Paya Lebar Methodist Girls' School (Secondary)
Department of Mathematics
2017 Preliminary Examination
Additional Mathematics Paper 2 (4047/2) Worked Solutions

1.(i) $p = -2$
$q = 4$
$r = 1$

1.(ii) $k + \frac{\pi}{2} = h$

2.(i) When $t = 0$, $M = 50$,

$50 = 120 - ke^0$
$k = 120 - 50$
$= 70$

2.(ii) $M = 120 - 70e^{\frac{1}{2}t}$

When $M = 0$, $0 = 120 - 70e^{\frac{1}{2}t}$

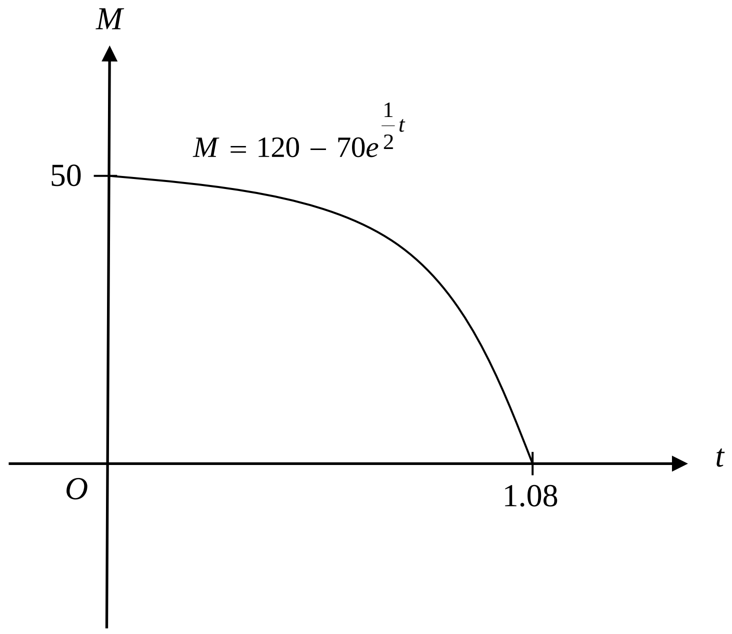
$e^{\frac{1}{2}t} = \frac{120}{70}$

$\frac{1}{2}t = \ln\left(\frac{120}{70}\right)$

$t = 2\ln\left(\frac{120}{70}\right)$

$t \approx 1.08$

2.(iii)



3.(i) Graph shows only one x-intercept.

3.(ii) Let $f(x) = x^3 - 3x^2 - x - 12$

$$f(4) = (4)^3 - 3(4)^2 - 4 - 12$$
$$= 0$$

Therefore, $x - 4$ is a factor of $f(x)$

3.(iii) $x^3 - 3x^2 - x - 12 = (x - 4)(x^2 + x + 3)$

$$x^3 - 3x^2 - x - 12 = 5(x - 4)$$
$$(x - 4)(x^2 + x + 3) = 5(x - 4)$$
$$(x - 4)(x^2 + x + 3 - 5) = 0$$
$$(x - 4)(x^2 + x - 2) = 0$$
$$(x - 4)(x - 1)(x + 2) = 0$$
$$x = 4 \quad \text{or} \quad x = 1 \quad \text{or} \quad x = -2$$

4.(i) $\frac{1}{2}\left(1 + \sqrt{3}\right) \times BC = \left(7 + \sqrt{192}\right)$

$$BC = \frac{2(7 + \sqrt{192})}{1 + \sqrt{3}}$$
$$= \frac{2(7 + 8\sqrt{3})}{1 + \sqrt{3}}$$
$$= \frac{14 + 16\sqrt{3})}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$$
$$= \frac{14 + 16\sqrt{3} - 14\sqrt{3} - 16(3)}{1 - 3}$$
$$= 17 - \sqrt{3} \text{ cm}$$

