H1 / H2 PHYSICS (2013)

TOPIC 10: WAVE MOTION

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Content

- Progressive waves
- Transverse and longitudinal waves
- Polarisation
- Determination of frequency and wavelength

Learning Outcomes

Candidates should be able to:

(a) show an understanding and use the terms displacement, amplitude, phase difference, period, frequency, wavelength and speed.

(b) deduce, from the definitions of speed, frequency and wavelength, the equation $v = f\lambda$.

(c) recall and use the equation $v = f\lambda$.

(d) show an understanding that energy is transferred due to a progressive wave.

(e) recall and use the relationship, *intensity* $\propto$ *(amplitude)*^{2}.

(f) analyse and interpret graphical representations of transverse and longitudinal waves.

(g) show an understanding that polarisation is a phenomenon associated with transverse waves.

(h) determine the frequency of sound using a calibrated c.r.o.

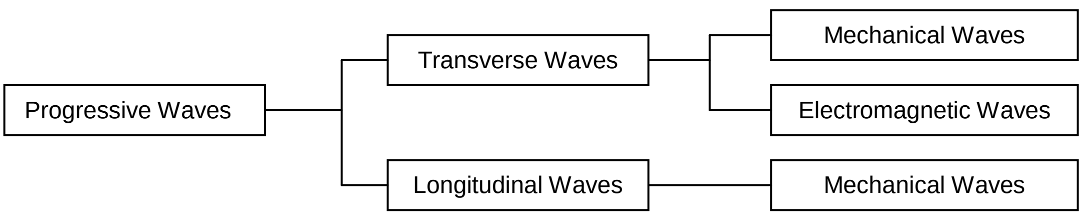
(i) determine the wavelength of sound using stationary waves.

10.1 Progressive Waves

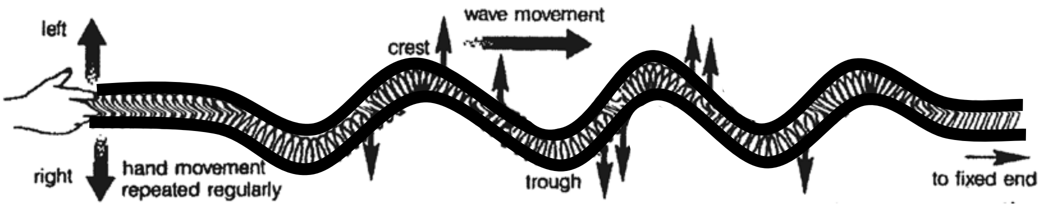
A ***progressive wave*** transfers energy from one point to another by means of vibrations or oscillations within the waves.

All waves involve a disturbance from an equilibrium position (i.e. an oscillation), and (for progressive waves,) the disturbance travels from one region of space to another. However, the particles do *not* move along with the disturbance.

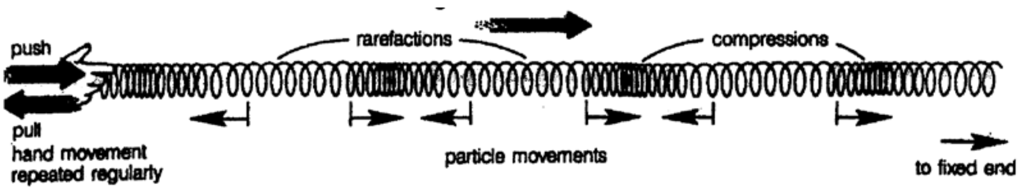
10.2 Transverse and Longitudinal Waves



Transverse wave: Transverse wave is a wave in which the oscillation of the particles in the wave is perpendicular to the direction of transfer of energy of the wave. (E.g.: electromagnetic waves)



Longitudinal wave: Longitudinal wave is a wave in which the oscillation of the particles in the wave is along the direction of transfer of energy of the wave. (E.g.: sound waves)



Mechanical wave: A wave that requires a medium for its transmission. (E.g.: sound waves and water waves)

Electromagnetic wave: Wave consisting of oscillating electric and magnetic fields that are at right angles to each other and to the direction of transfer of energy of wave. It does not require a medium for transmission. It can travel through a vacuum at the speed of light, i.e. 3×10^8 m s^{-1}. (E.g.: radio waves, visible light, X-rays and microwaves)

Electromagnetic Spectrum

Type of Radiation	Range of wavelength	Range of Frequency
Radio Waves	6 cm to 300 km	(1×10^3 to 5×10^9) Hz
Microwaves	10^{-3} m to 10^{-1} m	(10^9 to 3×10^{11}) Hz
Infrared Radiation	7.8×10^{-7} m to 0.5×10^{-3} m	(6×10^{11} to 4×10^{14}) Hz
Light (Visible Spectrum)	3.8×10^{-7} m to 7.8×10^{-7} m	(4×10^{14} to 8×10^{14}) Hz
Ultraviolet Radiation	6×10^{-10} m to 3.8×10^{-7} m	(8×10^{14} to 5×10^{17}) Hz
X-Rays	10^{-12} m to 10^{-9} m	From 3×10^{20} Hz
Gamma Rays	10^{-14} m to 10^{-10} m	(3×10^{18} to 3×10^{22}) Hz

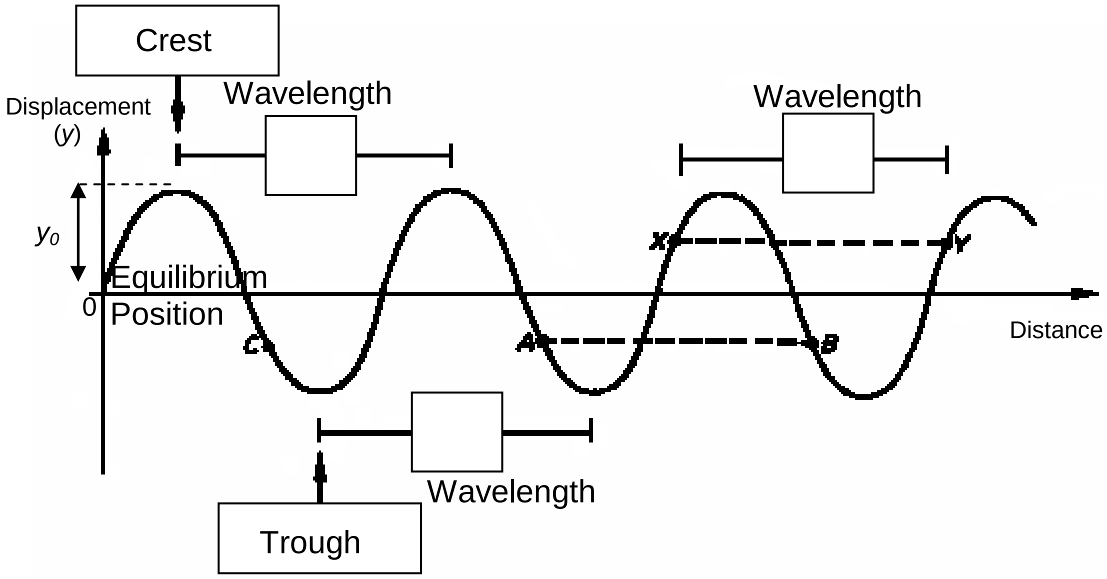
Note: Range of different radiations may overlap each other.

Visible Spectrum

Type of Radiation	Range of wavelength	Range of Frequency
Red $\lambda = 7 \times 10^{-7}$ m	(6.22 to 7.80) x 10^{-7} m	(3.84 to 4.82) x 10^{14} Hz
Orange	(5.97 to 6.22) x 10^{-7} m	(4.82 to 5.03) x 10^{14} Hz
Yellow	(5.77 to 5.97) x 10^{-7} m	(5.03 to 5.20) x 10^{14} Hz
Green	(4.92 to 5.77) x 10^{-7} m	(5.20 to 6.10) x 10^{14} Hz
Blue	(4.55 to 4.92) x 10^{-7} m	(6.10 to 6.59) x 10^{14} Hz
Violet $\lambda = 4 \times 10^{-7}$ m	(3.90 to 4.55) x 10^{-7} m	(6.59 to 7.69) x 10^{14} Hz

Source: *Longman* A-Level Course in Physics Volume 1
Refer to Annex A for more details on electromagnetic waves.

10.3 Characteristics of Waves



(a) Displacement (y): The distance in a specific direction from the equilibrium position of the simple harmonic motion.

(b) Amplitude (y_0 or A): Maximum possible displacement from the equilibrium point in either direction.

(c) Period (T): Time taken for a particle in the wave to undergo one complete oscillation. Unit: second. (equals to the time taken for the wave energy to travel a distance of one wavelength)

(d) Frequency (f): Number of oscillations per unit time made by the oscillating particle. $f = \frac{1}{T}$. Unit: Hertz. 1 Hz is equal to one oscillation (or one cycle) per second.

(e) Wavelength (λ): The shortest distance between any two successive points on a progressive waves which are vibrating in phase.

(f) Wave speed (v): Distance travelled per unit time by the wave energy in the direction of the transfer of the wave energy. It is related to the wavelength and the frequency according to the equation v = fλ.

From the definition of speed, Speed = $\frac{\text{Distance}}{\text{Time}}$

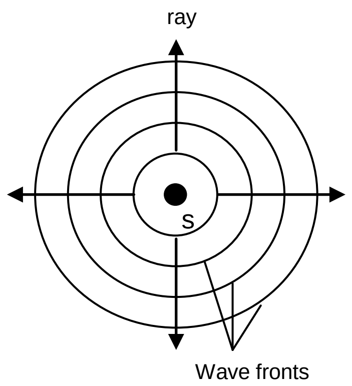
For one cycle, (i) the time taken is one period, *T*.
(ii) the distance travelled is a wavelength, λ.

Therefore, $v = \frac{\lambda}{T}$

Since $f = \frac{1}{T}$

Hence $v = f\lambda$

(g) Wave front: An imaginary line or surface joining points which are at the same state of oscillation (i.e. in phase). E.g.: a line joining crest to crest in a wave.



(h) Ray: The path taken by the wave. This is used to indicate the direction of transfer of wave energy. Rays are always at right angles to the wave fronts (i.e. wave fronts are always perpendicular to the direction of transfer of wave energy).

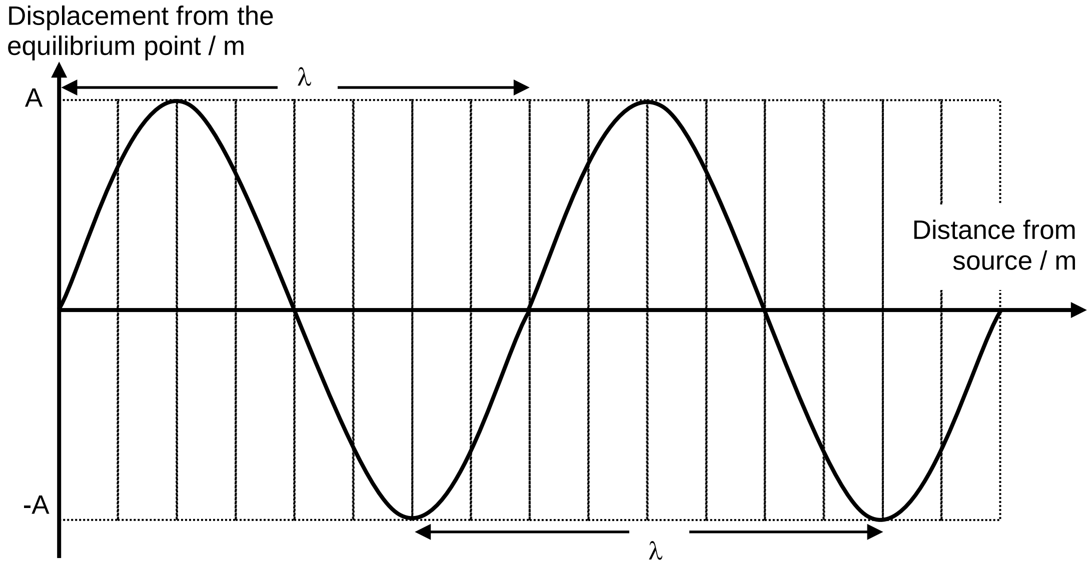
More details on the mathematical description of wave can be found in Annex B.

