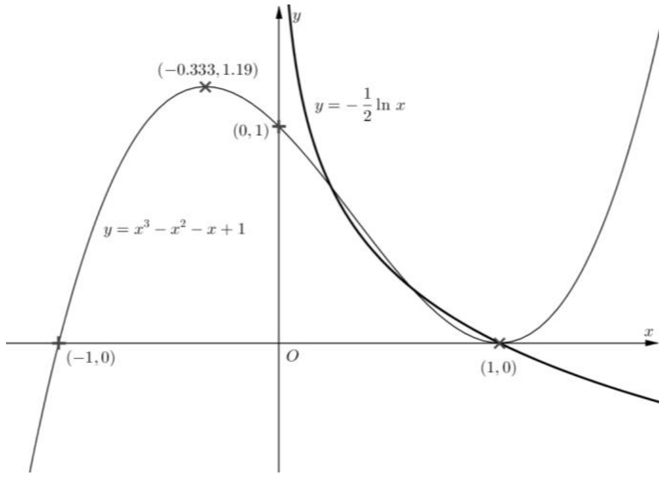
**2023 JC1 H1 REVISION SET A-2**
**COMPLETE SOLUTIONS**

<table>
<tr><td></td><td>GRAPHING TECHNIQUES</td></tr>
<tr><td>1</td><td>
DHS 2014 Promo Q3




$2x^3 - 2x^2 - 2x + 2 + \ln x = 0$

$2\left(x^3 - x^2 - x + 1\right) = -\ln x$

$x^3 - x^2 - x + 1 = -\frac{1}{2}\ln x$

From graph, since number of points of intersection between the graphs of $y = x^3 - x^2 - x + 1$ and $y = -\frac{1}{2}\ln x$ is 3, the number of solutions to $2x^3 - 2x^2 - 2x + 2 + \ln x = 0$ is 3.
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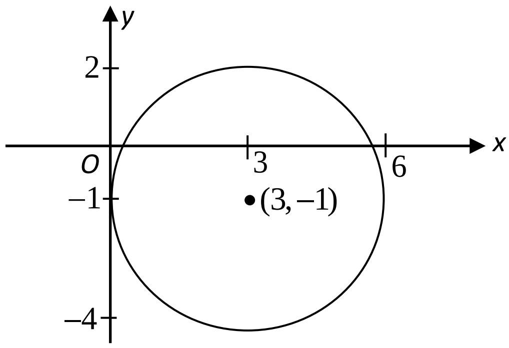
<tr><td>2</td><td>
$x^2 + y^2 - 6x + 2y + 1 = 0 \Rightarrow (x-3)^2 + (y+1)^2 = 9 \Rightarrow (x-3)^2 + (y+1)^2 = 9$

Coordinates of centre are <u>(3, –1)</u> and radius is <u>3</u>.



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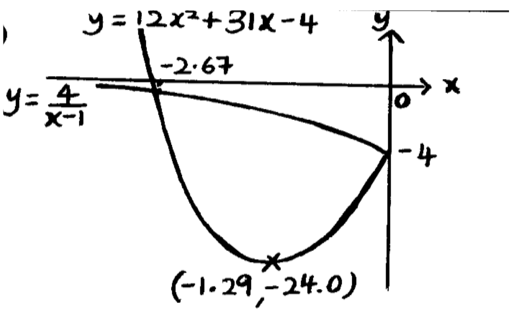