4.(ii) $(AC)^2 = \left(1 + \sqrt{3}\right)^2 + \left(17 - \sqrt{3}\right)^2$

$$= 1 + 2\sqrt{3} + 3 + 289 - 34\sqrt{3} + 3$$
$$= 296 - 32\sqrt{3} \text{ cm}^2$$

5.(a) $\alpha^2 + \beta^2 = -p$, $\alpha^2\beta^2 = 4$

$$(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta$$
$$= -p - 2(\sqrt{4})$$
$$= -p - 4$$

$$\alpha - \beta = \sqrt{-p-4}$$

5.(b) $\alpha^2 + \beta^2 = 5$, $\alpha^2\beta^2 = 4$

$$\alpha - 1 + 1 - \beta = \alpha - \beta$$
$$= \sqrt{5-4}$$
$$= 1$$

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$= 5 + 2(2)$$
$$= 9$$
$$\alpha + \beta = 3$$

$$(\alpha - 1)(1 - \beta) = \alpha + \beta - \alpha\beta - 1$$
$$= 3 - 2 - 1$$
$$= 0$$

A quadratic Eqn is $x^2 - x = 0$

6.(a)(i) $\log_5 32 = \dfrac{\lg 2^5}{\lg 5}$

$$= \frac{5\lg 2}{\lg \frac{10}{2}}$$
$$= \frac{5\lg 2}{\lg 10 - \lg 2}$$
$$= \frac{5m}{1-m}$$

6.(a)(i) $10^m = 2$

$$(10^m)^{10} = 2^{10}$$
$$2^{10} = 10^{10m}$$

6.(b)

$$2\log_3 x - \log_3 2 = \log_3(x-4)$$

$$\log_3\left(\frac{x^2}{2}\right) = \log_3(x-4)$$

$$\frac{x^2}{2} = x - 4$$

$$x^2 - 2x + 8 = 0$$

$$b^2 - 4ac = (-2)^2 - 4(1)(8)$$
$$= -28$$
$$< 0$$

there are no real solutions

7.(i) $\frac{dy}{dx} = -9(4-x)^2$

For stationary point, $\frac{dy}{dx} = 0$

$$-9(4-x)^2 = 0$$
$$x = 4$$
$$y = 5$$

stationary point = (4, 5)

