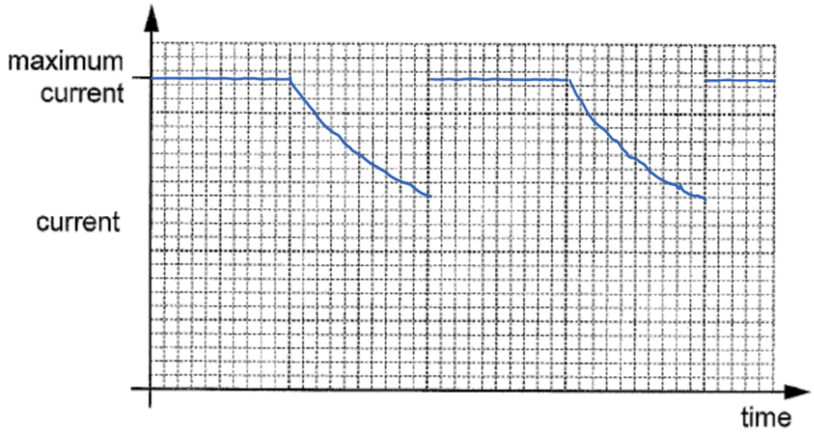
H3 A-Level 2020_Suggested Solutions

1	**(a)**	**(i)**	By Principle of Conservation of Linear Momentum, $\Sigma p_i = \Sigma p_f$ $0 = m_{Ra}v_{Ra} - m_\alpha v_\alpha \rightarrow \dfrac{v_{Ra}}{v_\alpha} = \dfrac{m_\alpha}{m_{Ra}} = \dfrac{4\text{ u}}{228\text{ u}} = \dfrac{1}{57}$
		(ii)	$\dfrac{\text{KE}_{Ra}}{\text{KE}_\alpha} = \dfrac{\frac{1}{2}m_{Ra}v_{Ra}^2}{\frac{1}{2}m_\alpha v_\alpha^2} = \dfrac{228\text{ u}}{4\text{ u}}\left(\dfrac{1}{57}\right)^2 = \dfrac{1}{57}$
	(b)	Total KE = 4.08 x 10^6 x 1.60 x 10^{-16} J Total KE = $\text{KE}_{Ra} + \text{KE}_\alpha = \dfrac{1}{57\text{ KE}_\alpha} + \text{KE}_\alpha = \dfrac{58}{57}\text{KE}_\alpha$ $\text{KE}_\alpha = \left(\dfrac{57}{58}\right)4.08\times10^6\times1.60\times10^{-19}\text{ J}$ Given that $\dfrac{q_\alpha}{m_\alpha} = 4.81\times10^7\text{ C kg}^{-1} \rightarrow m_\alpha = \dfrac{2\left(1.60\times10^{-19}\right)}{4.81\times10^7}\text{ kg}$ $\text{KE}_\alpha = \dfrac{1}{2}m_\alpha v_\alpha^2 \rightarrow v_\alpha = \sqrt{\dfrac{2\text{ KE}_\alpha}{m_\alpha}} = \sqrt{\dfrac{2\left(\frac{57}{58}\right)4.08\times10^6\times1.60\times10^{-19}}{\frac{2\left(1.60\times10^{-19}\right)}{4.81\times10^7}}}$ $= \sqrt{\left(\dfrac{57}{58}\right)\left(4.08\times10^6\right)\left(4.81\times10^7\right)} = 1.39\times10^7\text{ m s}^{-1}$	

2	**(a)**	Let t be the time that the ball remains on the circular plate. Horizontal displacement = x = 0.50t Vertical displacement = y = $\frac{1}{2}(g\sin 30)t^2$ $x^2+y^2=r^2$ $(0.50t)^2+\left(\frac{1}{2}(g\sin 30)t^2\right)^2=(0.20)^2$ Solving the quadratic equation yields t^2 = 0.06337. As a result, t = 0.252 s.
	(b)	Net force acting on the rolling ball: $F_{net}=ma_{CM}$ $mg\sin\theta-F=ma_{CM}\rightarrow a_{CM}=\frac{mg\sin\theta-F}{m}$ (1) Net torque on the rolling ball: $\tau=Fr=I_{CM}\alpha=I_{CM}\frac{a_{CM}}{r}\rightarrow a_{CM}=\frac{Fr^2}{I_{CM}}=\frac{Fr^2}{\frac{2}{5}mr^2}=\frac{5F}{2m}$ (2) Equating (1) and (2), we obtain $a_{CM}=\frac{mg\sin\theta-F}{m}=\frac{5F}{2m}\rightarrow F=\frac{2mg\sin\theta}{7}$ Now, we substitute F into (2) $a_{CM}=\frac{5F}{2m}=\frac{5}{2m}\frac{2mg\sin\theta}{7}=\frac{5g\sin 30}{7}=3.5\text{ m s}^{-2}$
	(c)	Moment of inertia for solid cylinder about its central axis is $\frac{MR^2}{2}$ Rotational Kinetic Energy = $\frac{1}{2}I_{CM}\omega^2=\frac{1}{2}\frac{MR^2}{2}\left(\frac{v^2}{R^2}\right)=\frac{0.080}{4}(1.84)^2=0.068\text{ J}$

