RAFFLES INSTITUTION
YEAR 5-6 PHYSICS DEPARTMENT

Tutorial 12 SUPERPOSITION

H2 PHYSICS 9749

Self - Check Questions

S1 What is the principle of superposition?

S2 How is a stationary wave formed?

S3 What is the phase difference between particles located between two adjacent nodes of a stationary wave? How about their amplitudes?

S4 What is the longest possible wavelength of a stationary wave set up in a string fixed at both ends? What about that of a stationary sound wave set up in a pipe with one end closed and with both ends open?

S5 What is diffraction? Is the effect of diffraction more pronounced when the size of the aperture is reduced or when it is enlarged?

S6 Why is diffraction of light not easily observed? (It was thought that light travels in a straight line.)

S7 What do you understand by the terms *coherence*, and *constructive* and *destructive interference*?

S8 What are the necessary conditions for two-source interference pattern to be observable?

S9 Explain why light from two monochromatic sources do not produce an observable interference pattern, but sound from two loudspeakers can.

S10 In a Young's double-slit experiment, what is the relation between the fringe separation x, the wavelength λ, the slit separation a and the slit to screen distance D? Under what conditions would the equation provide an accurate value of x?

S11 What is the grating formula? Define the symbols used and state the assumption used in its derivation.

S12 How do you calculate the maximum number of orders that can be observed when a beam of monochromatic light passes through a diffraction grating?

S13 State the expression which gives the angles at which the minima of a single-slit diffraction pattern occur. What condition is necessary for the expression to be valid?

S14 State the Rayleigh criterion for image resolution. Explain your symbols clearly.

S15 Explain why certain orders of the interference pattern of a diffraction grating are missing.

Self - Practice Questions

SP1 A suspension bridge is to be built across a valley where it is known that winds can gust at 5 s intervals. It is estimated that the speed of transverse waves along the span of the bridge is 400 m s^{-1}. Calculate the length of the bridge when there is danger of resonant vibrations at its fundamental frequency.

SP2 **J86/1/12**

A string of length L, fixed at both ends, is plucked at its midpoint and emits its fundamental note of frequency f_1. When the string is plucked at a different point, the first overtone frequency f_2 is also produced.

(i) What is the ratio f_2/f_1?

(ii) What is the speed of the transverse waves in the string in terms of f_2 and L?

SP3 An organ pipe of effective length 0.60 m is closed at one end. Given that the speed of sound in air is 300 m s^{-1}, determine the two lowest resonant frequencies.

SP4 A microwave transmitter emits waves which are reflected from a metal plate. The reflected wave and the incident wave superpose to form a standing wave. When a microwave detector is traversed along the standing wave, successive points at which the detector detected zero intensity were located 1.5 cm apart. What is the frequency of the waves?

SP5 Red light of wavelength 6.0×10^{-7} m is incident normally on a Young's double-slit, where the slits separation is 0.40 mm and the distance of the slits to the screen is 1.2 m.

(a) What is the separation of the fringes on the screen?

(b) What is the effect on the appearance of the fringes if

(i) the separation of the slits is decreased?

(ii) the screen is moved closer to the slits?

(c) What can be said about the fringe separation between higher order fringes?

SP6 A diffraction grating has 400 lines per mm and is illuminated normally by monochromatic light of wavelength 600 nm. Calculate

(a) the grating spacing,

(b) the angle to the normal at which the first order maximum is seen,

(c) the number of diffraction maxima observed.

SP7

Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P.