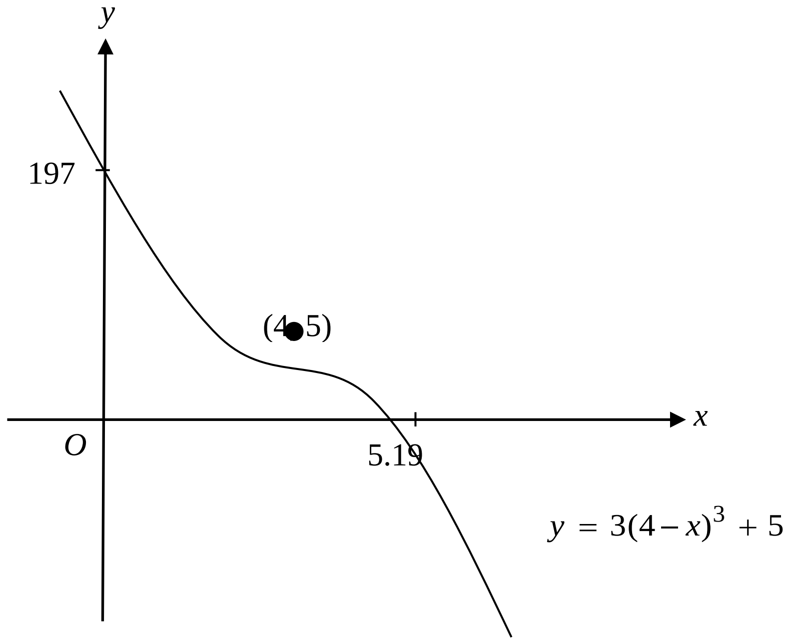
7.(ii) For $x < 4$, $\frac{dy}{dx} < 0$

For $x > 4$, $\frac{dy}{dx} < 0$

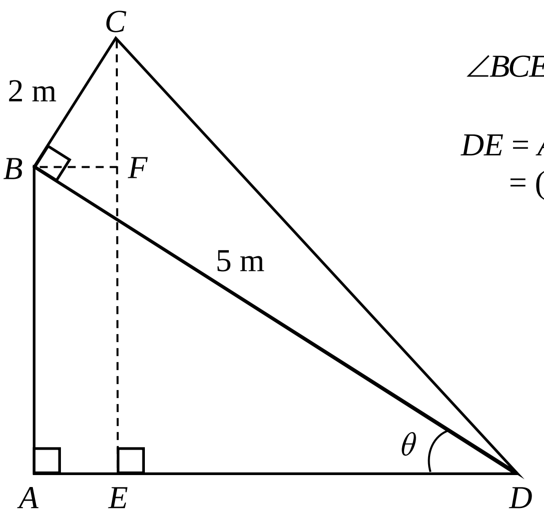
As x increases through 4, the sign of $\frac{dy}{dx}$ does not change.

The stationary point is a point of inflexion.

7.(iii)



8.(i)



$\angle BCE = \theta$

$DE = AD - AE$

$= (5\cos\theta - 2\sin\theta)$ m

8.(ii) $DE = R\cos(\theta + \alpha)$

$R = \sqrt{5^2 + 2^2}$

$= \sqrt{29}$

$\tan\alpha = \dfrac{2}{5}$

$\alpha = 21.8°$

$DE = \sqrt{29}\cos(\theta + 21.8°)$ m

8.(iii) $CE = AB + CF$

$= 5\sin\theta + 2\cos\theta$

$= \sqrt{29}\sin(\theta + 21.8°)$

8.(iv) area of triangle CDE $= \dfrac{1}{2} \times \sqrt{29}\cos(\theta + \alpha) \times \sqrt{29}\sin(\theta + \alpha)$

$= \dfrac{29}{2}\cos(\theta + \alpha)\sin(\theta + \alpha)$

$= \dfrac{29}{4}[2\cos(\theta + \alpha)\sin(\theta + \alpha)]$

$= \dfrac{29}{4}[\sin 2(\theta + \alpha)]$

$= \dfrac{29}{4}\sin(2\theta + 2\alpha)$

9.(i) In ΔFDB and ΔFAD

$\angle DFB = \angle AFD$ (common angle)

$\angle FDB = \angle FAD$ (alt seg theorem)

ΔFDB is similar to ΔFAD (all corr $\angle s$ are equal)

9.(ii) ΔFEB is similar to ΔFAE

$\frac{FE}{FA} = \frac{FB}{FE}$ (ΔFEB is similar to ΔFAE)

$FE^2 = FA \times GB$

$\frac{FD}{FA} = \frac{FB}{FD}$ (ΔFDB is similar to ΔFAD)

$FD^2 = FA \times GB$

$FD^2 = FE^2$

$FD = FE$

9.(iii) $\angle ABD = 90°$ ($ABF \perp BE$)

AD is a diameter. ($\angle$ in semicircle)

$\angle ABE = 90°$ ($ABF \perp BE$)

AE is a diameter. ($\angle$ in semicircle)

$\angle ADF = 90°$ (tan. $\perp$ rad.)

$\angle AEF = 90°$ (tan. $\perp$ rad.)

AF is a diameter. ($\angle$ in semicircle)

a circle with AF as a diameter passes through D and E.

10.(i) midpt of $PQ = \left(\frac{3-4}{2}, \frac{-1-2}{2}\right)$

$= \left(-\frac{1}{2}, -\frac{3}{2}\right)$

Gradient of $PQ = \frac{-1+2}{3+4}$

$= \frac{1}{7}$

Gradient of perpendicular bisector = – 7

Eqn of perpendicular bisector is

$y + \frac{3}{2} = -7(x + \frac{1}{2})$

$y = -7x - 5$ ------------ (1)

10.(ii) $y = -2x$ ----------------- (2)

Subst (1) into (2): $-2x = -7x - 5$

$x = -1$

From (2), $y = 2$

Centre of $C_1 = (-1, 2)$

radius $= \sqrt{(3+1)^2 + (-1-2)^2}$ 5

$= 5$ units

the equation of C_1 is $(x+1)^2 + (y-2)^2 = 25$

10.(iii) dist between R and centre

$= \sqrt{(2+1)^2 + (5-2)^2}$

$=$ 4.2426 units

$<$ 5 units

R lies inside the circle C_1.