<table>
<tr><td>3</td><td>(a)</td><td colspan="2">Take top to bottom. 9.2 cm: 460 V. Therefore Y-gain = 50 V cm^{-1}.

8.0 cm: $2T$ = 0.040 s. Therefore time-base = 5.0×10^{-3} s cm^{-1}.</td></tr>
<tr><td></td><td>(b)</td><td>(i)</td><td></td></tr>
<tr><td></td><td></td><td>(ii)</td><td>$V_{130} = 0.130 \times 1500 = 195$ V
$V = V_0 e^{-\frac{t}{RC}}$

At $t = 0.5T = 0.01$ s, $\dfrac{V}{V_0} = \dfrac{195}{230}$
$\dfrac{195}{230} = e^{-\frac{0.01}{1500C}}$

$C = 40.4\ \mu$F</td></tr>
<tr><td></td><td></td><td>(iii)</td><td>Total capacitance will drop, time constant will drop and the discharge is more rapid. Current will drop to under 130 mA in 0.01 s and the component is more likely to malfunction.</td></tr>
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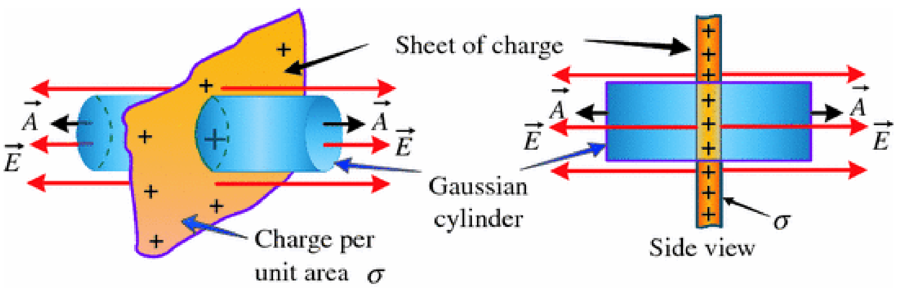
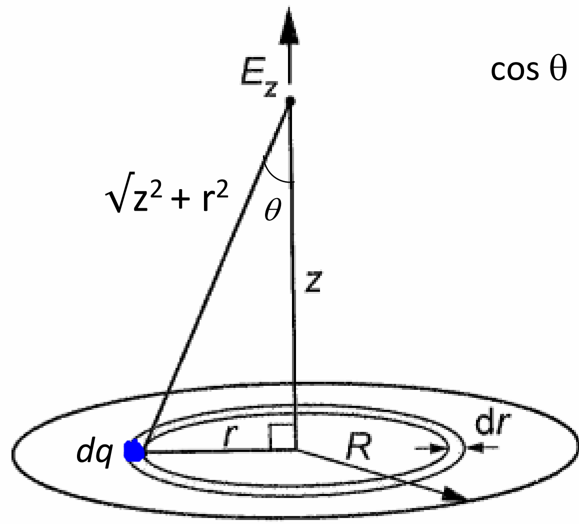
4	**(a)**	**(i)**	$\frac{\text{Units for W}}{\text{Units for q}} = \frac{\text{kg m}^2\ \text{s}^{-2}}{\text{A s}} = \text{kg m}^2\ \text{A}^{-1}\ \text{s}^{-3}$
		(ii)	$\varepsilon_0 = \frac{Cd}{A}$ and $C = \frac{Q}{V}$ Units for $\varepsilon_0 = \frac{\text{F m}}{\text{m}^2} = \frac{\text{F m}}{\text{m}^2} = \frac{\left(\text{A s} \big/ \text{kg m}^2\ \text{A}^{-1}\ \text{s}^{-3}\right)\text{m}}{\text{m}^2} = \frac{(\text{A s})\text{m}}{\text{kg m}^4\ \text{A}^{-1}\ \text{s}^{-3}} = \text{A}^2\ \text{s}^4\ \text{kg}^{-1}\ \text{m}^{-3}$
	(b)	Vernier caliper is used to measure diameter, because the measurement was stated to 0.01 cm. Micrometer screw gauge is used to measure separation, because the measurement was stated to 0.01 mm.	
	(c)	**(i)**	If V reaches breakdown V_b, charged accumulated is $q_b = CV_b = \frac{\varepsilon_0 A}{d} V_b$. Since $q_b = I\frac{T}{N}$, $I\frac{T}{N} = \frac{\varepsilon_0 A}{d} V_b$ Rearranging, $V_b = \frac{ITd}{N\varepsilon_0 A}$
		(ii)	$V_b = \frac{ITd}{N\varepsilon_0 A} = \frac{4.0\times10^{-6}(12\times60+14)(5.42\times10^{-3})}{200(8.85\times10^{-12})\left(\frac{\pi}{4}\times0.065^2\right)} = 2.71\times10^6\ \text{V}$
		(iii)	It is necessary to ionize the molecules in the air for an electrical current to flow. No molecules in vacuum, hence no current/breakdown.