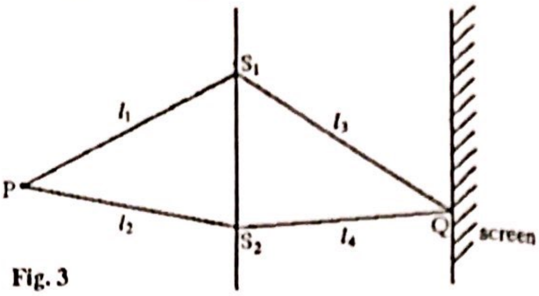


Fig. 3

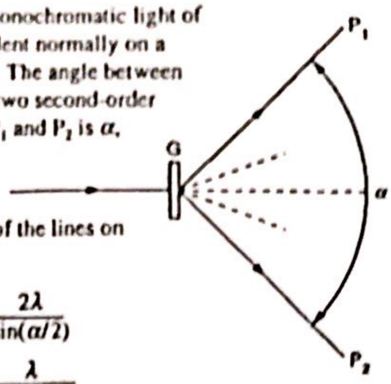
If, as shown in the diagram above (Fig. 3), the light is then allowed to fall on a screen, and if m is a positive integer, the condition for destructive interference at Q is that

A $(l_1 - l_2) = (2m + 1)\,\lambda/2$
B $(l_3 - l_4) = (2m + 1)\,\lambda/2$
C $(l_3 - l_4) = m\lambda$
D $(l_1 + l_3) - (l_2 + l_4) = (2m + 1)\,\lambda/2$
E $(l_1 + l_3) - (l_2 + l_4) = m\lambda$

N80/II/9

SP8

A parallel beam of monochromatic light of wavelength λ is incident normally on a diffraction grating G. The angle between the directions of the two second-order diffracted beams at P_1 and P_2 is α, as shown.



What is the spacing of the lines on the grating?

A $\dfrac{2\lambda}{\sin\alpha}$ C $\dfrac{2\lambda}{\sin(\alpha/2)}$

B $\dfrac{\lambda}{\sin\alpha}$ D $\dfrac{\lambda}{\sin(\alpha/2)}$

J86/I/11; N99/I/11

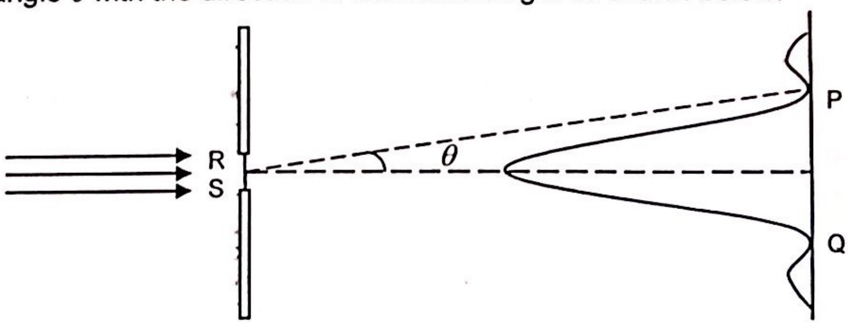
SP9

A parallel beam of white light (range of wavelengths 4.5×10^{-7} m to 7.5×10^{-7} m) is incident normally on a diffraction grating. The most deviated wavelength in the second order spectrum is diffracted through an angle of 60° from the direction of the incident beam. How many lines per metre are there on the grating?

A 5.8×10^5
B 9.6×10^5
C 11.6×10^5
D 19.2×10^5
E 2.3×10^6

J77/II/14

SP10 Monochromatic light of wavelength λ is incident normally on a single slit RS of width a. The diffraction pattern is formed on a screen PQ. The first minimum of this pattern makes an angle θ with the direction of the incident light as shown below.



Which of the following gives the correct expressions for the path difference SP–RP and for sin θ?

	SP–RP	sin θ
A	$\lambda/2$	$\lambda/2a$
B	$\lambda/2$	λ/a
C	λ	$\lambda/2a$
D	λ	λ/a

SP11 With reference to SP10, when the width of the slit is reduced,

A the intensities of all the peaks decreased and the width of the pattern remained unchanged.

B the intensities of all the peaks decreased and the width of the pattern decreased.

C the intensities of all the peaks remained unchanged and the width of the pattern increased.

