA (adj\ \angle s\ on\ a\ str.line)$ $= 180° - (180° - \angle ADF)$ $= \angle ADF$ $\therefore \Delta DFA$ is similar to ΔEFC (AA Similarity test)

12c	$\frac{DA}{EC} = \frac{DF}{EF}$ (corresponding sides of similar triangles are proportional) $DF \times EC = DA \times EF$ $By\ midpt\ theorem, DF = \frac{1}{2}AC$ $\frac{1}{2}AC \times EC = DA \times EF$