5	**(a)**	**(i)**	In the time taken for light to make the round trip, the cogwheel must advance by 0.5/720 of a revolution. $\frac{2L}{c}=\frac{0.5}{720}T$ $\frac{2L}{c}=\frac{0.5}{720}\frac{2\pi}{\omega}$ $c=\frac{1440\omega L}{\pi}$
		(ii)	$(3.15\times10^{5}\times10^{3})=\frac{1440\omega_0(8860)}{\pi}$ $\omega_0=77.6\text{ rad s}^{-1}$
		(iii)	$\omega_0, 3\omega_0, 5\omega_0$ COMMENT: The wheel must advance by 0.5/720 revolution, 1.5/720 revolutions, 2.5/720 revolutions, and so on
		(iv)	By averaging or otherwise, **random errors can be reduced** and a more accurate value for *c* can be obtained. COMMENT: There may also be merit in going for the 2nd or 3rd blocking of light if the teeth and notches are not of equal length.
	(b)		$\frac{2L}{c}=\frac{0.5}{18}T$ $\frac{2(88.6)}{3.00\times10^{8}}=\frac{0.5}{18}\frac{2\pi}{\omega}$ $\omega=295,485\text{ rad s}^{-1}$ $=295,485\div2\pi\times60$ $=2.82\times10^{6}\text{ rpm}$
	(c)	**(i)**	$\frac{\pi d^2}{4}+\frac{1}{2}(\frac{\pi D^2}{4}-\frac{\pi d^2}{4})=\frac{1}{2}(\frac{\pi D^2}{4}+\frac{\pi d^2}{4})=\frac{\pi}{8}(D^2+d^2)$
		(ii)	$\text{mass}=\rho V=(2.7)[\frac{\pi}{8}(6.0^2+5.5^2)](0.2)=14\text{ g}$
		(iii)	Smaller value. This approximation moved some mass nearer to the C.G. than they actually are. The nearer the mass is to the C.G., the smaller its contribution to the moment of inertia.

		(iv)	Moment of inertia, $I = \frac{1}{2}MR^2 = \frac{1}{2}(0.014)(\frac{0.0575}{2})^2 = 5.786 \times 10^{-6}$ kg m^2 Rotational KE, $E = \frac{1}{2}I\omega^2 = \frac{1}{2}(5.786 \times 10^{-6})(15000 \times \frac{2\pi}{60})^2 = 7.138$ J $VI =$ rate of increase of rotational KE $(9.0)I = \frac{7.138}{4.2}$ $I = 0.189$ A
		(v)	Some power may be lost dues to friction in the axle of the wheel. OR The calculation assumes the rotational KE was increased at a constant rate.