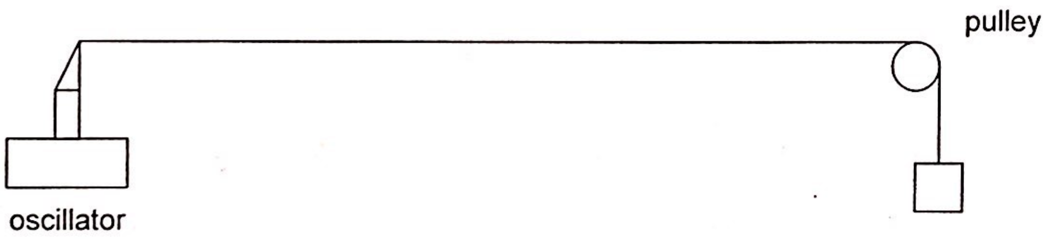
D the intensities of all the peaks decreased and the width of the pattern increased.

Discussion Questions

Stationary Waves

D1 ***J99/3/4***

One end of a horizontal string is attached to an oscillator and the other end is passed over a pulley. The string is kept under tension by means of a weight as shown below.



As the frequency of the oscillator is increased, stationary waves are produced on the string at certain frequencies.

(a) Draw the stationary wave on the string when the frequency is such that the distance between the oscillator and the pulley corresponds to two wavelengths of the wave on the string.

(b) On your diagram, label the position of nodes **N** on the string.

(c) Explain why a stationary wave is observed on the string only at particular frequencies of vibration of the oscillator.

D2 A 60.0 cm long, vertical hollow tube is initially submerged just below the surface of water. As it is raised, the tube first resonates to a vibrating tuning-fork of frequency 512 Hz when the air column above the water level is 14.8 cm. The next resonance is observed when the length of air column is 48.0 cm. Determine

(a) the end-correction, and

(b) the lowest frequency at which the tube will resonate when it is open at both ends.

D3 A loudspeaker S, which is emitting sound of frequency 600 Hz, is placed in front of a reflector R to produce a stationary wave. A sensitive microphone M is moved between R and S, and the signal from M is monitored on a CRO.

CRO signals show that the first peak-signal is detected when M is located at the 130 mm mark on a ruler placed between R and S. The third peak-signal is detected when M is at the 698 mm mark.

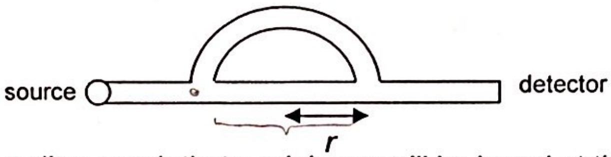
Calculate

(a) the speed of sound in air, and

(b) the positions of the two minima that lie between these two peak-signals.

Two-Source Interference

D4 Sound waves of wavelength 0.42 m enter a tube where a semi-circular pipe of radius r is linked to a straight pipe as shown below.



What is the smallest radius r such that a minimum will be heard at the detector?

D5 In a Young's double-slit experiment, the distance between slits is 0.50 mm and the slits are 1.0 m from the screen. Two interference patterns can be seen on the screen when lights of wavelengths 480 nm and 600 nm are incident perpendicularly to the slits. What is the separation between the third-order interference fringes on the screen?

D6 **J85/3/8 (modified)**

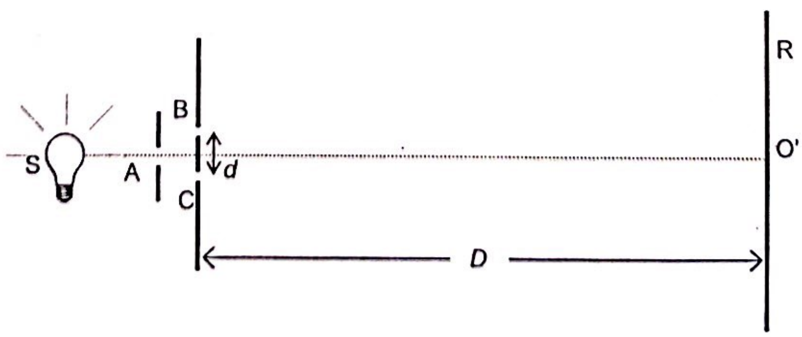
(a) What do you understand by

(i) coherence

(ii) interference

between two separate wave trains?