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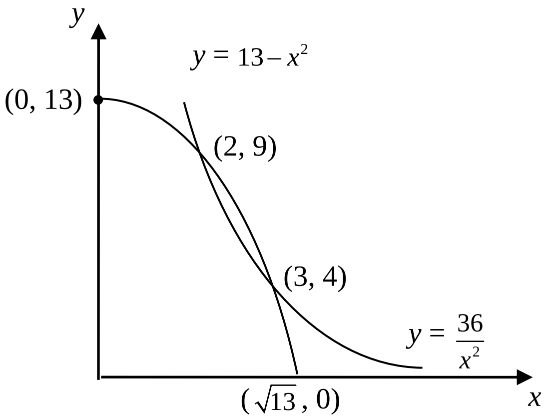
<tr><td>4</td><td>
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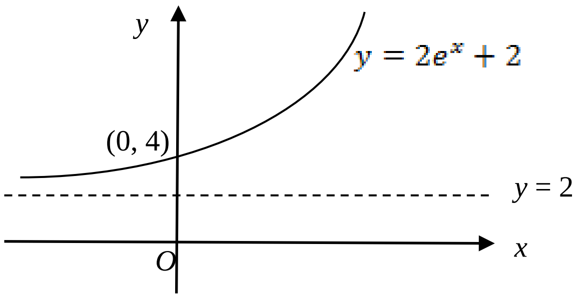
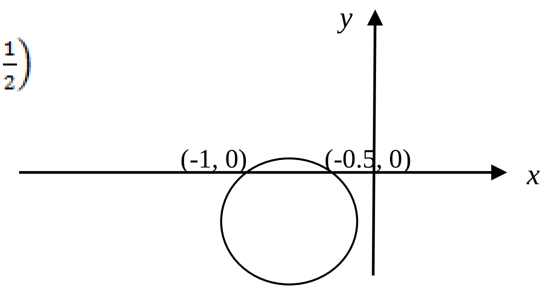
<tr><td>5 (i)</td><td>
</td></tr>
<tr><td>5 (ii)</td><td>$2x^2 + 3x + 2y^2 + 2y + 1 = 0$<br><br>Circle with centre = $\left(-\frac{3}{4}, -\frac{1}{2}\right)$<br><br>Axes of symmetry :<br><br>$x = -\frac{3}{4}, y = -\frac{1}{2}$<br>
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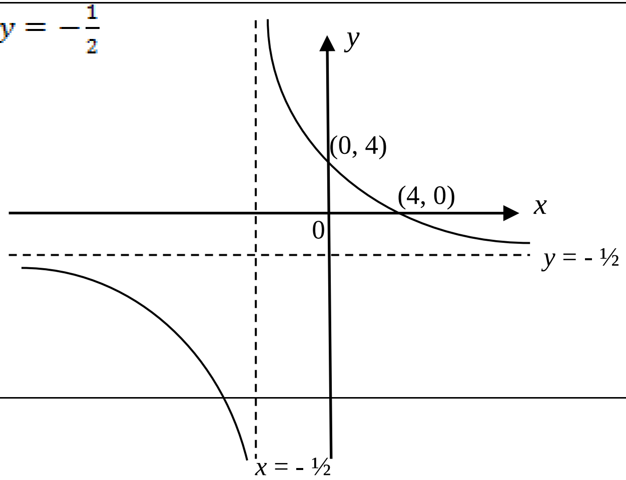
<tr><td>5 (iii)</td><td>Asymptotes : $x = -\frac{1}{2}, y = -\frac{1}{2}$<br>
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<tr><td>6</td><td>$y = \dfrac{ax+c}{x-b} = a + \dfrac{c+ab}{x-b}$<br><br>Asymptotes are $x = b$ and $y = a$<br>Comparing with $x = 1$ and $y = 2$, <u>b = 1</u> and <u>a = 2</u><br>$\therefore \quad y = \dfrac{2x+c}{x-1}$<br>At intersection with x-axis, $y = 0$ and $x = -3/2 \Rightarrow 0 = 2(-3/2) + c$ so <u>c = 3</u></td></tr>
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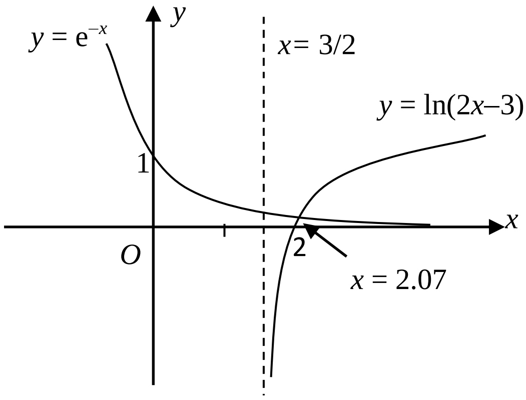
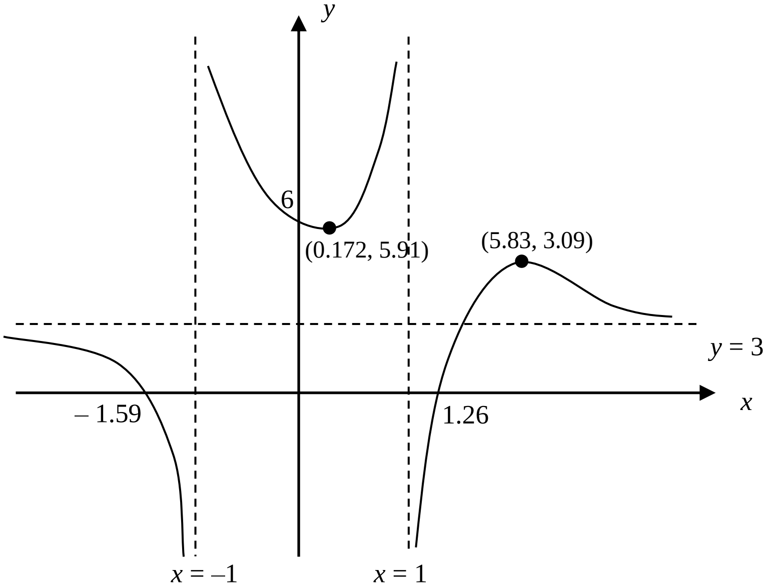
| **7** | $y=\ln(kx-3)$<br>Vertical asymptote : $kx-3=0 \Rightarrow x=\dfrac{3}{k}$<br>$x$-intercept: $y=0 \Rightarrow kx-3=1 \Rightarrow x=\dfrac{4}{k}$ $\left(\dfrac{4}{k},0\right)$<br><br>$e^x \ln(2x-3)=1 \Rightarrow \ln(2x-3)=e^{-x}$<br>Sketch the graph of $y = e^{-x}$ to the same diagram.<br>From the GC, the two graphs intersect at $x =$ 2.07. |
| **8** | (i) Asymptotes are $y = 3$, $x = -1$ and $x = -1$<br>(ii)<br> |
| **9** | $y=\dfrac{4x+1}{2x-2}=2+\dfrac{5}{2x-2}$<br>Asymptotes $x = 1$ and $y = 2$<br>Intersections with axes (0, −1/2) and (−1/4, 0)<br><br>$x=\log_2(4x+1)-\log_2(2x-2)$<br>$=\log_2\dfrac{4x+1}{2x-2}$<br>$\therefore \dfrac{4x+1}{2x-2}=2^x$ |