6	**(a)**	**(i)**	Surface charge density σ = charge per unit area Hence $Q = \sigma(\pi R^2)$ $\therefore \quad \sigma = \dfrac{Q}{4\pi R^2}$
		(ii)	Assume the circular thin disc to be an infinitely large uniformly charged circular sheet and by symmetry we say that the field at 'z' is a uniform outgoing field E_z and solve it by taking a gaussian cylinder with the top and bottom above and below the disc with $z \le R$.  Applying Gauss's Law, let electric field at position (0,0,z) be E_z $\oint E_z.dA = \dfrac{Q}{\varepsilon_o}$ $E_z \oint dA = \dfrac{\sigma A}{\varepsilon_o}$ $2AE_z = \dfrac{\sigma A}{\varepsilon_o}$ $E_z = \dfrac{\sigma}{2\varepsilon_o} = 2\pi\sigma(\dfrac{1}{4\pi\varepsilon_o})$ $= 2\pi k\sigma$ where $k = (\dfrac{1}{4\pi\varepsilon_o}) = 9.0\times10^9$ kg m^3 C^{-2} Comment: All necessary steps must be shown since this is a "show" question. It is necessary to include a diagram with the Gaussian surface drawn with the E field lines drawn
		(iii)	since $E = -\dfrac{dV}{dr}$ let V_z be the electric potential at (0,0,z), $V_z = -\int_0^z E\,dr = -Ez$ Comment: The minus sign should not be omitted as it signifies the relationship between V and E.

(b) **(i)**



$$\cos\theta = \frac{z}{\sqrt{z^2+r^2}}$$

Consider infinitesimal charge element $dq = \sigma\pi r^2$ and $dE_z = dE \cos\theta$

let E_z be the electric field at (0,0,z),

$$dE_z = \frac{kdq}{(\sqrt{z^2+r^2})^2}\cos\theta \quad where\ k = \frac{1}{4\pi\varepsilon_o}$$

$$dE_z = \frac{kdq}{(\sqrt{z^2+r^2})^2}\frac{z}{\sqrt{z^2+r^2}}$$

$$dE_z = \frac{kzdq}{(z^2+r^2)^{\frac{3}{2}}}$$

$$\int_0^R dE_z = \int_0^R \frac{kzdq}{(z^2+r^2)^{\frac{3}{2}}}$$

$$E_z = \int_0^R \frac{kz\sigma(2\pi rdr)}{(z^2+r^2)^{\frac{3}{2}}} = \pi k\sigma z\int_0^R \frac{2r\,dr}{(z^2+r^2)^{\frac{3}{2}}}$$

$$let\ u = z^2 + r^2, du = 2r\,dr$$

$$E_z = \pi k\sigma z\int_{z^2}^{z^2+R^2} \frac{du}{(u)^{\frac{3}{2}}} = \left[\frac{\pi k\sigma z}{-\frac{1}{2}}u^{-\frac{1}{2}}\right]\Bigg|_{z^2}^{z^2+R^2}$$

$$= 2\pi k\sigma z\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right] \text{(shown)}$$

		(ii)	When $z \leq R$, $z^2 + R^2 \approx R^2$ $E_z = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{R^2}}\right]$ $= \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{R}\right] \approx \frac{\sigma}{2\varepsilon_o}[1-0] = \frac{\sigma}{2\varepsilon_o}$ Comment: The simplification of the expression and the approximation to be made must be clearly shown in the working.
		(iii)	When $z \geq R$, $\frac{R}{z} \leq 1$ $E_z = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2 + R^2}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{\sqrt{z^2\left(1 + \frac{R^2}{z^2}\right)}}\right]$ $= \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{z}{z\sqrt{\left(1 + \frac{R^2}{z^2}\right)}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\sqrt{\left(1 + \left(\frac{R}{z}\right)^2\right)}}\right]$ $\approx \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\sqrt{\left(1 + \left(\frac{R}{z}\right)^2\right)}}\right] \approx \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\left[1 + \left(\frac{R}{z}\right)^2\right]^{\frac{1}{2}}}\right]$ $= \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\left[1 + \left(\frac{R}{z}\right)^2\right]^{\frac{1}{2}}}\right] = \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{\left[1 + \frac{1}{2}\left(\frac{R}{z}\right)^2\right]}\right]$ $= \frac{\sigma}{2\varepsilon_o}\left[1 - \frac{1}{1}\right] = 0$ Comment: The binomial expansion must be used in the simplification of the expression.

