YISHUN INNOVA JUNIOR COLLEGE
JC 1 PROMOTIONAL EXAMINATION
Higher 2

CANDIDATE NAME

CG

MATHEMATICS

9758/01

Paper 1

30 September 2024

Additional Materials: Printed Answer Booklet
List of Formulae and Results (MF27)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your CG, index number and name on the work you hand in.

Answer **all** questions.
Write your answers on the Printed Answer Booklet. Follow the instructions on the front cover of the answer booklet.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are **not** allowed in a question, you must present the mathematical steps using mathematical notations and not calculator commands.
You must show all necessary working clearly.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

1 A concert sells tickets under three different age categories: 'Senior Citizen', 'Adult' and 'Child'. Three groups of people *A*, *B* and *C* went to the concert on 3 different days. For Group *A*, all tickets were purchased at the original prices. For Group *B*, 'Senior Citizen' tickets were purchased at a 20% discount, while tickets under the other two categories were purchased at the original prices. For Group *C*, 'Child' tickets were purchased at half the 'Adult' original price, while 'Senior Citizen' and 'Adult' tickets were purchased at the original prices. The numbers in each age category for each group and the total price of the tickets for each group, are given in the following table.

Group	Senior Citizen	Adult	Child	Total Price
A	5	12	6	\$2440
B	15	10	5	\$2660
C	8	10	12	\$2560

(a) Express this information as 3 linear equations and hence find the original price of a ticket in each of the age categories. [3]

(b) The Lee family, consisting of 2 senior citizens, 2 adults and 3 children, went to the concert. Assuming their tickets were purchased at the original prices, find the total price they paid. [1]

2 Given that $\sum_{r=1}^{n} r^3 = \frac{n^2}{4}(n+1)^2$, find an expression in terms of n for $\sum_{r=n+1}^{2n}\left(4r^3+n\right)$. Leave your answer in fully factorised form. [4]

3 **(a)** By first expressing x^2-2x+3 in completed square form, show that x^2-2x+3 is positive for all real values of x. [2]

(b) Without the use of a calculator, solve the inequality $\frac{\left(x^2-2x+3\right)(1-x)}{x^2-x-2}<0$. [3]

(c) Hence solve the inequality $\frac{\left(x-2\sqrt{x}+3\right)\left(1-\sqrt{x}\right)}{x-\sqrt{x}-2}<0$. [3]

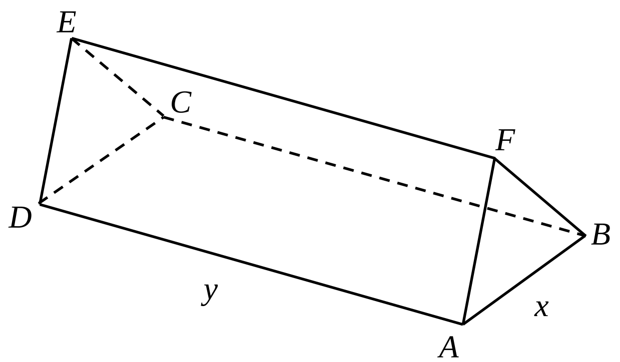
4 **(a)** Given that $y=x^{\cos x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x only. [3]

(b) A curve has equation $x^2y-xy^2=6$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. [2]

(ii) Find the exact coordinates of the point on the curve where $\frac{\mathrm{d}y}{\mathrm{d}x}=1$. [5]

5 The diagram below shows a closed prism with a rectangular base *ABCD*. Triangles *ABF* and *DCE* are equilateral, congruent, and perpendicular to the base. The lengths of *AB* and *AD* are x cm and y cm respectively. The total surface area of the prism is 300 cm^2.



(a) Show that the volume of the prism, V cm^3, is given by

$$V = 25\sqrt{3}x - \frac{1}{8}x^3.$$ [4]

(b) Hence find the maximum possible volume of the prism. [4]

6 A curve C has parametric equations

$$x = a\left(t^2 - t\right), \qquad y = a\left(1 - 2t^2\right), \qquad \text{for } -3 < t \le 2,$$

where a is a positive constant.

(a) Sketch the graph of C, stating the coordinates of the end-points. [2]

(b) Find, in terms of a,

(i) the equation of the normal at the point where $t = -1$, [4]

(ii) the exact coordinates of the point where the normal cuts C again. [4]

7 The functions f and g are defined by

$$\text{f} : x \mapsto \text{e}^x + 4, \ \ x \in \mathbb{R}, \ \ x \le 1,$$
$$\text{g} : x \mapsto \sin x, \quad x \in \mathbb{R}, -\pi < x < \pi.$$

(a) Find $\text{f}^{-1}(x)$ and state the domain and range of f^{-1}. [4]

(b) Determine whether g^{-1} exists. [1]