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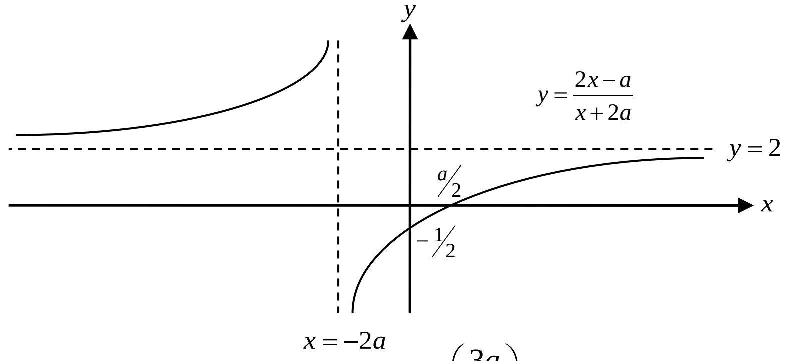
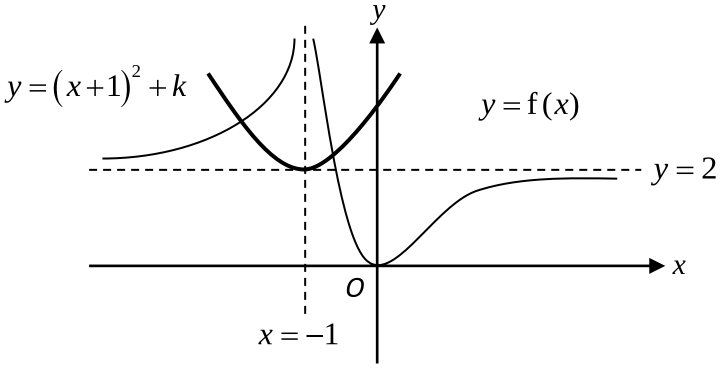
| | At intersection, $x = -0.769$ (3 sf) or $x = 2.10$ (3 sf)<br>But $4x + 1$ and $2x - 2$ must both be positive, so $x > 1$ only.<br>Therefore, $x$ cannot be $-0.769$, giving **$x = 2.10$** only (3 sf). |
| **10 (a)** | <br>Sub $x = \frac{3a}{2}$ into equation of $H$: $y = \dfrac{2\left(\frac{3a}{2}\right) - a}{\left(\frac{3a}{2}\right) + 2a} = \dfrac{4}{7}$<br>Hence the point $\left(\frac{3a}{2}, \frac{4}{7}\right)$ lies on $H$. |
| **10 (b)** | $x^2 + 2x + (k+1) = (x+1)^2 + (k+1) - 1 = (x+1)^2 + k$<br>Curve is quadratic with minimum point at $(-1, k)$. Hence, from the graph, there will always be two points of intersection, giving two $x$-coordinates. Hence there will always be 2 real roots for $k \geq 2$.<br> |