		(iv)	When $z \leq R$, $E_z = \dfrac{\sigma}{2\varepsilon_o}$ The electric field at (0.0.z) is independent of z, equivalent to the case for an infinite charged sheet, the electric field lines are parallel lines emerging from the surface and constant everywhere (R is ∞). When $z \geq R$, $E_z = 0$. The electric field at (0,0,z) of the disc is equivalent to the electric field of a point charge when R = 0. Comment: Comments on the physical significance and recognition of the common scenarios that led to the expressions are expected.

7	**(a)**	**(i)**	(Because of symmetry, the gravitational effect due to the Earth is equivalent to that of the same mass placed at the Earth's centre of mass.) At the equator, *the person will be farther from the Earth's centre of mass*. By Newton's Universal Law of Gravity, the gravitational force on the person will be smaller. Hence, the person's measured weight will be *smaller* at the equator.
		(ii)	*At the equator, part of the gravitational force provides the centripetal force* to keep the person in rotation about the Earth's axis. Hence, the normal contact force on the weighing scale will be smaller and the measured weight will be *smaller* at the equator
	(b)	**(i)**	At the North Pole, the measured weight is the person's true weight. Hence, $$m = \frac{W}{g} = \frac{775}{9.81} = 79.0 \text{ kg.}$$ The Earth moves about its own axis at a period of 24 hours = 24 x 3600 = 86 400 s. The difference at the equator is due to the centripetal force. Hence, the difference is $$F_c = m\,\omega^2\,r = m\left(\frac{2\pi}{T}\right)^2 r = (79.0)\left(\frac{2\pi}{24\times 3600}\right)^2 (6380 \times 10^3) = 2.67 \text{ N.}$$
		(ii)	For the person to be apparently weightless at the equator, the gravitational force there would give a centripetal acceleration equal to g = 9.81 m s^{-2}. $$F_g = F_c$$ $$g = a_c = \omega^2 r = \left(\frac{2\pi}{T}\right)^2 r$$ Plugging in g = 9.81 m s^{-2} and r = 6380 x 10^3 m, we obtain $$T = 5067 \text{ s} = 1.41 \text{ hours.}$$
	(c)	**(i)**	Principle of Conservation of Angular Momentum states that when the net external torque acting on a system about a point is zero, the total angular momentum of the system about that point is constant.
		(ii)	When ice-skater pulls in her arms to chest, her moment of inertia decreases. By PCAM, total angular momentum of her about her centre of mass is constant, thus, her angular velocity will increase.
		(iii)	By PCAM , Moment of inertia of disc X final angular velocity = Moment of inertia of sphere X initial angular velocity $$\frac{Mr_2^2}{2T_2} = \frac{2Mr_1^2}{5T_1}$$ And by volume of disc = volume of sphere

			$$\frac{hr_2^2}{1}=\frac{4r_1^3}{3}$$ By solving the above 2 eqns, $$\frac{4r_1^2}{6hT_2}=\frac{2r_1^2}{5T_1}$$ $$h=\frac{5T_1r_1}{3T_2}$$
		(iv)	Total mass = $\frac{hr_2^2\pi}{1}=\frac{4\pi r_1^3}{3}$ By $h=\frac{r_1}{3}\rightarrow r_2^2=4r_1^2$ For weightlessness to occur, gravitational force $\le$ centripetal force $\frac{GM}{r_2^2}\le r_2\omega_2^2$ $\frac{GM}{r_2^2}\le\frac{r_2 4\pi^2}{T_2^2}$ $T_2^2\le\frac{r_2^3 4\pi^2}{GM}=\frac{32r_1^3\pi^2}{GM}$
		(v)	T_2 should be less than 5067 s. $T_2^2\le\frac{32\pi^2r_1^3}{GM}=\frac{32\pi^2r_1^3}{G\rho V_{disc}}=\frac{32\pi^2r_1^3}{G\rho\pi r_2^2h}$ $T_2^2\le\frac{32\pi^2r_1^3}{G\rho\pi\frac{4}{5}r_1^2\frac{T_2}{T_1}h}=\frac{32\pi^2r_1^3}{G\rho\pi\frac{4}{5}r_1^2\frac{T_2}{T_1}\frac{r_1}{3}}=\frac{32\pi}{G\rho\frac{4}{3}}=\frac{24\pi}{G\rho}$ $T_2^2\le\frac{24\pi}{6.67\times10^{-11}(5510)}\rightarrow T_2=14317$ s Since $T_2>5067$ s, the person does not feel weightlessness.
		(vi)	The squared of distances between the particles of disc and the point A is larger than before, meaning the effective squared of distance between the centre of mass of the disc and the point A increases. Since the gravitational field strength is inversely proportional to the effective squared of distance between the centre of mass of disc and the point A, it will be smaller too.