(b)



The figure above illustrates the Young's double-slit experiment. A source of light S illuminates a narrow slit A, which acts as a source for the narrow slits B and C, and produces fringes on the screen. With light of wavelength λ, bright fringes of separation s are formed on the screen. State the relation between λ, s, d, and D. Suggest suitable values for d and D.

Describe and explain what happens to the fringes if

(i) both slits B and C are made narrower whilst keeping *d* constant,

(ii) the light emerging from slit B is reduced in intensity to half that from slit C,

(iii) a thin sheet of transparent plastic is inserted between slit B and slit A,

(iv) slits B and C are both covered with sheets of polaroid with their polarising axis aligned, and that in front of B is slowly rotated.

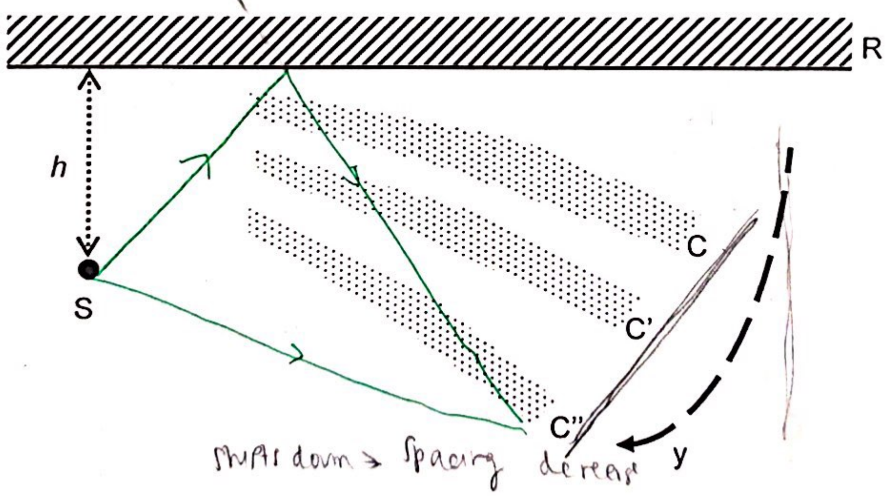
(c) When the wavelength of the source is 500 nm, the center of the 120th dark fringe counting from O' lies at R. Upon replacement of the source by one of unknown wavelength, R is found to be the location of the 90th bright fringe (take O' as the 0th order maximum). Find the wavelength of the unknown source.

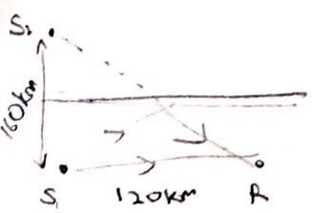
D7 **J79/3/1**

A source S of continuous waves a distance *h* from a plane reflector R produces regions of high intensity such as C, C' and C" as shown below. Account for this.

When the frequency of S is changed slowly, the regions C, C' and C" move in the direction y as shown. Account for this, and deduce whether the frequency has been increased or decreased. Assume that no phase change occurs upon reflection with R.

Young's Double Slit





$\sin\theta = \frac{n\lambda}{d}$

$D \gg d$

In the Appleton's experiment, S was a radio transmitter on the Earth's surface and R was the Heaviside layer – a reflecting layer in the atmosphere 80 km above the ground. When the transmitted wavelength is slowly changed from 200 m to 180 m, a receiver on the ground 120 km away from S observed fluctuations in the received signal strength. Calculate the number of signal maxima observed during this change of frequency.

Diffraction Grating

D8 In the spectrum of white light obtained by using a certain diffraction grating, the second and third orders partially overlap. What wavelength in the third-order spectrum will appear at the angle corresponding to a wavelength of 650 nm in the second-order spectrum?

D9 When a cadmium light is viewed through a diffraction grating having 500 lines per mm, the first order spectral lines were observed at the stated angles.

colour	angle / °
red	18.78
green	14.74
light blue	13.89
dark blue	13.53

(a) Find the wavelengths of each of these lines.

(b) What is the maximum order of each colour?

D10 J93/3/2

A beam of red light from a laser is shone normally on to a diffraction grating. Bright light is seen emerging at certain angles as shown in Fig. 10(a).