10.4 Graphical Representations of Transverse and Longitudinal Waves

10.4.1 Displacement-Distance Graph

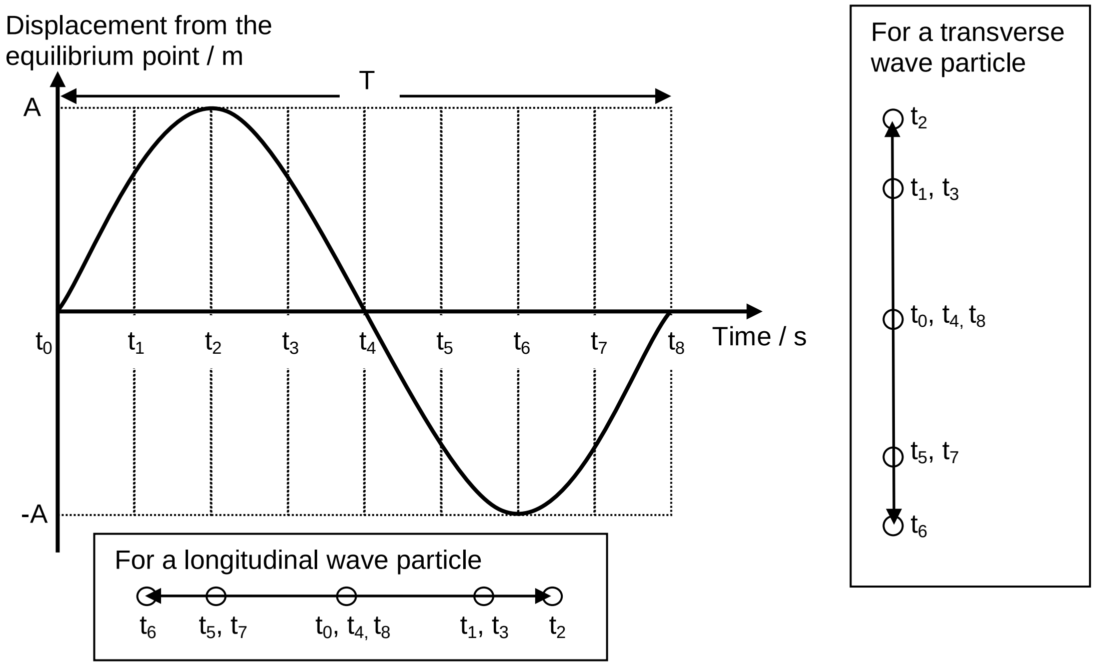
The displacement-distance graph shows how the displacements of the particles vary with the distance from the source at *a particular instant in time*.

The *wavelength* can be determined from the graph. (one complete sine curve or cosine curve)



10.4.2 Displacement-Time Graph

The displacement-time graph shows how the displacement of *a single wave particle* varies with time.

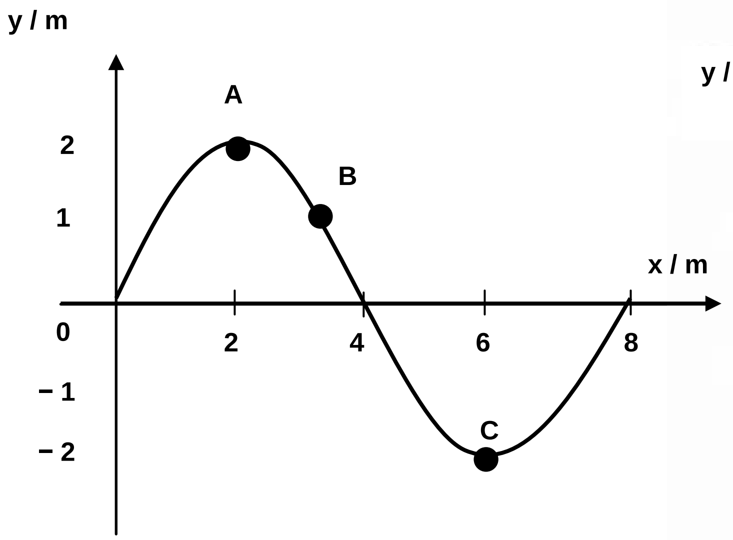


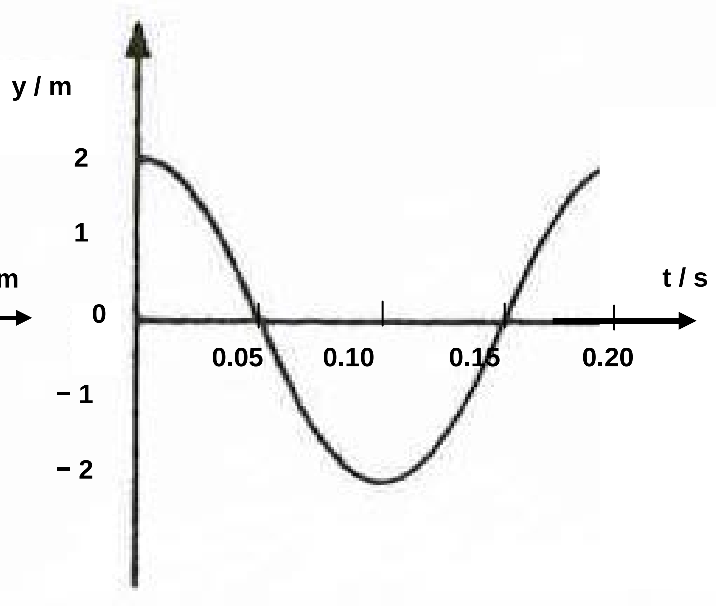
For a longitudinal wave, the vertical axis represents horizontal displacement, e.g. positive represents rightwards and negative represents leftwards.

All the particles move in a similar manner with the same *amplitude* and *frequency* as the wave. The *period* can be determined from the graph. (one complete sine curve or cosine curve)

E.g. 1

Displacement-distance graph	Displacement-time graph
(displacement of <u>all</u> particles at an <u>instant</u>)	(displacement of <u>one</u> particle e.g. Particle A with respect to <u>time</u>)





1) What is the displacements of Particle A, B and C respectively? **2 m, 1 m , −2 m**

2) What is the amplitude of the wave? **2m**

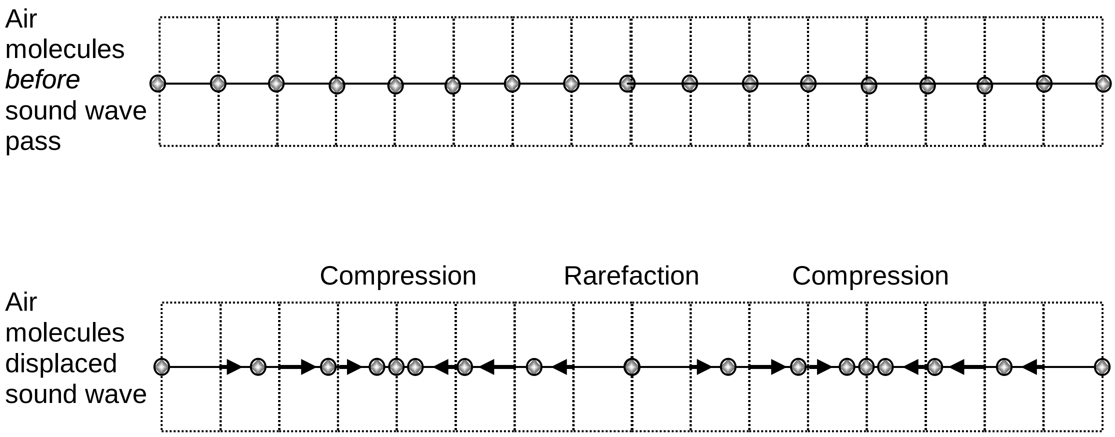
3) What is the wavelength of the wave? **8 m**
(Note: wavelength is the distance of one <u>sine</u> or one <u>cosine</u> curve on the displacement - <u>distance</u> graph)

4) What is the period of the wave? **0.20 s**
(Note: period is the duration of one <u>sine</u> or one <u>cosine</u> curve on the displacement – <u>time</u> graph)

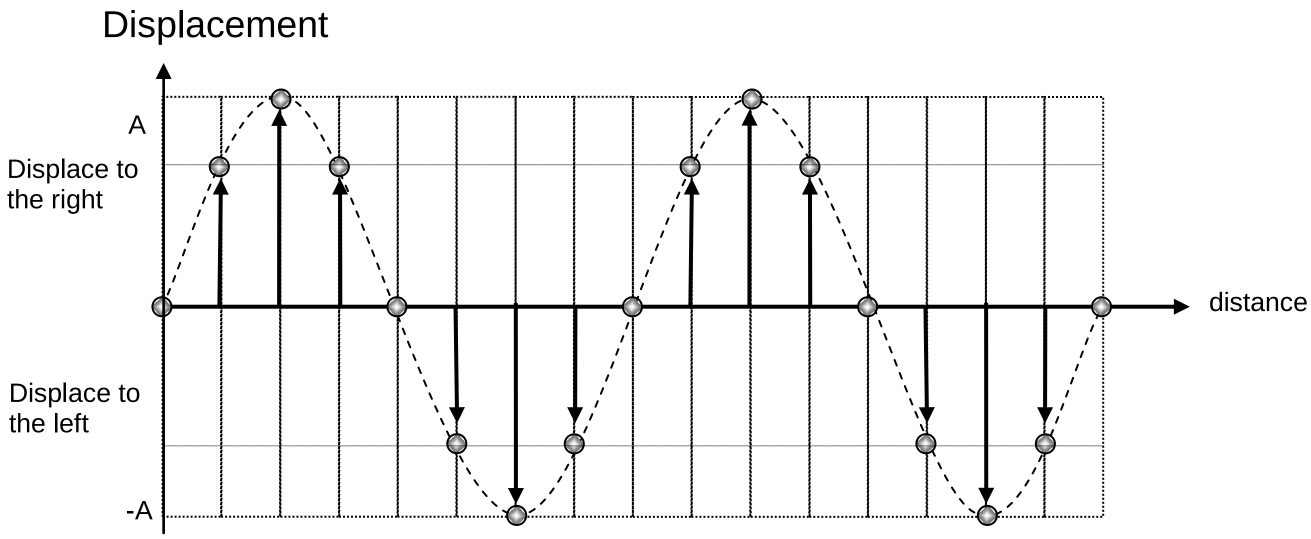
5) What is the frequency of the wave? **5 Hz**

6) What is the speed of the wave? **40 ms^{-1}**

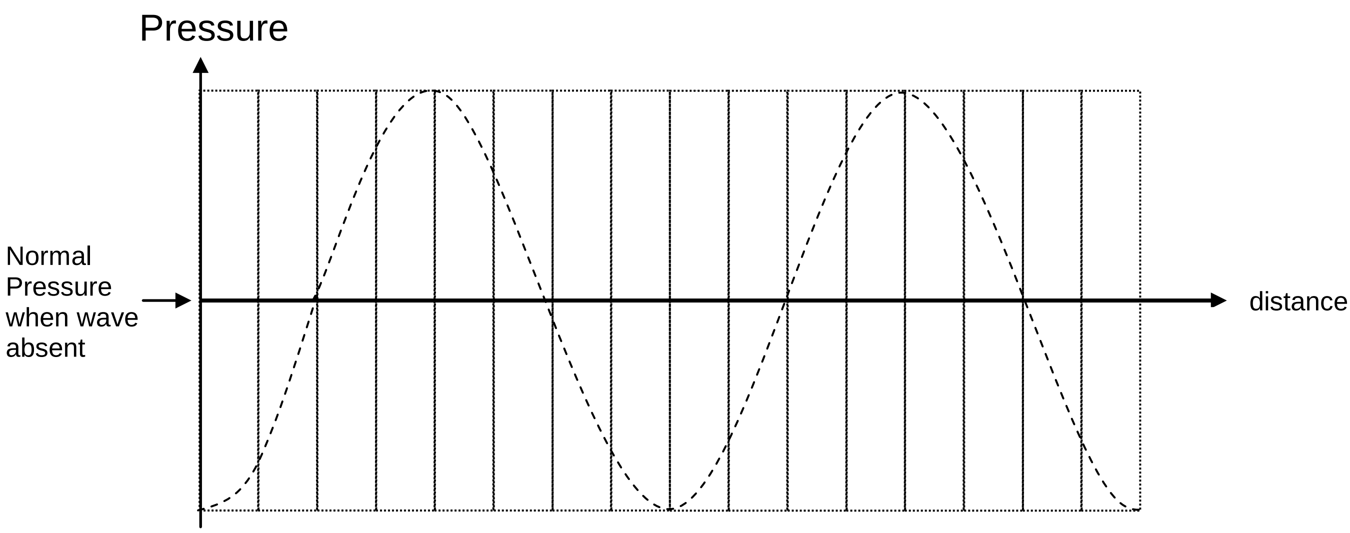
E.g. 2 Sketch the displacement-distance graph and the pressure-distance graph



Representation for longitudinal wave



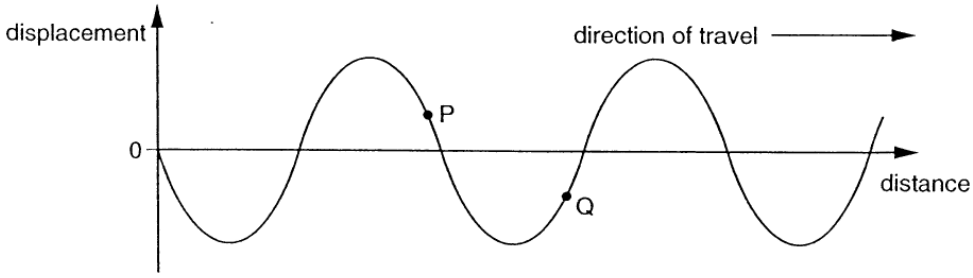
Displacement-distance graph for longitudinal wave



Pressure-distance graph for longitudinal wave

E.g. 3

A transverse progressive wave travels along a rope. The graph shows the variation of displacement with distance along the rope, at a certain time. The wave is travelling to the right.