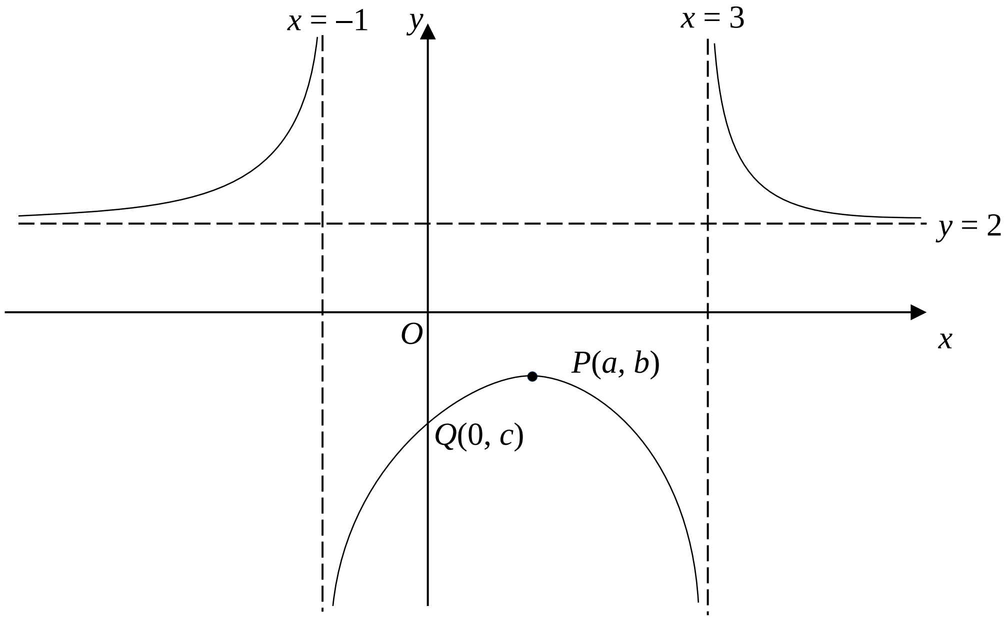
(c) Explain why the composite function fg exists. [2]

(d) Find $\text{fg}(x)$ and state the domain of fg. [2]

(e) Determine the exact range of fg. [1]

8 **(a)** Describe a series of transformations that will transform the curve with equation $y = x^2$ onto the curve with equation $y = (3x+2)^2 + 1$. [3]

(b) The diagram shows a sketch of the curve $y = \mathrm{f}(x)$ with asymptotes $x = -1$, $x = 3$ and $y = 2$. The curve has a maximum point $P(a, b)$ and passes through $Q(0, c)$.



On separate diagrams, sketch the following graphs indicating the coordinates of the points corresponding to P and Q, and the equations of the asymptotes, if any.

(i) $y = \mathrm{f}\left(|x|\right)$ [3]

(ii) $y = \mathrm{f}'(x)$ [3]

9 **(a)** Find $\int \sin^2 3\theta \, \mathrm{d}\theta$. [2]

(b) Find $\int x(\ln x)^2 \, \mathrm{d}x$. [3]

(c) Evaluate $\displaystyle\int_1^3 \frac{x^2}{\sqrt{2x^3 - 1}} \, \mathrm{d}x$ exactly. [3]

(d) Use the substitution $u = \mathrm{e}^x$ to evaluate $\displaystyle\int_0^{\ln\sqrt{3}} \frac{\mathrm{e}^{3x}}{\mathrm{e}^{2x} + 1} \, \mathrm{d}x$ exactly. [4]

10 **(a)** The line l_1 has cartesian equation $\dfrac{x+1}{2} = y-4 = \dfrac{2-z}{3}$.

The plane p_1 has cartesian equation $x-4z=5$.

(i) Find a vector equation of l_1. [2]

(ii) Find the acute angle between l_1 and p_1. [2]

(iii) Find the position vector of the point of intersection of l_1 and p_1. [2]

(b) The plane p_2 contains the line l_2 with vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}),\ \lambda \in \mathbb{R}$, and the point $R(-1, 2, 2)$.

(i) Find the shortest distance from R to l_2. [3]

(ii) Find a cartesian equation of p_2. [3]

(iii) Find a vector equation of the line of intersection between p_1 and p_2. [1]

11 **(a)** Zack saves \$$x$ on 1 January 2024. On the first day of each subsequent month, he saves \$10 more than in the previous month, so that he saves $\$(x+10)$ on 1 February 2024, $\$(x+20)$ on 1 March 2024, and so on. Find the value of x if he saves \$25 000 in total on 1 December 2026. [3]

(b) Dan opens a savings account. He deposits \$200 on 1 January 2024 and continues to deposit \$200 on the first day of each subsequent month. The interest rate of the savings account is 0.2% per month, so that on the last day of each month the amount in the account on that day is increased by 0.2%.

(i) Find how much the \$200 deposited on 1 January 2024 is worth at the end of 31 December 2024. [1]

(ii) Show that, at the end of the nth month (where January 2024 is the first month, February 2024 is the second month, and so on), the total amount in the account is

$$\$\ 100200\left(1.002^{n} - 1\right).$$ [4]

(iii) In which month and year will the amount in Dan's account first exceed \$4500? Explain whether this occurs on the first or last day of the month. [4]

BLANK PAGE