8	(a)	An inertial reference frame is a frame that moves at a constant velocity and has no acceleration. Newton's laws are obeyed in inertial frames of reference.	
	(b)	The total momentum of a system is zero in zero-momentum frame (centre of mass frame). In this frame, the velocities of a system of particles simply change directions while keeping their speeds (magnitude) the same after elastic collisions	
	(c)	(i)& (ii)	Choose rightwards as positive. v_m : speed of *m* after the elastic collision v_{5m} : speed of 5*m* after the collision By PCOLM, $mv + 5m(0) = mv_m + 5mv_{5m}$ ---(1) By RSOA = RSOS, $v - 0 = v_{5m} - v_m$ ---(2) Solving (1) and (2), we obtain $v_m = -\frac{2v}{3}$ and $v_{5m} = \frac{v}{3}$. The negative sign indicates the mass *m* moves towards left after the collision. **Method II: Zero-momentum frame, aka centre of mass frame (CM)** Here are some notations which will be repeatedly used in this frame $u_{E,CM}$: speed of Earth frame in CM frame $u_{CM,E}$: speed of CM frame in Earth frame $u_{CM,E} = \frac{mv + 5m(0)}{m + 5m} = \frac{v}{6}$ We can also write $u_{E,CM} = -\frac{v}{6}$ Using Galilean transformations, For mass *m*, $u_{m,CM} = u_{m,E} + u_{E,CM} = v - \frac{v}{6} = \frac{5v}{6}$ (speed of mass *m* before the collision in CM frame) $v_{m,CM} = -\frac{5v}{6}$ (speed of mass *m* after the collision in CM frame) $v_{m,E} = v_{m,CM} - u_{E,CM} = -\frac{5v}{6} - \left(-\frac{v}{6}\right) = -\frac{2v}{3}$ (speed of mass *m* after the collision in Earth frame) For mass 5*m*, $u_{5m,CM} = u_{5m,E} + u_{E,CM} = 0 - \frac{v}{6} = -\frac{v}{6}$ (speed of mass 5*m* before the collision in CM frame) $v_{5m,CM} = \frac{v}{6}$ (speed of mass 5*m* after the collision in CM frame) $v_{5m,E} = v_{5m,CM} - u_{E,CM} = \frac{v}{6} - \left(-\frac{v}{6}\right) = \frac{v}{3}$ (speed of mass 5*m* after the collision in Earth frame)

(iii) Let's label the 5*m* mass on the right as Object A and the same mass (5*m*) on the left as Object B.

2nd elastic collision between mass *m* and Object B (mass 5*m*):

$v_{m,2nd}$: speed of *m* after the 2nd elastic collision

$v_{5m,B}$: speed of 5*m* (object B) after the collision

By PCOLM, $m\left(-\frac{2v}{3}\right)+5m(0)=mv_m+5mv_{5m,B}$ ---(1)

By RSOA = RSOS, $-\frac{2v}{3}-0=v_{5m,B}-v_m$ ---(2)

Solving (1) and (2), we obtain $v_{m,2nd}=+\frac{4v}{9}$ and $v_{5m,B}=-\frac{2v}{9}$.

The negative sign indicates the mass 5*m* (object B) moves towards left after the collision and the mass *m* moves toward right.

Since $v_{m,2nd}=+\frac{4v}{9}>v_{\text{Object A}}=\frac{v}{3}$ (from (c)(ii)) , the third collision will take place between mass *m* and Object A (5*m*).

3rd elastic collision between mass *m* and Object A (mass 5*m*)

$v_{m,3rd}$: speed of *m* after the 3rd elastic collision

$v_{5m(A)}$: speed of 5*m* (object A) after the collision

By PCOLM, $m\left(+\frac{4v}{9}\right)+5m\left(\frac{v}{3}\right)=mv_{m,3rd}+5mv_{5m(A)}$ ---(1)

By RSOA = RSOS, $+\frac{4v}{9}-\frac{v}{3}=v_{5m(A)}-v_{m,3rd}$ ---(2)

Solving (1) and (2), we obtain $v_{m,3rd}=+\frac{7v}{27}$ and $v_{5m(A)}=+\frac{10v}{27}$.