In which direction are P and Q moving?

	movement of P	movement of Q
A	downwards	downwards
B	downwards	upwards
C	upwards	downwards
D	upwards	upwards

Ans: C

10.5 Phase and Phase Difference

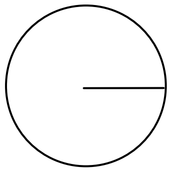
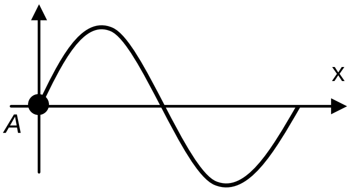
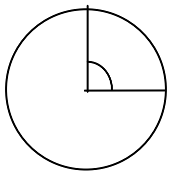
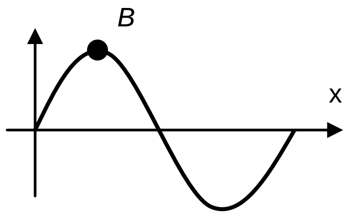
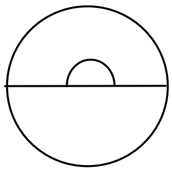
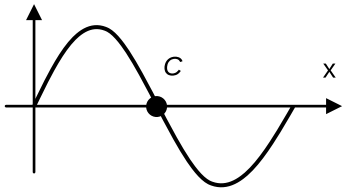
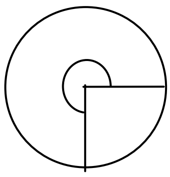
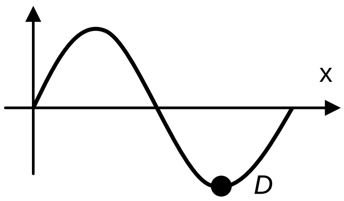
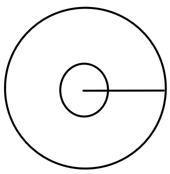
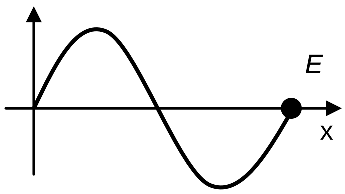
Phase is an angle, in degrees (°) or radians (rad) which gives a measure of the fraction of a cycle that has been completed by an oscillating particle or by a wave. One cycle corresponds to 360 ° or 2 π rad.

Phase difference ($\Delta\phi$) is a measure of how much one wave is out of step with another or how much one particle in a wave is out of step with another. It is expressed in terms of angles from 0 ° to 360 ° or 0 to 2π radians.

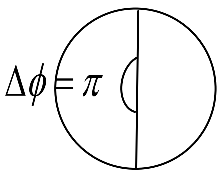
Two particles are *in phase* if they are in step with one another. (i.e. phase difference = *zero*).
Two particles are *out of phase* if they are *not* in step with one another. (phase difference ≠ 0).
Two particles are in *anti-phase*, if they are out of phase by half cycle (phase difference = 180° or π rad.)

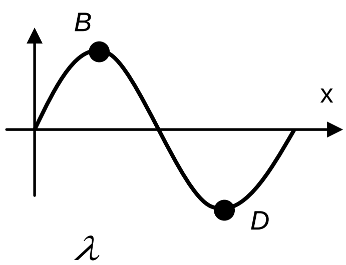
Note: Particles oscillating in phase need not have the same amplitude but they must have the same frequency.

Exercise: Find the phase of the Point *A*, *B*, *C*, *D* and *E* on the wave.

	Point on the wave	Phase Φ (relating to circular motion)
		**0**
		$\frac{\pi}{2}$
		π
		$\frac{3\pi}{2}$
		2π

Find the phase difference between *B* and *D*

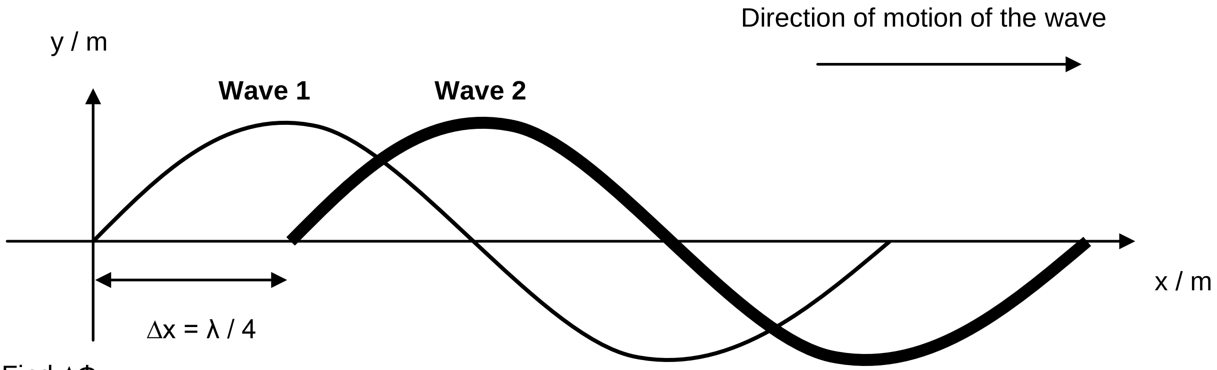




$$\Delta\Phi = \Phi_D - \Phi_B = \frac{3\pi}{2} - \frac{\pi}{2} = \pi$$

$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} \qquad \frac{\Delta\phi}{2\pi} = \frac{\frac{\lambda}{2}}{\lambda} \qquad \Delta\phi = \pi$$

Phase difference between 2 waves on displacement-distance graph: $\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$



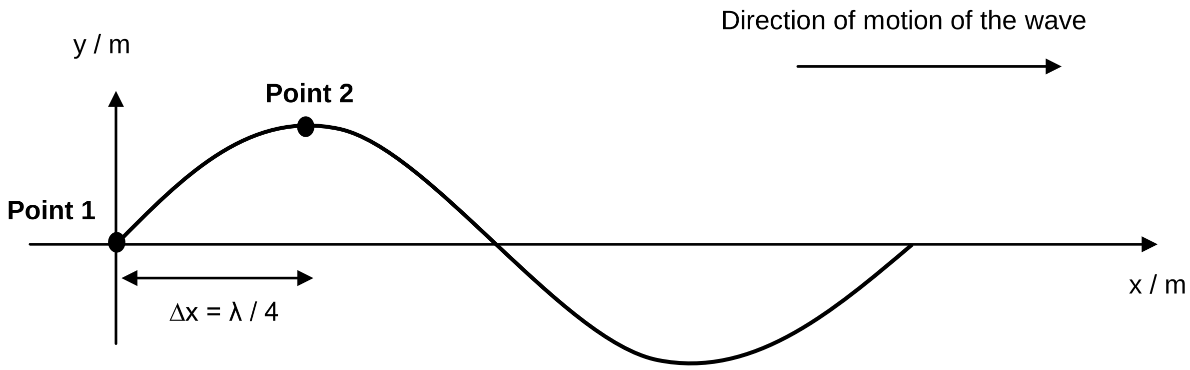
Find ΔΦ

ΔΦ = π / 2

Wave 2 is ___leading___ Wave 1 by ___**π / 2**___

Wave 1 is ____lagging___ Wave 2 by___**π / 2**___

Phase difference between 2 points on the same wave on displacement-distance graph:
$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$



Find ΔΦ

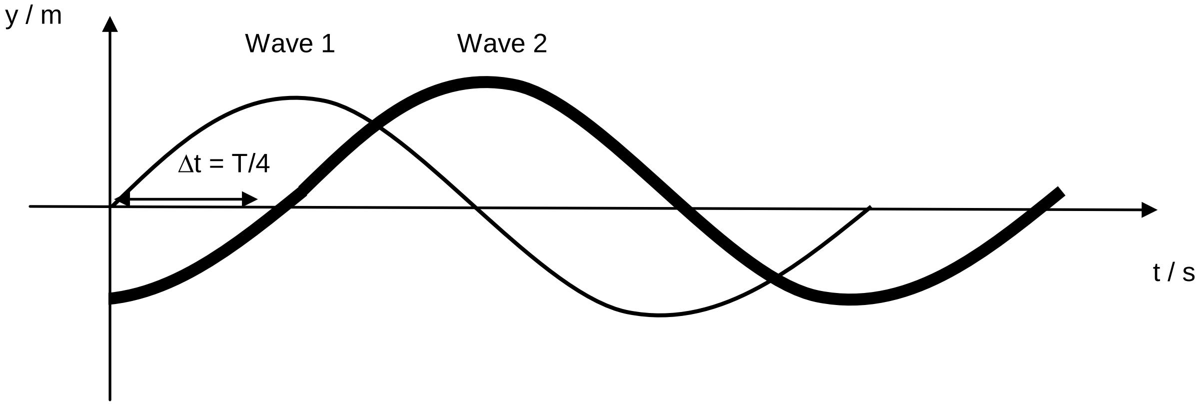
ΔΦ = π / 2

Point 2 is ___lagging___ Point 1 by ___**π / 2**___

Point 1 is ____leading___ Point 2 by___**π / 2**___

Phase difference between 2 waves on displacement-time graph: $\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T}$

Proof: $\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{v\Delta t}{\lambda} = \frac{f\lambda\Delta t}{\lambda} = \frac{\Delta t}{T}$