Use the principle of superposition to suggest a qualitative explanation of this effect.

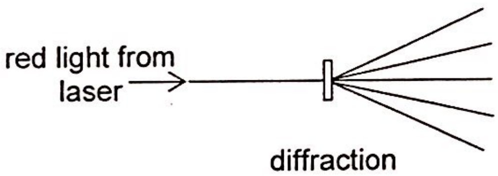


Fig. 10(a)

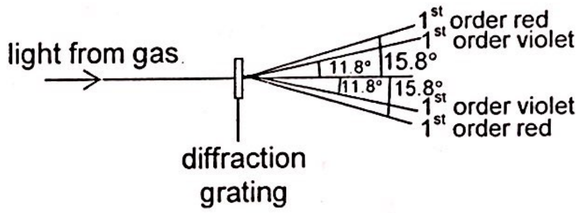


Fig. 10(b)

A diffraction grating with a grating spacing of 2.20×10^{-6} m is used to examine the light from a glowing gas. It is found that the first order violet light emerges at an angle 11.8° and the first order red light at an angle 15.8° as shown in Fig. 10 (b)

(a) Calculate the wavelengths of these two colours.

(b) Describe and explain what will be observed at an angle of 54.8°.

(c) Without making any further calculations, draw a sketch similar to Fig. 10(b) showing the whole pattern observed.

Rayleigh Criterion

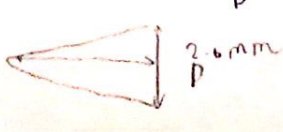
D11 Two closely spaced monochromatic point sources of light are viewed through a single slit of width *w*.

(a) Sketch the appearance of the images observed for small and large values of *w*.

(b) Sketch a graph showing the intensity variations along a line through the centres of the images for a critical value of *w* when the images are just resolvable.

(c) Suggest a way to improve the resolution of an imaging system which is used to observe small objects or short distances.

DR2 Estimate the size of the smallest object the eye can resolve at 25 cm.
$\lambda = 500$ nm
D = 2.0 mm (size of the pupil during the day)



2.0mm D

@ 20mm = 250mm

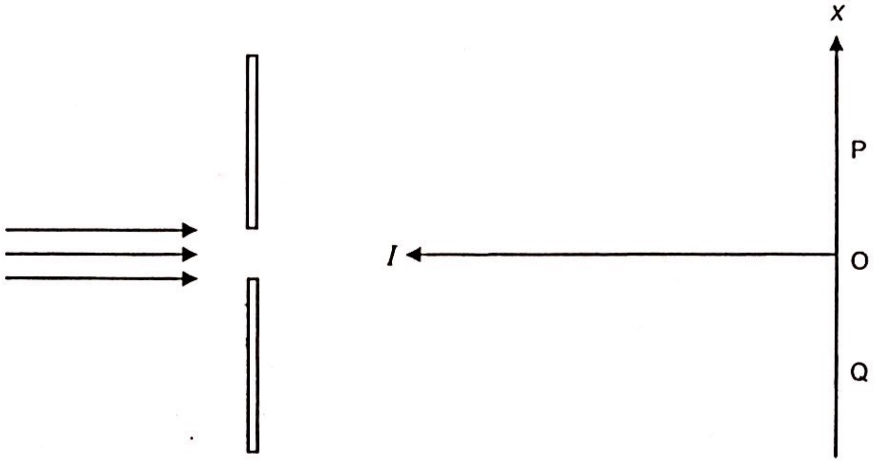
$\theta = \frac{2}{250} = \frac{1}{125}$

$\frac{1}{125} \approx \frac{\lambda}{b}$

$b = 125\lambda = 6.25 \times 10^{-5}$ m
λ check tut

Single Slit Diffraction

D12 Monochromatic light of wavelength λ is incident normally on a narrow slit of width w. A screen PQ is set up some distance from the slit to observe the diffraction pattern.



(a) What do you understand by the diffraction of waves?

(b) How is Huygen's principle used to explain diffraction in terms of secondary wavelets?