We know that after the 2nd collision, the Object B (mass 5*m*) is moving **left** with a speed of 2*v*/9. After the 3rd collision, the mass *m* continues to move **right** with a speed of 7*v*/27. Since both travel in opposite directions, there will not be a 4th collision between *m* and 5*m* (object B).

Method II: Zero-momentum frame, aka centre of mass frame (CM)

2nd elastic collision between mass *m* and Object B (mass 5*m*):

$$u_{CM,E}=\frac{m\left(-\frac{2v}{3}\right)v+5m(0)}{m+5m}=-\frac{v}{9} \text{ ---------- } u_{E,CM}=+\frac{v}{9}$$

			Using Galilean transformations, For mass m, $u_{m,CM} = u_{m,E} + u_{E,CM} = -\frac{2v}{3} + \frac{v}{9} = -\frac{5v}{9}$ (speed of mass m before the 2nd collision in CM frame) $v_{m,CM} = \frac{5v}{9}$ (speed of mass m after the 2nd collision in CM frame) $v_{m,E} = v_{m,CM} - u_{E,CM} = \frac{5v}{9} - \left(+\frac{v}{9}\right) = \frac{4v}{9}$ (speed of mass m after the 2nd collision in Earth frame) For Object B (mass 5m) $u_{5m(B),CM} = u_{5m(B),E} + u_{E,CM} = 0 + \frac{v}{9} = \frac{v}{9}$ (speed of Object B (5m) before the collision in CM frame) $v_{5m(B),CM} = -\frac{v}{9}$ (speed of Object B (5m) after the collision in CM frame) $v_{5m(B),E} = v_{5m(B),CM} + u_{E,CM} = -\frac{v}{9} - \left(+\frac{v}{9}\right) = -\frac{2v}{9}$ (speed of Object B (5m) after the collision in Earth frame) Since $v_{m,2nd} = +\frac{4v}{9} > v_{\text{Object A}} = \frac{v}{3}$ (from (c)(ii)) , the third collision will take place between mass m and Object A (5m). **3rd elastic collision between mass *m* and Object A (mass 5*m*):** $u_{CM,E} = \frac{m\left(\frac{4v}{9}\right)v + 5m(\frac{v}{3})}{m+5m} = +\frac{19v}{54}$ --------- $u_{E,CM} = -\frac{19v}{54}$ Using Galilean transformations, For mass m, $u_{m,CM} = u_{m,E} + u_{E,CM} = \frac{4v}{9} - \frac{19v}{54} = \frac{5v}{54}$ (speed of mass m before the 3rd collision in CM frame) $v_{m,CM} = -\frac{5v}{54}$ (speed of mass m after the 3rd collision in CM frame) $v_{m,E} = v_{m,CM} - u_{E,CM} = -\frac{5v}{54} - \left(-\frac{19v}{54}\right) = \frac{7v}{27}$ (speed of mass m after the 3rd collision in Earth frame) We know that after the 2nd collision, the Object B (mass 5m) is moving **left** with a speed of 2v/9. After the 3rd collision, the mass m continues to move **right** with a speed of 7v/27. Since both travel in opposite directions, there will not be a 4th collision between m and 5m (object B).
		(iv)	% of loss in KE of mass m = $\frac{\frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{7v}{27}\right)^2}{\frac{1}{2}mv^2} = 93.3\%$
	(d)	**(i)**	The ball **5** rolls down and it gains large KE mainly from the work done by the strong attractive force from the magnet but gains little KE from loss in GPE. The magnetic force is inversely related to the distance between ball and the magnet. Hence, the ball **5** will have a high momentum and energy just before colliding with the magnet.

			When it collides with the magnet, the momentum and energy of ball **5** will be transferred to ball **4**. The balls have identical masses, and any collision among them is elastic. After a few collisions among ball-bearings **4**,**3**, **2** and **1**, the momentum and energy of ball **5** will eventually be transferred to the ball **1** which moves at a high speed.
		(ii)	The ball **5** rolls down and it gains KE mainly from the work done by the strong attractive force from the magnet. The magnetic force is inversely related to the distance between ball and the magnet. The gain in KE and the work done by magnetic force will be smaller since the average force is smaller for the same distance travelled in Fig. 8.4 ad 8.5. Thus, the ball **1** will have this smaller KE and will move off with a smaller speed.