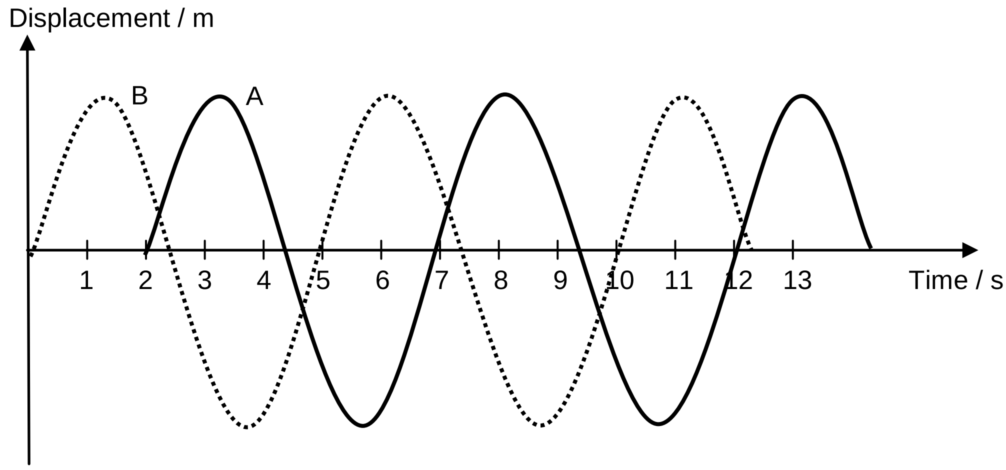
Find ΔΦ

ΔΦ = π / 2

Wave 2 is ___lagging______ Point 1 by ______________**π / 2**____

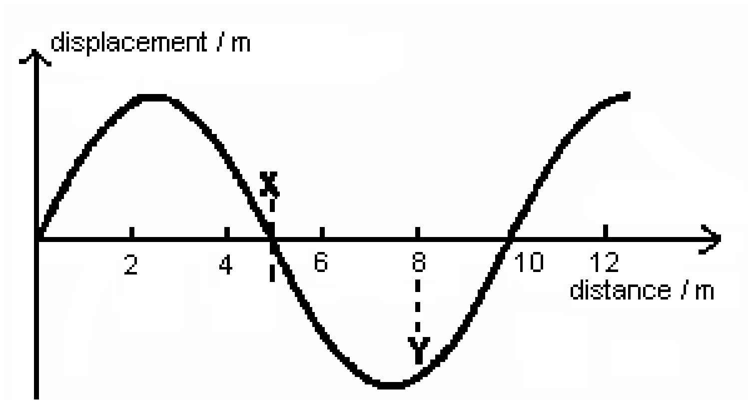
Wave 1 is ___leading______ Point 2 by______________**π / 2**___

E.g. 4 From the displacement-time graph below determine the phase difference for the waves A and B. State which wave is leading.



$$\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T} \Rightarrow \frac{\Delta\phi}{2\pi} = \frac{2}{5} \Rightarrow \Delta\phi = \frac{4\pi}{5}$$ $\Rightarrow$ **B leads A** by $\frac{4\pi}{5}$

E.g. 5 From the displacement-distance graph below, determine the phase difference between the particle at X and the particle at Y. State which particle is leading.



$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\frac{\Delta\phi}{2\pi} = \frac{3}{10}$$

$$\Delta\phi = \frac{3\pi}{5}$$

X leads Y by $\frac{3\pi}{5}$

E.g. 6 A sound wave of frequency 400 Hz is travelling in air at a speed of 320 m s^{-1}. Determine the phase difference between two points on the wave that are 0.20 m apart in the direction of travel.

V = f λ , λ = $\frac{v}{f} = \frac{320}{400}$ = 0.80 m , $\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{0.20}{0.80} \Rightarrow \Delta\phi = \frac{\pi}{2}$

10.6 Energy and Intensity of Waves

Recalling from the previous topic, the energy associated with an oscillation is proportional to the square of its amplitude. Recall SHM: Total Energy $E = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2} m (2\pi f)^2 x_0^2$
(Refer to Annex C for details)

Intensity (I) of a wave is the wave energy incident per unit time per unit area normal to the direction of energy transfer at any given point of the wave at a distance. Unit: W m^{-2}

Since $I \propto E$ and $E \propto f^2 x_O^2$, then $I \propto f^2 x_O^2$

For constant frequency f, $I \propto x_O^2$ $\quad$ $I \propto A^2$ $\quad$ where A is amplitude.

$$\text{Intensity} = \frac{\text{Energy}}{\text{Time} \times \text{Area}} = \frac{\text{Power}}{\text{Area}} \qquad I = \frac{P}{\text{Area}}$$

3 Dimensions(3D) Transmission

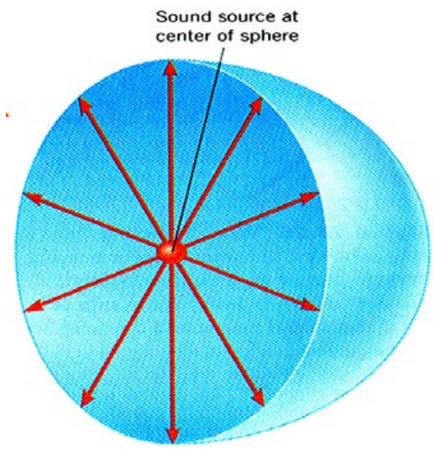
Consider a point source which emits energy with power P
Energy is transferred radially outwards in 3 dimensions (3D), carried on an expanding spherical surface.

At a distance r from the source,

$$I = \frac{P}{\text{Area}} \qquad I = \frac{P}{4\pi r^2}$$

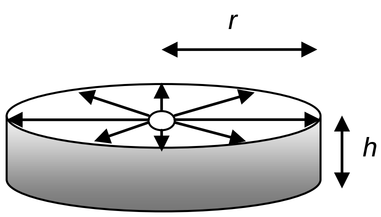
Where $4\pi r^2$ is the spherical surface area

Hence the intensity of a wave, $I \propto \frac{1}{r^2}$ (3D)



The further the distance from the point source, the lower the intensity of energy!

2 Dimensions(2D) Transmission



Consider a point source which emits energy with power P
Energy is transferred radially outwards in 2 dimensions (2D), for example, wave energy on the surface of the pond.

$$I = \frac{P}{\text{Area}} \qquad I = \frac{P}{2\pi r h}$$

Where $2\pi r h$ is the surface area along the circumference of the cylindrical shape

Hence the intensity of a wave, $I \propto \frac{1}{r}$ (2D)

E.g. 7 A 20 W loudspeaker is emitting sound at its full power in all directions. Find:

(a) the intensity of sound at a distance 10 m away

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{20}{4\pi(10)^2} = 0.0159 \text{ Wm}^{-2}$$

(b) the power received by a square microphone of length 2.0 cm placed at a distance 10 m away from the loudspeaker,

Since $I = \frac{P}{A}$, $P = I A = (0.0159)(0.02)(0.02) = 6.36 \times 10^{-6}$ W

(c) the amplitude of the vibrations at 10 m, given that at 2.0 m, the amplitude of the vibrations is 4.0 cm.

Since $I \propto A^2$ and $I \propto \frac{1}{r^2} \Rightarrow A^2 \propto \frac{1}{r^2} \Rightarrow A \propto \frac{1}{r}$

$$\frac{A_2}{A_1} = \frac{r_1}{r_2} = \frac{2}{10} = \frac{1}{5}$$

$A_2 = \frac{1}{5} \times 4.0 = 0.8$ cm.

E.g. 8 A point source of sound emits energy equally in all directions at a **constant rate** and a person 8 m from the source listens. After a while, the intensity of the source is halved. If the person wishes the sound to seem as loud as before, how far should he now be from the source?

$$I = \frac{P}{A} \qquad I = \frac{P}{4\pi r^2} \qquad I \alpha \frac{P}{r^2} \qquad I = k\frac{P}{r^2}$$

To get the same intensity when power is halved,

$$I_1 = k\frac{P_1}{r_1^2} \text{ ----- (1)} \qquad I_2 = k\frac{P_2}{r_2^2} \text{ ----- (2)}$$

$$\frac{(1)}{(2)}: \frac{I_1}{I_2} = \frac{P_1}{P_2}\frac{r_2^2}{r_1^2}$$

Sub in $I_2 = I_1$, $P_2 = \frac{1}{2} P_1$, we get

$$r_2 = \frac{r_1}{\sqrt{2}} = \frac{8}{\sqrt{2}}$$

$$r_2 = 5.66\, m$$

10.7 Determine the frequency of sound using a calibrated C.R.O.

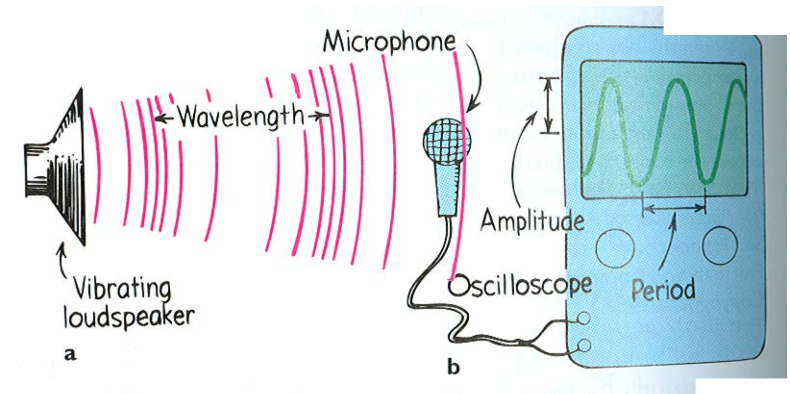
A signal (of unknown frequency) can be fed through a microphone into a cathode-ray oscilloscope (C.R.O.).

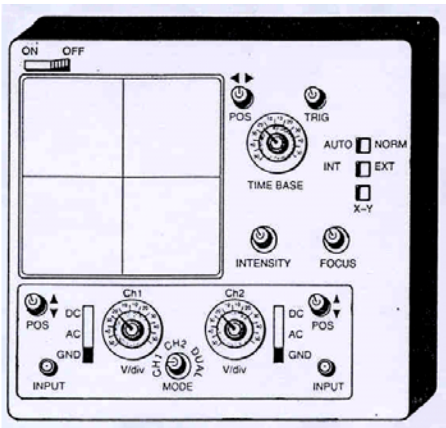
With the time-base of the C.R.O. turned on, the trace on the C.R.O. screen will be a display of the displacement against time.

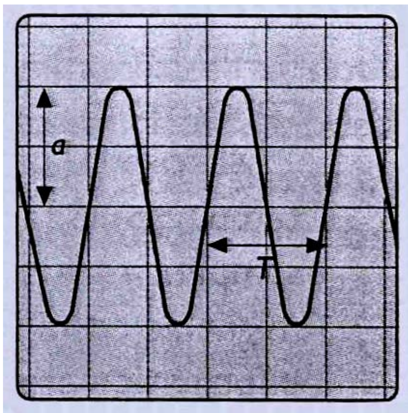
The time-base can be adjusted to appropriate value so that the period can be determined from the screen. Then the frequency of the signal can be calculated using $f = \frac{1}{T}$.

10.7.1 Functions of C.R.O.

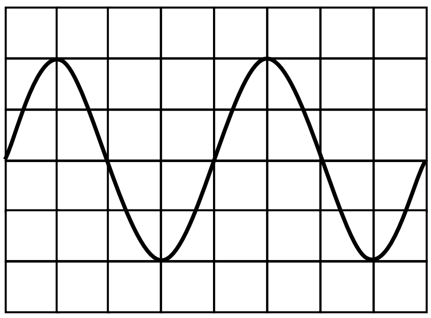
The microphone connected to the c.r.o. is used to pick up the sound produced by the loudspeaker. The input sensitivity setting of the *y*-plates (in V cm^{-1}) and the time-base (in ms cm^{-1}) setting to the *x*-plates can be adjusted so as to display an appropriate number of cycles on the c.r.o. screen. The screen is divided into squares marked 1 cm x 1 cm. (Refer to Annex D for more details)







E.g. 9 The following stationary wave pattern is obtained using a C.R.O. whose screen is graduated in centimetre squares. Given that the time-base is adjusted such that 1 cm on the horizontal axis of the screen corresponds to a time of 1.0 ms, find the period and frequency of the wave.