10.(iv) Centre of $C_2 = (1, 2)$

radius $=$ 5 units

the equation of C_2 is $(x-1)^2 + (y-2)^2 = 25$

11.(i) $p = 7$

11.(ii) when the scooter changes its direction of motion, $v = 0$

$$7 - 8\sin\frac{1}{3}t = 0$$

$$\sin\frac{1}{3}t = \frac{7}{8}$$

basic angle = 1.0654

$$\frac{t}{3} = 1.0654,\ \pi - 1.0654$$

$$t = 3.1963, 6.2284$$

$$\approx 3.20, 6.23$$

11.(iii)

$$s = \int (7 - 8\sin\frac{1}{3}t)\,\mathrm{d}t$$

$$= 7t + 8(3)\cos\frac{1}{3}t + C$$

$$= 7t + 24\cos\frac{1}{3}t + C$$

when $t = 0$, $s = 0$,

$$0 = 24\cos 0 + C$$

$$C = -24$$

$$s = 7t + 24\cos\frac{1}{3}t - 24$$

when $t = 2$, $s = 7(2) + 24\cos\frac{2}{3} - 24$

$= 8.8612$

when $t = 3$, $s = 7(3) + 24\cos 1 - 24$

$= 9.9672$

distance moved in 3rd second $= 9.9672 - 8.8612$

$= 1.1059$

≈ 1.11 m

12.(a) $\frac{d}{dx}\left(3x^2\ln x\right) = 3x^2\left(\frac{1}{x}\right) + 6x\ln x$

$$= 3x + 6x\ln x$$

12.(b) $\int\left(3x + 6x\ln x\right)dx = 3x^2\ln x + C$

$$6\int x\ln x\,dx = 3x^2\ln x - \int 3x\,dx + C$$

$$= 3x^2\ln x - \frac{3x^2}{2} + C'$$

$$\int x\ln x\,dx = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + C''$$

12.(c)(i) At A, $y = 0$,

$x\ln x = 0$

$x = 0$ or $\ln x = 0$

(rej) $x = 1$

12.(c)(ii) $\int_{\frac{1}{2}}^{1} x\ln x\,dx = \left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_{\frac{1}{2}}^{1}$

$$= \frac{1}{2}(1)^2\ln(1) - \frac{1}{4}(1)^2 - \left[\frac{1}{2}\left(\frac{1}{2}\right)^2\ln\left(\frac{1}{2}\right) - \frac{1}{4}\left(\frac{1}{2}\right)^2\right]$$

$$= -\frac{1}{4} - \left[-\frac{1}{8}\ln 2 - \frac{1}{16}\right]$$

$$= \frac{1}{8}\ln 2 - \frac{3}{16}$$

$$\int_1^2 x\ln x\ \mathrm{d}x = \left[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2\right]_1^2$$

$$= \frac{1}{2}(2)^2\ln(2) - \frac{1}{4}(2)^2 - \left[\frac{1}{2}(1)^2\ln(1) - \frac{1}{4}(1)^2\right]$$

$$= 2\ln 2 - 1 - \left[-\frac{1}{4}\right]$$

$$= 2\ln 2 - \frac{3}{4}$$

total shaded area = $-\int_{\frac{1}{2}}^{1} x\ln x\ \mathrm{d}x + \int_1^2 x\ln x\ \mathrm{d}x$

$$= -\left(\frac{1}{8}\ln 2 - \frac{3}{16}\right) + 2\ln 2 - \frac{3}{4}$$

= 0.737 sq unit