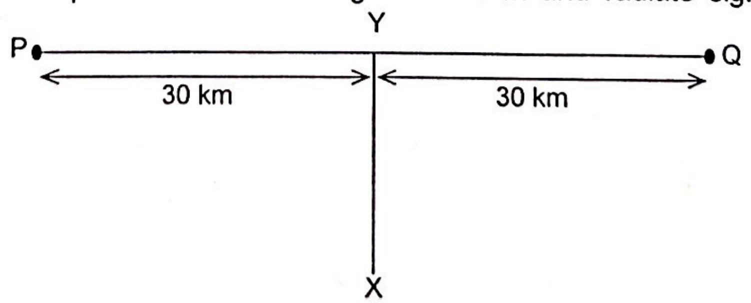
(c) Deduce, from first principles, the direction of the first diffraction minimum for plane waves incident normally on the slit.

(d) Draw a graph of intensity I against distance x from the central point O along the line PQ on the screen.

(e) Draw another graph on the same axes to represent the diffraction pattern when the width of the slit is halved.

Additional Question

D13 A ship at X is equidistant from two shore-based radio transmitters P and Q as shown below. Both transmitters operate on a wavelength of 300 m and radiate signals of equal amplitude.



(a) In the figure, the ship at X detects no signal. What information does this give about the signals from P and Q?

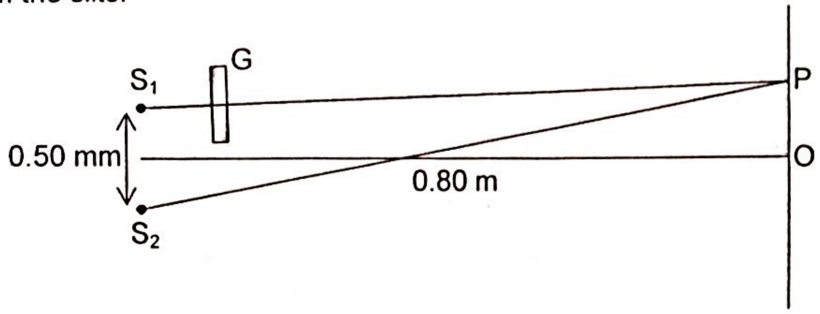
(b) The ship moves in a straight line from X to Y. Throughout the journey the amplitude of the signal detected by the ship is zero. Explain this.

(c) The ship moves in the direction YQ until the signal detected has amplitude twice that from either transmitter alone. How far has the ship moved?

(d) When the ship sails from Y to the harbor alongside transmitter Q, the detected signal rises and falls in amplitude. Calculate how many dips in intensity will be detected between Y and Q, inclusive.

Challenging Questions

C1 In the figure below, S_1 and S_2 are two coherent monochromatic light sources separated by a distance of 0.50 mm. Position O is equidistant from S_1 and S_2, and the screen is 0.80 m from the slits.



When a thin parallel-sided piece of glass G of thickness 3.6×10^{-6} m is placed in front of S_1 as shown, the centre of the fringe system moves from O to a point P.

Calculate OP, if the wavelength of the two sources is 6.0×10^{-7} m and the refractive index of the glass is 1.5.

Ans: 2.88×10^{-3} m

C2 An oil film floating in water is illuminated by white light at normal incidence. The film is 280 nm thick and has a refractive index of 1.45. By considering interference of light rays which are reflected at the oil-water and oil-air boundary, find the dominant colour in the reflected light. Take the refractive indices of air and water to be 1.00 and 1.30, respectively.

Note: When light enters a denser medium from a less-dense medium, it will undergo a 180° phase shift. No such phase shift occurs when light enters an optically less dense medium from a denser one.