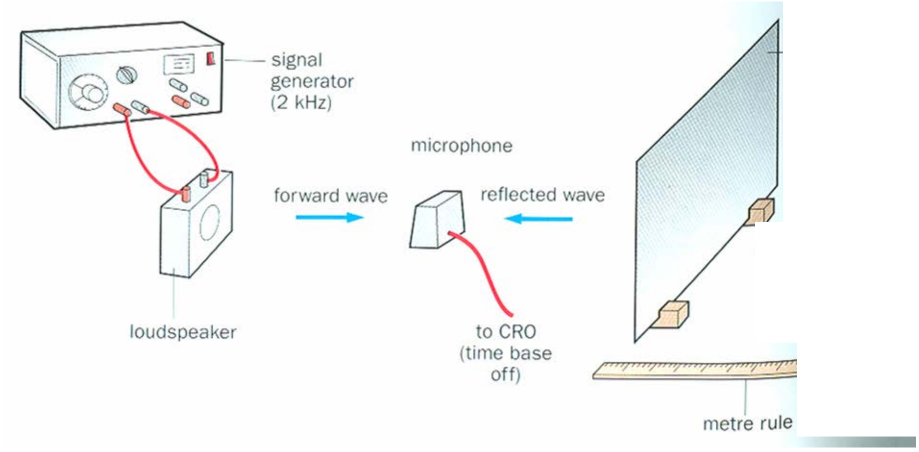


Since 1 cm represents 1.0 ms
Then 4 cm represents 4.0 ms

Therefore period T = 4.0 ms

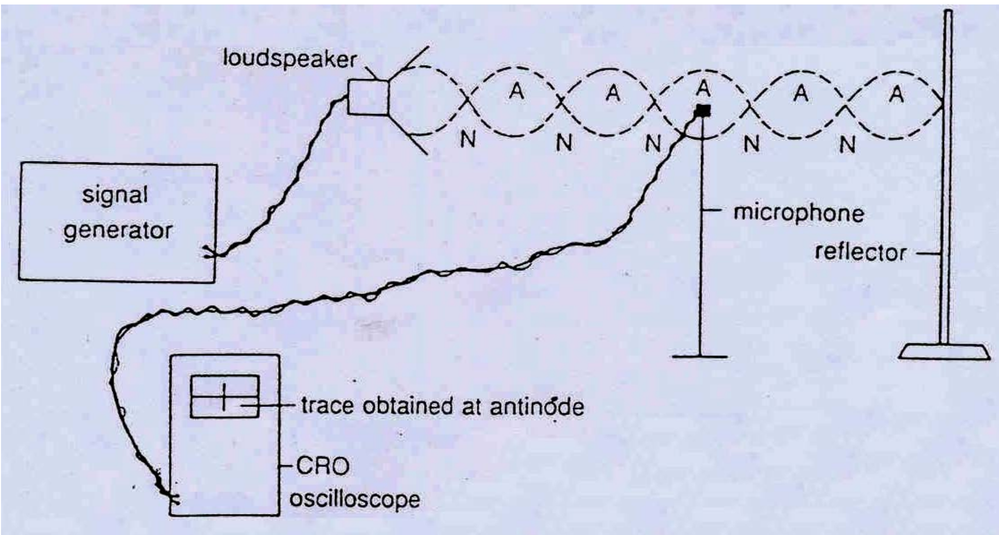
$$f = \frac{1}{T} = \frac{1}{4.0 \times 10^{-3}} = 250 Hz$$

10.8 Determine the wavelength of sound using stationary waves*



1. The loudspeaker delivers a sound via signal generator. The incident wave is directed towards and is reflected at the reflector plate.

2. S*uperposition* (more details in next chapter) of the incident wave and the reflected wave produces a *stationary wave* between the loudspeaker and the reflector. When two progressive waves of the same type of equal amplitude, equal frequency, equal wavelength, equal speed travelling in opposite directions meet and undergo superposition with each other, a stationary wave is formed. Vibrational energy of the wave is stored and not transmitted from one point to another. Wave has amplitudes of oscillation varying from maximum at antinodes to zero amplitude at nodes.



3. A microphone, connected to the c.r.o. (with time base switched off), is moved along the straight line between the loudspeaker and the reflector.

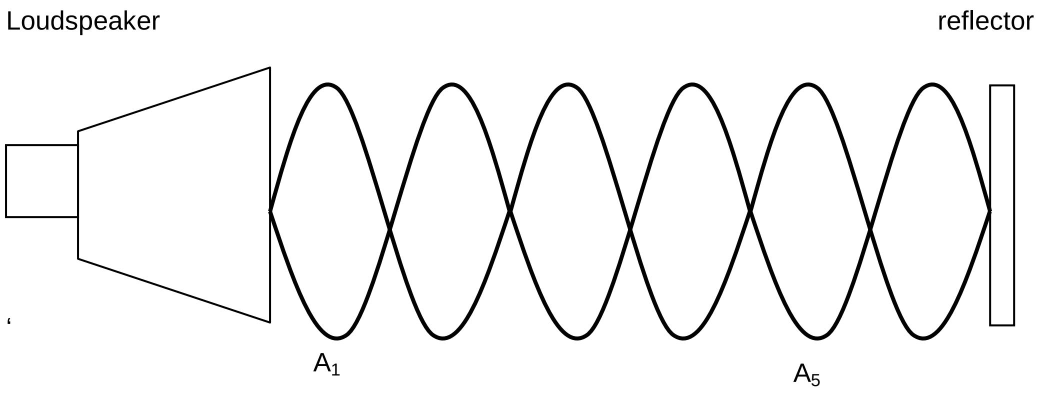
4. When the microphone is at positions of minimum amplitude (*nodes* N) the signal displayed on the c.r.o. is a minimum. When the microphone is at positions of maximum amplitude (*antinodes* A), the signal displayed on the c.r.o. is a maximum.

5. Several positions of the *antinodes* and *nodes* would be observed. The average distance *d* between two a<u>djacent</u> *antinodes* is then determined.

6. The wavelength of the sound can be calculated from $d = \frac{\lambda}{2}$

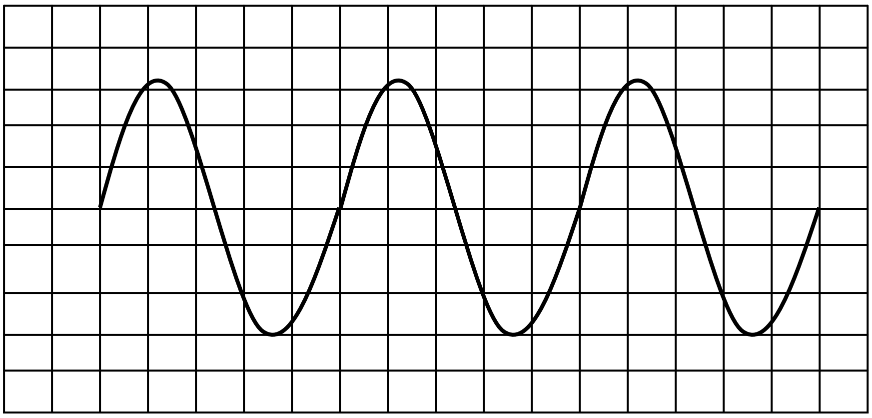
7. If the frequency of the sound *f* is known, then the speed of the sound wave $v = f\lambda = f\,(2d)$

E.g. 10 Find the speed of the sound wave given the following information:



When the microphone moved from the loudspeaker to the reflector, it was found that the distance between antinodes, A_1 and A_5 is 6.6 m.

When the microphone is place at A_5, the following waveform is shown on the CRO with a time-base = 2 ms / div



From 1st Figure

Distance between 2 consecutive antinodes is ½λ

4(½λ) = 6.6 m

2λ = 6.6 m

λ = 3.3 m

From 2nd Figure

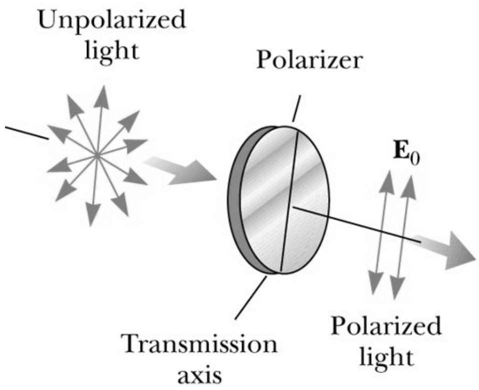
$T = 5 \times 2 \times 10^{-3} = 10 \times 10^{-3}$ s

$f = \dfrac{1}{T} = \dfrac{1}{10 \times 10^{-3}}$ = 100 Hz

$v = f\lambda = (100)(3.3) = 330\ ms^{-1}$

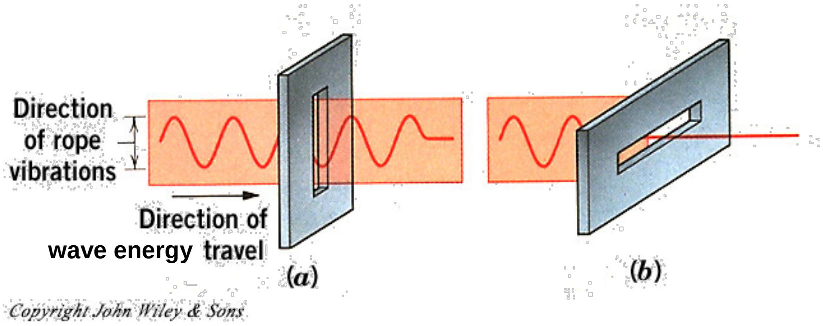
10.9 Polarisation

If the oscillations in a wave are confined to one direction only, in a plane normal to the direction of transfer of energy of the wave, the wave is said to be polarised in that direction.

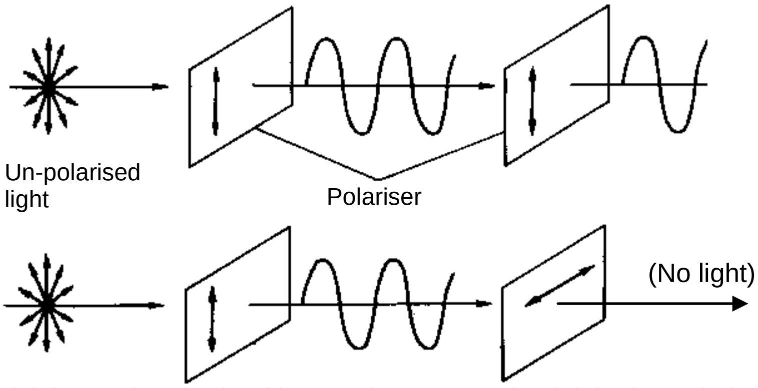


Only transverse waves may be polarised. Light is an example of a transverse wave that can be polarised.

Consider a rope that is set into vibration in a vertical plane such that it forms a transverse wave. If we place a vertical slit in the path of the wave, it will pass through unaffected. However, if we place a horizontal slit in its path, it will not pass through.



Such slits are similar to polarisers. A polariser (or Polaroid) is a thin sheet of crystalline material containing long chains of molecules arranged parallel to each other.



Plane-polarised light can be obtained by passing un-polarised light through the polarisers.

10.9.1 recall and apply Malus' Law, $I = I_o\cos^2\theta$, to calculate the intensity of light transmitted by a plane polarizer.

An ideal polariser passes 100% of the incident light polarised parallel to its polarising axis and blocks completely all light polarised perpendicular to this axis. An ideal polariser should transmit 50% of all incident unpolarised light. This is because the incident light is a random mixture of all states of polarisation, the vertical and horizontal components are, on average, equal.

Suppose we place two polarizers between a light source and a photocell as shown in the Figure. The first polariser has its polarizing axis held vertical. We can rotate the second polarizer (also known as the analyser) so that its polarizing axis makes an angle θ with the vertical. The transmitted intensity is a maximum when $\theta = 0^o$ (i.e. when the polarizing axes of the filters are parallel). The transmitted intensity is zero when $\theta = 90^o$ (i.e. when the filters are 'crossed'). At intermediate values of the angle θ, only the parallel component of amplitude $E\cos\theta$ is transmitted. Below is the working to find the transmitted intensity at intermediate values of the angle θ:

Let I_o be the intensity of light incident on the ideal polarisers and I is the amount transmitted at an angle θ. Recall that the intensity of an electromagnetic wave is proportional to the square of the *amplitude* of the wave, $I_0 \propto E^2$ ------------------- (1)

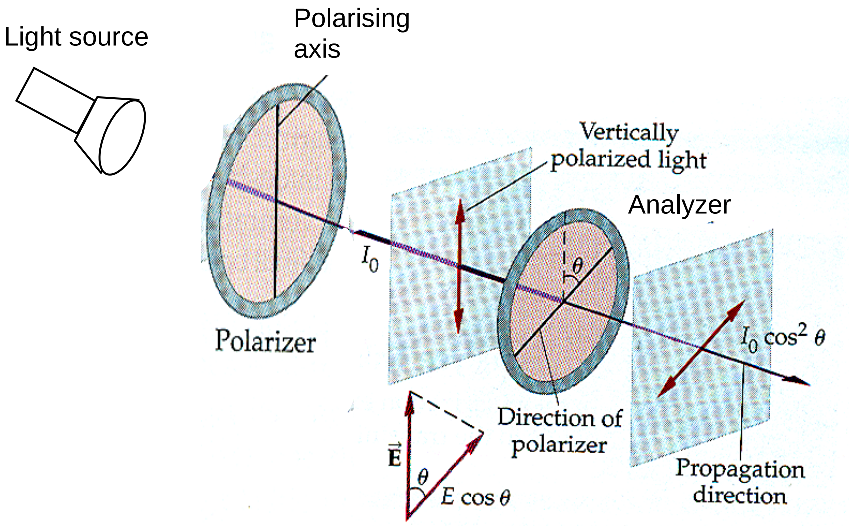
$$I \propto (E\cos\theta)^2 \quad \text{or} \quad I \propto E^2\cos^2\theta \quad \text{-------------------} \quad (2)$$

$$\frac{(2)}{(1)}: \qquad \frac{I}{I_0} = \cos^2\theta$$

Thus the intensity of the light transmitted through the analyzer is can be found by Malus' Law

$$I = I_o\cos^2\theta \qquad \textbf{(6.1)}$$

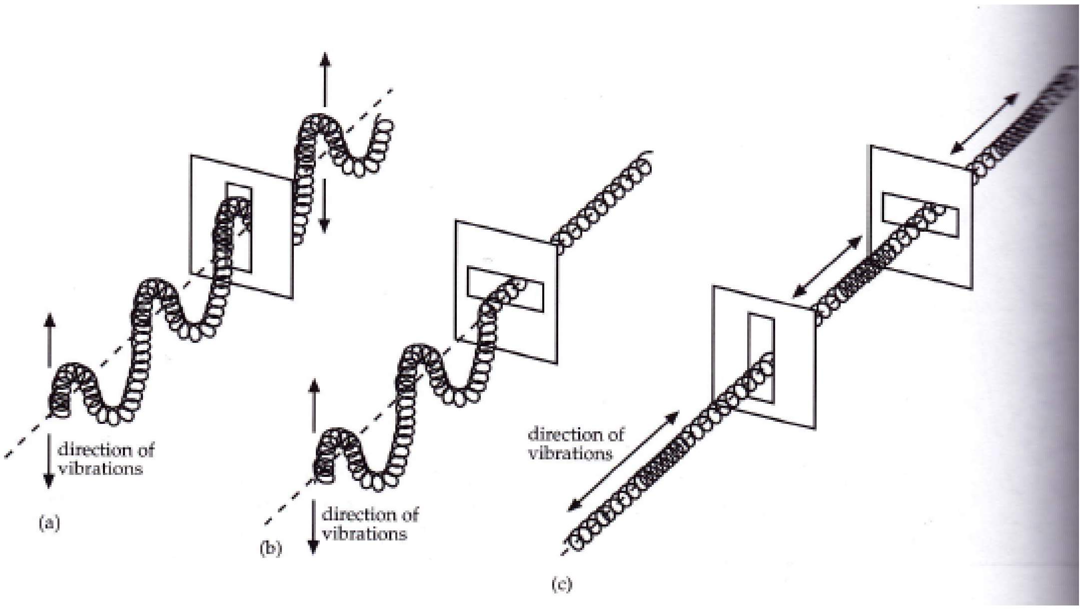
Malus' law applies only if the incident light passing through the analyzer is already linearly polarized.



See Annex E for more details

E.g. 11 Explain why it would not be possible to polarise sound waves.

Only transverse waves can be polarised. However sound waves are longitudinal waves and do *not* have oscillations in the plane normal to the direction of transfer of energy of the wave. Thus it cannot be polarised.



E.g. 12

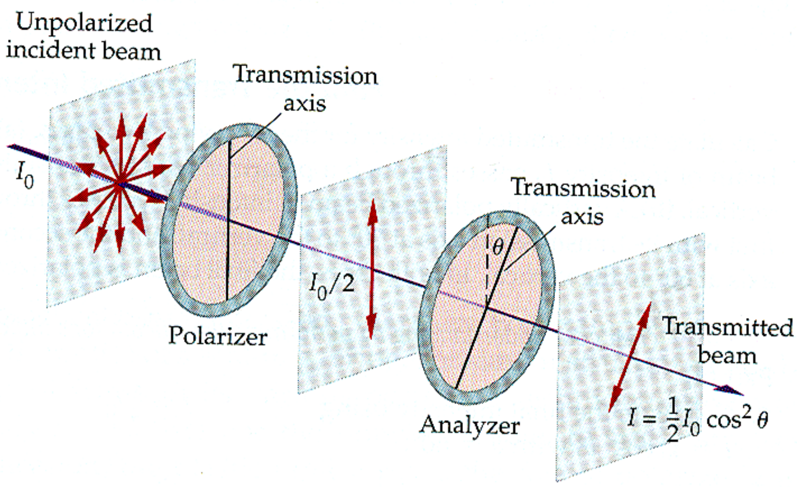
Given that unpolarized light with intensity I_o is incident on a pair of filters, find the intensity of transmitted light after the second filter if the angle between their polarizing axes is 30^o.

The first polariser transmits 50% of all incident unpolarised light thus $I_1 = \frac{1}{2} I_0$

After passing through the first polariser, the light is linearly polarised.

Apply Malus' Law for the second polariser, $I_2 = I_1 \cos^2 \theta$

The intensity transmitted by the second polariser is $I_2 = \frac{I_o}{2}\cos^2(30^o) = \frac{I_o}{2}(\frac{3}{4}) = \frac{3}{8} I_o$.



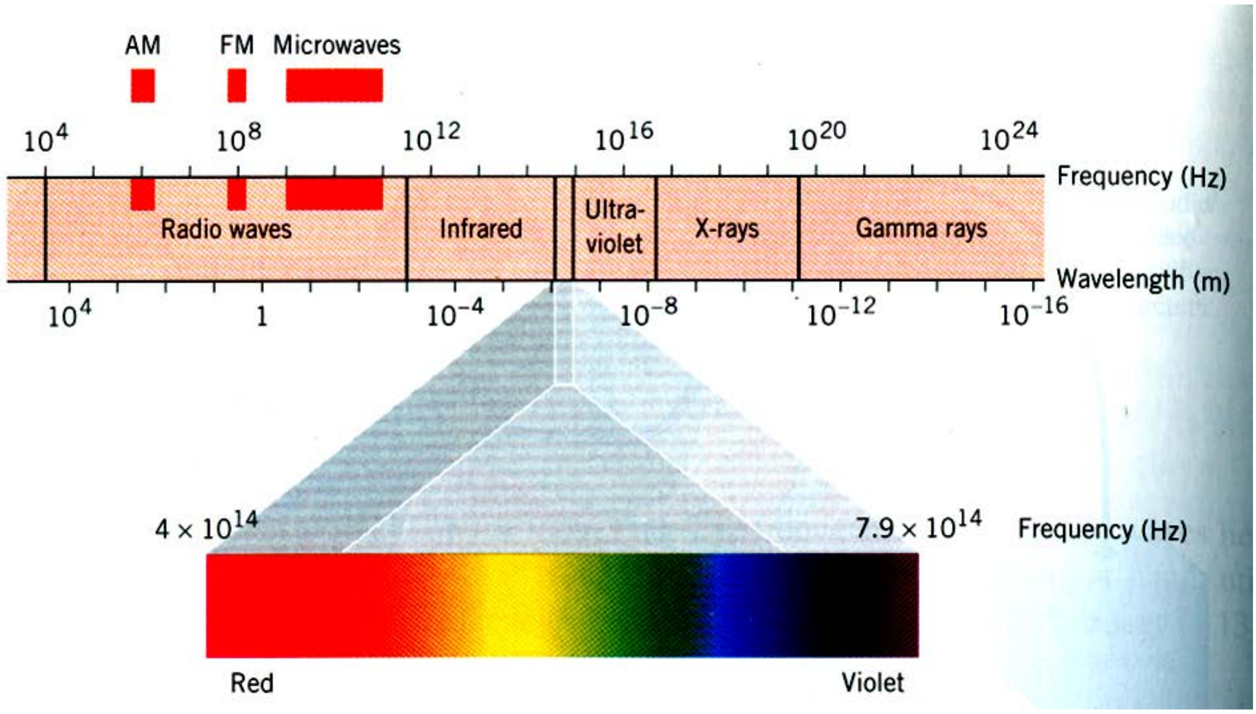
Annex A
Electromagnetic Waves

Electromagnetic waves consist of varying electric and magnetic fields. The two fields are perpendicular to each other, always in phase and vibrate with the same frequency (which is the frequency of the wave itself). The two fields are also perpendicular to the motion of the wave.

Properties of electromagnetic waves:

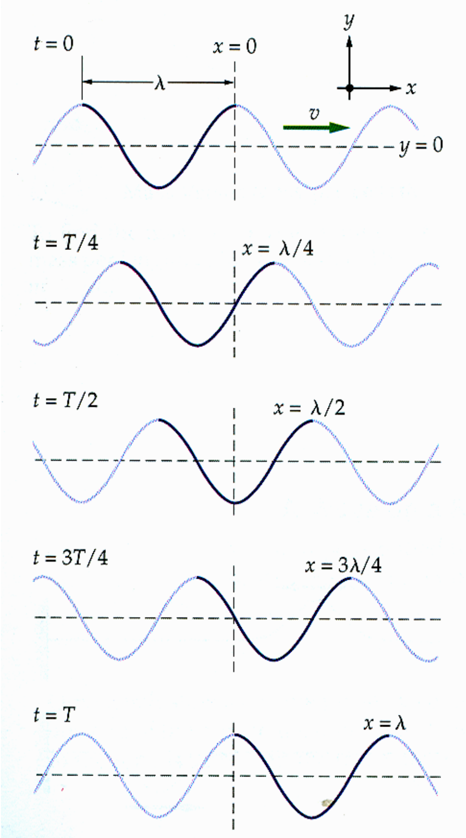
1) All electromagnetic waves travel in vacuum with the same speed (= 3.0×10^8 m s^{-1})
2) All electromagnetic waves transfer energy from one place to another and can be absorbed/emitted by matter.

Electromagnetic waves differ from one another in their frequency. The classification of electromagnetic waves according to frequency is the electromagnetic spectrum.



Electromagnetic Wave		Source of wave	Properties
Frequency	Wavelength		
Radio waves 10^4 to 10^9 Hz	Range from less than 1 cm to hundreds of metre	Oscillating electrons in special circuits coupled to radio aerials	Different wavelengths find specialized uses in radio communications.
Microwaves 10^8 to 10^{12} Hz	Range from 1 mm to 30 cm	Special electronic devices e.g. klystron	Radar communications in aircraft navigation.
Infra-red 10^{10} to 10^{14}Hz	Extends about 1 mm beyond the red light in the visible spectrum.	Produced by hot bodies and molecules' rotation and vibration energies	Cause sensation of warmth.
Visible Light 10^{14} to 10^{15} Hz	Range from 400 nm to 700 nm	Re-arrangement of outer orbital electrons in atoms and molecules.	Initiate chemical reactions
Ultra-violet 10^{15} to 10^{17}Hz	Range from 10 nm to 400 nm	Orbital electrons of atoms in high voltage gas discharge tubes and the mercury vapour lamp.	Ionise atoms in atmosphere and cause many chemical reactions e.g. tanning of human skin.
X-rays 10^{15} to 10^{25}Hz	Range from 0.1 nm to 10 nm	(a) Rapid deceleration of fast-moving electrons e.g. by tungsten target. (b) Changes in energy of innermost orbital electrons.	Used in radiography – treatment of certain forms of cancer.
γ-rays 10^{17} to 10^{32}Hz	Range from 1 pm to 100 pm	Changes of energy levels in the nucleus	Highly penetrating and produce serious damage when absorbed by living tissues.

Annex B Mathematical Description of a Wave



When a wave travels through a medium, it displaces the particles of the medium from their undisturbed positions. We want to consider the mathematical equation that describes the displacement *y* of a particle as a function of time *t* and position *x* from the source of the wave.

The displacement *y* of the particle (wave) on position *x* is of the form

$$y = A\cos\left(\frac{2\pi}{\lambda}x\right) \qquad (1)$$

The wave repeats itself when *x* increases by an amount equal to its wavelength λ, so replacing *x* by ($x + \lambda$) should give the same value of *y*.

$$A\cos\left[\frac{2\pi}{\lambda}(x+\lambda)\right] = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\right) = A\cos\left(\frac{2\pi}{\lambda}x\right)$$

Equation (1) is correct in that it describes a vertical displacement that repeats itself with a wavelength λ.