Ans: 541 nm (green)

Answers

SP1 1000 m
SP2 (i) 2, (ii) f_2L
SP3 125 Hz, 375 Hz
SP4 1.0×10^{10} Hz
SP5 (a) 1.8 mm, (b) (i) fringe separation increases, (ii) fringe separation decreases, (c) the fringe separation x increases for higher order fringes.
SP6 (a) 2.5×10^{-6} m (b) 13.9° (c) 9
SP7 D **SP8** C **SP9** A **SP10** D **SP11** D
D2 (a) 1.8 cm, (b) 267 Hz
D3 (a) 341 m s^{-1}, (b) 272 mm, 556 mm
D4 $\cancel{7.2 \times 10^{-4}}$ m 0.184 m
D5 $\cancel{7.4 \times 10^{-4}\ \text{m}}$ 7.2×10^{-4} m
D6 (c) 664 nm
D7 44
D8 433 nm
D9 (a) 644nm, 509 nm, 480 nm, 468 nm, (b) 3, 3, 4, 4
D10 (a) 4.50×10^{-7} m, 5.99×10^{-7} m
D13 (c) 75 m, (d) 200

Self - Check Questions

S1 The principle of superposition states that when two or more waves of the same kind meet at a point in space, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

S2 A stationary wave results from the superposition of two progressive waves of the same frequency, amplitude and speed, travelling along the same line but in opposite directions.

S3 These particles are in phase (i.e. phase difference = 0), but they have different amplitudes. Their amplitudes may vary from 0 at the nodes to the max at the antinodes.

S4 Twice the length of the string, four times the length of the pipe, twice the length of the pipe.

S5 Diffraction is the bending of waves after passing through an aperture or round an obstacle. It is more pronounced when the aperture size is reduced.

S6 Diffraction effects are only significant when the aperture size is comparable to the wavelength of the waves. The wavelengths of visible light are very short (~10^{-7} m) compared the size of 'ordinary' openings such as doors and windows. Hence, diffraction of light is not easily observed

S7 Coherence refers to the condition in which two sources or waves have a constant phase difference.

Constructive (Destructive) interference occurs when two waves meet in *phase (anti-phase)* to produce a *maximum (minimum)* amplitude which is equal to the *sum (difference)* of the amplitudes of the two waves.

S8
- The waves must be **coherent**.
- The waves must have the **same amplitude**.
- The waves must **overlap** and be of the **same type**.
- For transverse waves, they must be **unpolarised** or **polarised** in the same plane.

S9 Separate light sources of the same frequency do not produce coherent waves because light pulses from the sources have short durations and abrupt change in phase. Since the human eye cannot cope with the rapid changes in phase, the pattern is not observable.

On the other hand, two loudspeakers connected to the same signal generator produce long continuous wave-trains which have a constant phase difference. Hence, the interference pattern does not fluctuate.

S10 $x = \dfrac{\lambda D}{a}$; valid when $a \ll D$ and near the zeroth-order where $\theta < 6°$.

S11 $d \sin\theta = n\lambda$, where d is the slit-separation, n is the order being observed, θ is the angle of the n^{th}-order maximum from the straight-through, λ the wavelength of incident waves. This equation assumes that the rays through the slits are parallel, ie $d \ll D$, where D is the distance between the grating and the screen.

S12 From $\sin\theta = \dfrac{n\lambda}{d} < 1 \Rightarrow n < \dfrac{d}{\lambda}$.

S13 $w \sin\theta = n\lambda$, where n is a positive integer.

The equation is only valid when the distance of the screen from the single-slit is much larger than the width of the single-slit as the derivation assumed parallel rays.

S14 Rayleigh criterion states that two images are just resolved by a slit when $\theta \approx \dfrac{\lambda}{w}$, where θ is angular separation of the images subtended at the slit, λ is the wavelength of light incident on the slit and w is the slit-width.

S15 This is because the positions of the maxima of the diffraction grating interference pattern coincided with the minima of the single-slit diffraction pattern.

Self - Practice Questions

SP1 Frequency of wind = 0.2 Hz. Hence, resonance or a standing wave is produced when the fundamental frequency of the bridge is 0.2 Hz. The resonant wavelength is thus

$$\lambda = \frac{v}{f} = \frac{400}{0.2} = 2000 \text{ m}$$

If this is the fundamental mode, then $\lambda = 2L$ so that $L = 1000$ m.