This is only part of the equation. We have not yet to describe how the wave changes with time. This is shown above for the wave at $t = 0$, $t = \frac{T}{4}$, $t = \frac{T}{2}$, $t = \frac{3T}{4}$ $t = T$. Note that the peak in the wave that was originally at $x = 0$ at $t = 0$ moves to $x = \frac{\lambda}{4}$, $x = \frac{\lambda}{2}$, $x = \frac{3\lambda}{4}$ $x = \lambda$ for the times given. The position *x* of this peak can be written as follows:

$$x = \lambda\frac{t}{T}$$

In general, if the position of a given point on a wave at $t = 0$ is $x(0)$ and its position at time *t* is $x(t)$, the relation between these two positions is

$$x(t) - \lambda\frac{t}{T} = x(0)$$

So, to take into account the time dependence of the wave, we replace *x* in equation (1) with $x - \lambda\frac{t}{T}$ to give the wave equation

$$y = A\cos\left[\frac{2\pi}{\lambda}\left(x - \lambda\frac{t}{T}\right)\right] \quad \text{or} \quad \boxed{y = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right)}$$

This equation depends on both time *t* and position *x*. The wave repeats itself whenever position increases by wavelength λ or time increases by period *T*.

Annex C

Intensity of waves (derivation – more for enrichment)

Waves (progressive) transmit energy from one place to another.

As waves travel through a medium, the energy is transmitted as *vibrational* energy from particle to particle of the medium.

For a sinusoidal wave of frequency f, the particles move in SHM as the wave passes, so each particle has an **energy** E,

$$E = \frac{1}{2} m(2\pi f)^2 x_O^2$$
$$= 2\pi^2 m f^2 x_O^2$$

where x_o is the amplitude of its motion.

Since $m = \rho V$, where ρ is the density of the medium and V its volume,

and $V = Al$, where A is the cross-sectional area through which the wave travels and l is the distance the wave travels in a time t,

and that $l = vt$, where v is the speed of the wave,

$$m = \rho V = \rho Al = \rho A\, vt$$

Hence,

$$E = 2\pi^2 \rho A\, vt\, f^2 x_O^2$$

The energy transported by a wave is proportional to the square of the amplitude.

$$\boldsymbol{E \propto x_O^2}$$

Since **power** P is the average rate of transfer of energy,

$$P = \frac{E}{t} = 2\pi^2 \rho A\, v\, f^2 x_O^2$$

The **intensity / Intensity** (I) of a wave is the wave energy incident per unit time per unit area normal to the direction of energy transfer. Hence,

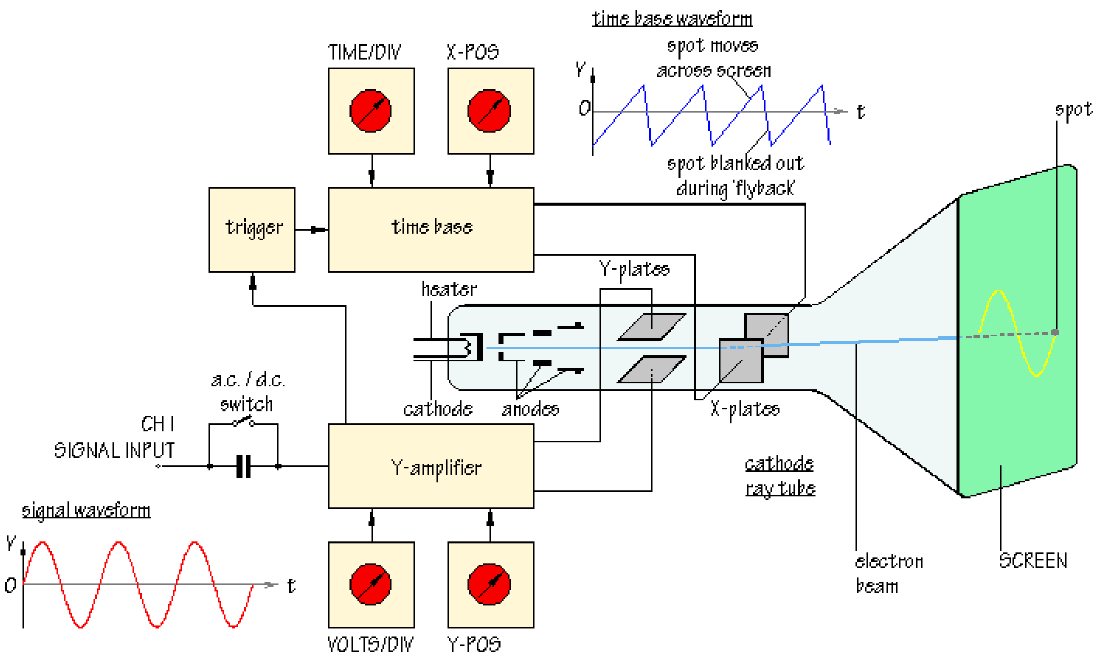
$$I = \frac{P}{A} = 2\pi^2 \rho\, v\, f^2 x_O^2$$
$$I \propto f^2 x_O^2$$

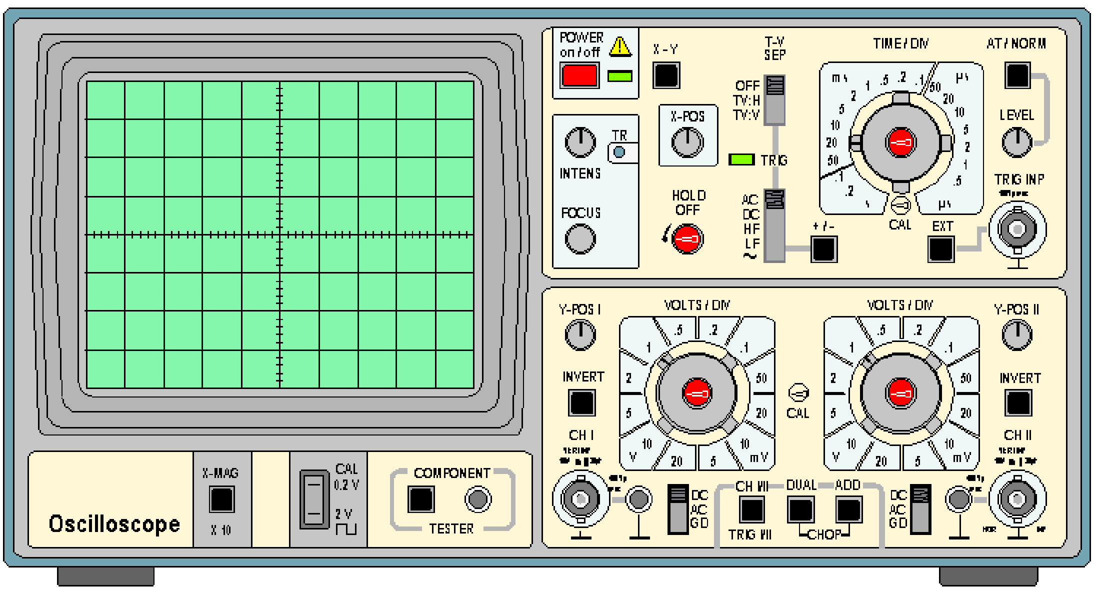
For constant frequency *f*,
The intensity of a wave is proportional to the square of the wave amplitude.

$$\boldsymbol{I \propto x_O^2 \quad \text{OR} \quad I \propto A^2}$$

Annex D Cathode Ray Oscilloscope

The Cathode Ray Oscilloscope is an electronic instrument used in making electrical measurements. It consists of a cathode ray tube, which is a vacuum tube in which electrons are accelerated and deflected by 2 sets of plate placed at right angles to each other. The charge on the horizontal plates is programmed to change in such a manner that the beam sweeps across the face of the tube at a constant rate. The sound wave is picked up by a microphone, which changes the sound signal to an electric signal that is applied to the vertical plates. The combined effect of the horizontal and vertical plates causes the beam to sweep horizontally and up and down at the same time, resulting in a sinusoidal waveform displayed on the oscilloscope screen.





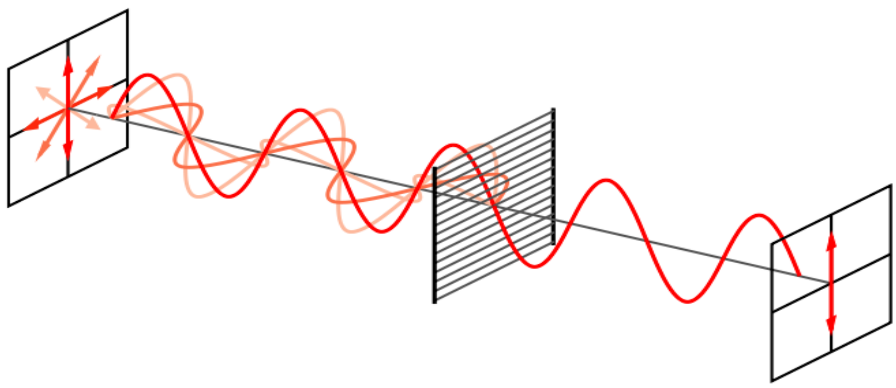
Annex E Concept of polarisation and how polarisers work

Polaroid is a material that polarizes light through selective absorption by orientated molecules. This material is fabricated in thin sheets of long-chain hydrocarbons, which are stretched during manufacture so that the molecules align. After the sheet is dipped into a solution containing iodine, the molecules become good electrical conductors. However, the conduction takes place primarily along the hydrocarbon chains, because the valence electrons can move easily along the chains. As a result, the molecules readily absorb light whose electric field vector is parallel to their lengths, and transmit light whose electric field vector is perpendicular to their lengths. It is common to refer to the direction perpendicular to the molecular chain as the transmission axis. In an ideal polarizer, all the light with electric field parallel to the transmission axis is transmitted, and all the light with electric field perpendicular to the transmission axis is absorbed.

Polarization is a characteristic of all transverse waves. Light, a form of electromagnetic waves consist of electric and magnetic fields, oscillating perpendicular to each other and to the direction of propagation. By convention, the direction of polarization is taken to be that of the electric field vector. Light is generally unpolarized, such as incandescent light bulbs and fluorescent light. The light emitted from an individual atom is polarized but when light from a vast number of atoms with random orientations is combined, there is no preferred polarization direction. Thus, the emitted light is a random mixture of waves linearly polarized in all possible transverse directions called **unpolarized light** or natural light.

Radio waves, on the other hand, are usually polarized because an antenna with a definite orientation generates them. For instance, the vertical rod antennas that are used for cellular telephones in automobiles emit waves that are polarized in the vertical direction. Nonetheless, visible light can be polarized, as all other forms of electromagnetic radiation by passing it through a **polarizing filter** (known commercially as ‘Polaroid’ which is widely used for sunglasses and camera lenses). Light passes through a polarizing filter and emerges with only one allowed electric field orientation. If the polarized light then meets a second filter, with its polarizing direction at right angles to that of the first filter, no light can pass. Two such Polaroid sheets are said to be ‘crossed’.

Perhaps, the easiest to understand how a polarizer works in a practical sense is if we use microwaves as an example. Microwaves has wavelengths of a few centimeters, thus a grid of horizontal wires spaced few centimetres apart will block horizontally polarized microwaves because all the energy in the horizontal component of its electric vector is being absorbed making electrons move left and right along the wires. The electric vector orientated in the vertical direction pass through almost unaffected, since electrons cannot move through the air between the wires (Figure below).



Tutorial 10 - Waves

Self-Attempt Questions

1. (a) Explain, using clearly labelled sketches where appropriate, the meaning of the following terms when applied to wave motion.

    (i) displacement

    (ii) amplitude

    (iii) wavelength

    (iv) frequency

    (v) period

    (vi) phase difference between 2 waves

   (b) Derive an expression for a wave's speed of in terms of its frequency and wavelength.

   (c) Explain what is meant by (i) progressive wave

   (ii) Intensity of a wave

   (d) Distinguish between longitudinal waves and transverse wave.

2. (a) Sketch a displacement-time graph for a *point* in a sinusoidal wave with a frequency of 100 Hz and an amplitude of 10 mm.

   (b) Sketch on the same axes, a second graph for a point on another wave with twice the frequency and half the amplitude.

   (c) Given that both waves travel at 330 m s^{-1}, sketch the "displacement-distance" graph on the same axes for both waves.

   (d) State the type of wave and suggest a more precise label for the graph in (c).

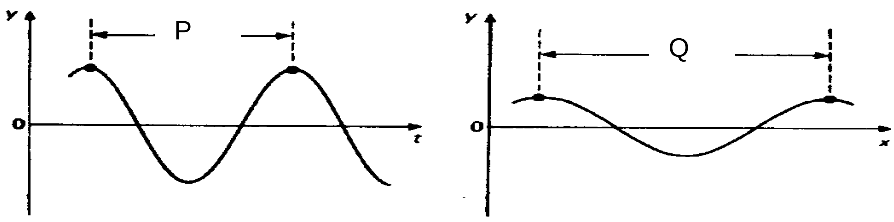
3. Visible light has wavelengths between 400 nm and 700 nm, and its speed in a vacuum is 3.0×10^8 m s^{-1}. Calculate the minimum and maximum frequency of visible light.

4. Ms Kitty has a huge circular bowl of diameter 400 mm that contains water at rest. One day, she tapped the side of the bowl gently and discovered that a complete circular pulse can be produced on the surface of the water which travels inwards with a speed of 250 mm s^{-1}. Determine the radius of the pulse and its direction of travel (inwards or outwards), 1 second after the pulse is produced.

5. The same progressive wave is represented by the two graphs shown below.

Displacement *y* against time *t* for constant position

Displacement *y* against position *x* for constant time

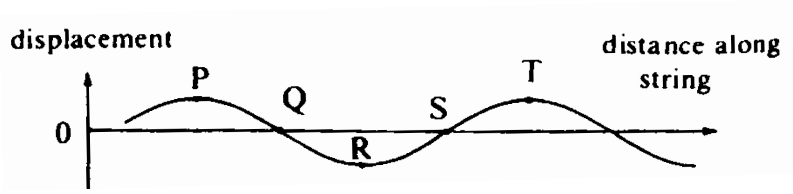


Write an expression for the speed of propagation of the wave.

6. The figure below shows an instantaneous position of a string as a transverse progressive wave travels along it from *right to left*.

Indicate the directions of the velocities of the points 1, 2 and 3 on the string.

7. The graph below shows the shape at a particular instant of part of a transverse wave travelling along a string.



State whether the following statements are true or false:

(a) The speed of the element at P is a maximum

(b) The displacement of the element at Q is always zero

(c) The energy of the element at R is entirely kinetic

(d) The energy of the element at S is entirely potential

Discussion Questions

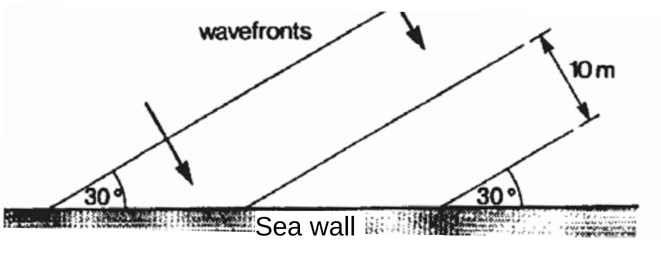
8. (a) TYS Pg (10)3 Q6 [2008 P1 Q20] (b) TYS Pg (10)4 Q8 [2010 P1 Q20]

9. When a mass is dropped in still water, the amplitude of the resulting ripples decreases with increasing distance from the point of entry of the mass. Explain why.

10. A certain wave has a wavelength of 1.00 m. Determine the distance between two points on this wave with a phase difference of $\frac{\pi}{4}$ rad.

[J93/II/2b]

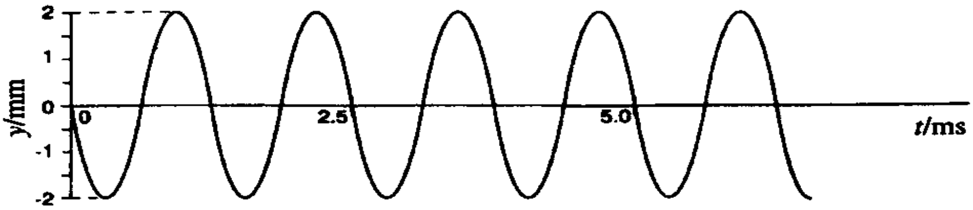
11. Parallel water waves of wavelength 10 m strike a straight sea wall. The wave fronts make an angle of 30° with the wall as shown in the figure below.



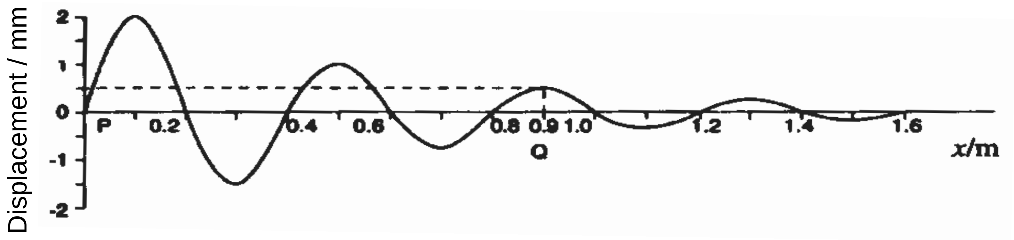
Determine the difference in phase at any instant between the waves at two points 5 m apart along the wall.

[N95/I/10 mod]

12. (a) A progressive wave moves past two points P and Q, which are separated by a distance of 0.90 m. A graph showing how the displacement *y* at P varies with time *t* is shown in figure below.



Another graph showing how the displacement of the wave at time t = 0 varies with distance x from P is also shown below.



Using data from the graphs, deduce for this wave

(i) the speed

(ii) the phase difference between the oscillations at P and those at Q

(iii) the ratio $\dfrac{\text{amplitude at P}}{\text{amplitude at Q}}$

(iv) the ratio $\dfrac{\text{intensity at P}}{\text{intensity at Q}}$

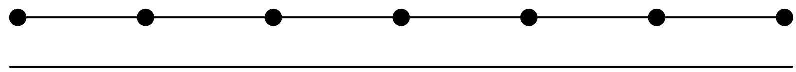
(b) Light waves, sound waves in air and surface water waves are different forms of waves. Suggest, with a reason, which of these might be the wave being considered in (a).

(c) (i) Suggest an experimental method to obtaining the first graph.

(ii) Discuss whether the same method could be used for the second graph.

[N96/III/2 part]

13. (a) The figure below illustrates seven particles equally spaced along a horizontal line in still air.



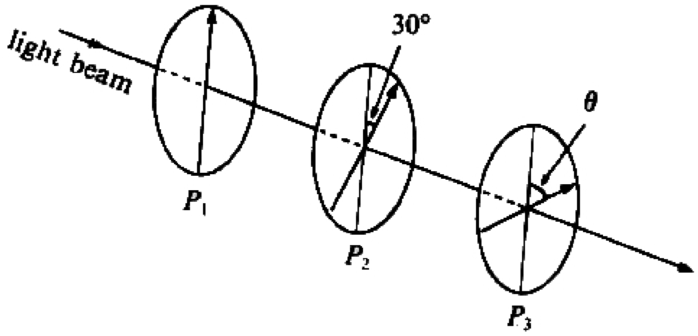
A sound wave is sent through the air rightwards along the horizontal line. Given that the wavelength of the wave spans the distance between the first and the seventh particle, draw on the second line the possible positions of the seven particles in the wave relative to their undisturbed positions

(b) Use the information from your diagram to draw two graphs, one to show how the displacement d of the particle varies with the horizontal distance x and one to show how the pressure p in the air varies with x.

(c) State the phase difference between the displacement wave and the pressure wave.

14. (a) Thin parallel wires are stretched across a wooden frame. Explain why vertically polarised microwaves are transmitted through the frame most strongly when the wires are horizontal.

(b) A beam of un-polarised light passes through three Polaroid P_1, P_2 and P_3 as shown. P_1 and P_2 have polarising axes at 30° to each other, and the polarising axis of P_3 can be varied at an angle θ to that of P_1.



Find the values of θ between 0^o and 360^o for which the intensity of the emergent light is at a minimum.

Challenging Questions

15. A helium-neon laser emits an intense, monochromatic beam. Typical beam divergence is about 1.2 milliradians. The beam is plane-polarised and exhibits a particularly low level of optical noise…

    (a) Explain what is meant by each of the following terms in the passage

        (i) monochromatic

        (ii) plane-polarised

        (iii) optical noise

    (b) If the laser beam is directed normally at a wall 6.0 m away, estimate the diameter of the spot of the light on the wall.

    (c) Describe briefly how you would confirm that the beam is plane polarised.

16. Two radio transmitters emit electromagnetic waves of frequency 9.0×10^7 Hz. Calculate

    (a) the internodal distance in the standing wave set up along the line joining the transmitters

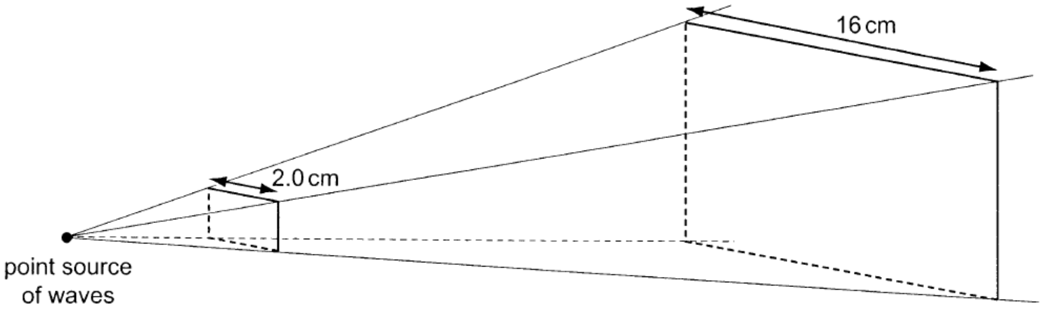
    (b) the rate the nodes are passed by a mobile receiver which is moved along the line joining the transmitters at a speed of 10 m s^{-1}.

(Ans: 1.67 m, 6 nodes per second)

17. A pair of crossed polaroid sheets will not transmit light. Show why some light is transmitted when a third polaroid sheet held at 45º to the crossed axes is inserted between two crossed polaroids.

2008 P1 Q20

20 Waves from a point source pass through an area that is 2.0 cm wide, as shown.



Within this area, the intensity of the waves is *I* and their amplitude is *A*. The waves reach a second area of width 16 cm.

What will be the intensity and amplitude of the waves when they reach the second area?

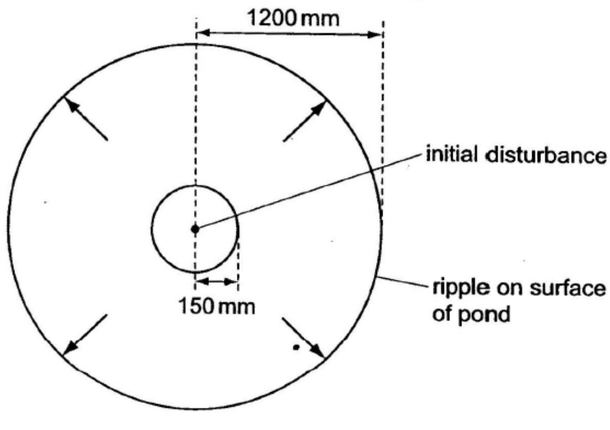
	intensity	amplitude
A	$\frac{I}{8}$	$\frac{A}{4}$
B	$\frac{I}{64}$	$\frac{A}{4}$
C	$\frac{I}{64}$	$\frac{A}{8}$
D	$\frac{I}{256}$	$\frac{A}{16}$

2010 P1 Q20

20 Ripples on the surface of a pond spread out in circles from the point of an initial disturbance.

Assume that the energy of the wave is spread over the entire circumference of the ripple.

For one such ripple, the amplitude of the ripple at a distance of 150 mm from the disturbance is 2.0 mm.



What will be the amplitude of the ripple at a distance of 1200 mm from the disturbance?
(Assume that no energy is lost in the propagation of the ripple.)

A 0.031 mm **B** 0.13 mm **C** 0.25 mm **D** 0.71 mm