SP2 (i) Since the speed of the wave propagating along the string is a constant,

$$v_1 = v_2 \Rightarrow f_1\lambda_1 = f_2\lambda_2 \Rightarrow \frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2} = 2$$

(ii) $v = f_2 \times \lambda_2 = f_2 L$.

SP3 Since this is a closed pipe, the two lowest frequencies will be the first $\left(L = \frac{1}{4}\lambda_1\right)$ and the third $\left(L = \frac{3}{4}\lambda_3\right)$ harmonics. The wavelengths are λ_1=4L and λ_2=4L/3 respectively.

Hence, the frequencies are:

First Harmonic: $f_1 = v/\lambda_1 = 300 \div 2.4 = 125$ Hz

Third Harmonic: $f_3 = v/\lambda_3 = 300 \div 0.8 = 375$ Hz

SP4 Successive nodes are 1.5 cm apart. Hence, $\lambda = 2 \times 1.5 = 3.0$ cm.

From $v = f \times \lambda$,

$\Rightarrow \quad 3.0 \times 10^8 = f \times 0.030$

$\therefore \quad f = 1.0 \times 10^{10}$ Hz

SP5 (a) Separation of fringes $x = \dfrac{\lambda D}{a} = \dfrac{6.0 \times 10^{-7} \times 1.2}{0.40 \times 10^{-3}} = 1.8 \times 10^{-3}$ m

(b) **(i)** If a is decreased, then the fringe separation x increases.

(ii) If D is decreased, then the fringe separation x decreases.

(c) The formula in (a) is not applicable to higher-order fringes which appear beyond 6° from the straight-through. The fringe separation between higher order fringes gets larger.

SP6 (a) N = 400 lines per mm = 400 000 lines per metre.

Grating spacing $d = 1/N = 1 \div 400\,000 = 2.5 \times 10^{-6}$ m

(b)
$$d \sin\theta = n\lambda$$
$$2.5 \times 10^{-6} \sin\theta = 1 \times \left(600 \times 10^{-9}\right)$$
$$\theta = 13.9^\circ$$

(c) $\sin\theta = \dfrac{n\lambda}{d} \quad \Rightarrow \dfrac{n\lambda}{d} < 1$

$$\Rightarrow n < \frac{d}{\lambda} = \frac{2.5 \times 10^{-6}}{6 \times 10^{-7}} = 4.17$$

Hence, n = 4. Adding the maxima of the 'other side' and that at the straight-through position, the total number of maxima is 4 + 4 + 1 = 9

SP7 As point source P is not equidistant from slits S_1 and S_2, path difference Δx of the 2 rays must be determined from source P to point of observation Q:

$$\Delta x = (l_1 + l_3) - (l_2 + l_4)$$

Since destructive interference takes place at point Q, the path difference Δx must be an odd multiple of half-wavelength:

$$\Delta x = (2m + 1)\,\lambda/2 \text{ or } (m + \tfrac{1}{2})\,\lambda \text{ (D)}$$

SP8 $n = 2$, $\theta = \alpha/2$

Hence, $d = \dfrac{n\lambda}{\sin\theta} = \dfrac{2\lambda}{\sin(\alpha/2)}$. (C)

SP9 From $d \sin\theta = n\lambda$, it can be seen that the longest wavelength deviates the most.

$$\Rightarrow d = \frac{n\lambda}{\sin\theta} = \frac{2(7.5\times10^{-7})}{\sin 60°} = 1.73\times10^{-6}\ \text{m}$$

$\therefore N = 1/d = 5.77\times10^{5}$ (A)

SP10 For the first minimum to occur at P, the path difference between wavelets from R and the mid-point to P is $\lambda/2$. Similarly, the path difference between wavelets from the mid-point and S to P is also $\lambda/2$. Hence, the path difference between SP and RP is λ. Angle θ for the first minimum is such that $\sin\theta = \lambda/a$. (D)

SP11 When the width of the slit is reduced, less energy passes though the slit and hence the intensity of all peaks decreases.

Since $\sin\theta = \lambda/w$, when w decreases, θ increases and hence the width of the pattern also increases. (D)