HUA YI SECONDARY SCH PRELIMINARY EXAMINAT	IOOL 4-G3 / TION 2024 5-G2			
NAME				
CLASS	INDEX NUMBER			
MATHEMATICS	4052/02			
PAPER 2 Candidates answer on the Question Paper.	PAPER 2 19 August 2024 2 hour 15 minutes Candidates answer on the Question Paper.			
MARK	ING			
SCHE	ME			
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Setter: Ms Jasmine Tan

Mathematical Formulae

Compound interest

Total Amount =
$$P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone =
$$\pi rl$$

Surface area of a sphere = $4\pi r^2$

Volume of a cone =
$$\frac{1}{3}\pi r^2 h$$

Volume of a sphere
$$=\frac{4}{3}\pi r^3$$

Area of triangle $ABC = \frac{1}{2}ab\sin C$

Arc length = $r\theta$, where θ is in radians

Sector area =
$$\frac{1}{2}r^2\theta$$
, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

Mean =
$$\frac{\sum fx}{\sum f}$$

Standard deviation =
$$\sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 (a) Express as a single fraction in its simplest form $\frac{2}{7-3x} - \frac{1}{6-x}$.

$$\frac{2}{7-3x} - \frac{1}{6-x} = \frac{2(6-x) - (7-3x)}{(6-x)(7-3x)}$$
$$= \frac{12 - 2x - 7 + 3x}{(6-x)(7-3x)} - --M1 \quad \text{[expand numerator correctly]}$$
$$= \frac{5+x}{(6-x)(7-3x)} - --A1$$

(**b**) It is given that
$$v = \frac{3-5w}{w+2} + 9$$
.

(i) Find v when
$$w = -6$$
.

$$v = \frac{3-5(-6)}{-6+2} + 9 = \frac{3}{4} \quad or \quad 0.75 \quad ---B1$$

1		Г17
Answer	• • • • • • • • • • • • • • • • • • • •	

(ii) Rearrange the formula to make *w* the subject.

$$v = \frac{3-5w}{w+2} + 9$$

(w+2)(v-9) = 3-5w ----M1 [remove bases]
wv-9w+2v-18 = 3-5w
wv-4w = 21-2v ----A1 [group w-terms on 1 side]
$$w = \frac{21-2v}{v-4}$$

= $-\frac{21-2v}{4-v}$ ----A1

Answer $w = \dots [3]$

(c) Solve the equation
$$\frac{5}{x-2} - \frac{3}{x^2-4} = \frac{1}{7}$$
.

Give your solutions correct to two decimal places.

 $\frac{5}{x-2} - \frac{3}{x^2 - 4} = \frac{1}{7}$ $\frac{5(x+2)-3}{(x+2)(x-2)} = \frac{1}{7} - --M1 \quad \text{[puts to common base]}$ $7(5x+10-3) = x^2 - 4$ $x^2 - 35x - 53 = 0 - --M1 \quad \text{[simplifies accurately quad. eqn.]}$ $x = \frac{-(-35) \pm \sqrt{(-35)^2 - 4(1)(-53)}}{2(1)} - -M1$ $x = -1.45389 \quad \text{or} \quad x = 36.45389$

x = -1.45 ---- A1 or x = 36.45 ---- A1 [accept 2 d.p. only]

Answer $x = \dots, x = \dots$ [5]

2 C is the point (-9,1) and D is the point (7,4).

$$\overrightarrow{CE} = \begin{pmatrix} -2 \\ 8 \end{pmatrix}.$$

(a) Calculate the length of the line *CD*.

Length
$$CD = \sqrt{(-9-7)^2 + (1-4)^2} - --M1$$

= $\sqrt{265} = 16.3 - --A1$

Answer units [2]

(b) Determine the coordinates of point *E*.

$$\overrightarrow{CE} = \overrightarrow{OE} - \overrightarrow{OC}$$
$$\binom{-2}{8} + \binom{-9}{1} = \overrightarrow{OE}$$
$$\therefore E(-11,9) \quad ---B1$$

Answer E(.....) [1]

(c) Find the equation of the line *DE*.

Leave your answer in the form ax + by = c, where a, b and c are constants.

Gradient
$$DE = \frac{4-9}{7-(-11)} = -\frac{5}{18} - --M1(ECF)$$

 $y = -\frac{5}{18}x + c$
Subst. $D(7,4)$
 $c = \frac{107}{18} - --M1$

So, equation of line *DE*:
$$\frac{5}{18}x + y = \frac{107}{18}$$

 $5x + 18y = 107$ ---- A1

(i) State the fifth term of the sequence.

$$5^{\text{th}} \text{ term} = \frac{21}{25} \quad ---B^{1}$$

(ii) Find an expression, in terms of n, for the *n*th term, T_n , of this sequence.

(b) Elijah finds a number grid from his board game.The diagram shows part of a number grid.

A rectangle outlining four numbers, as shown, can be placed anywhere on the grid.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	77	÷ -				

(i) If *p* represents the number in the top right corner of the rectangle, write down an expression, in terms of *p*, for the number in the bottom left corner of the rectangle.

(ii) Show that the difference between the products of the numbers in the opposite corners of the rectangle is always -7.

Answer

[2]

(p-1)(p+7) - p(p+6) ---M1 [form BOTH set of products, take difference] = $p^2 + 7p - p - 7 - p^2 - 6p ---A1$ [expand all terms correctly] = -7 (shown)

.....

(iii) Elijah says it is impossible for the sum of the four numbers in the rectangle to be 199.

Justify with relevant working, why he is correct.

Answer

[3]

Sum =
$$p-1+p+p+6+p+7$$

= $4p+12$ --- M1 [simplied sum correctly]

If sum = 199, then we must see that p is an integer,

4p+12 = 199 4p = 187 ---M1 [isolated *p* - term] p = 46.75

From above, we see that <u>p is not an integer</u>, so 199 cannot be the sum of four numbers on the grid. ----- A1 (attempt to explain and make relation to the workings)

.....

4 (a) Sophie earns a monthly salary of \$6875.

She gives 15% of this amount to her parents.

She puts 35% of the remainder into a savings account.

Calculate the amount she has left after giving to her parents and putting into her savings account. Leave your answer correct to the nearest dollar.

Amount given to parents = $15\% \times 6875 = \$1031.25$ Amount left after giving parents = \$5843.75

← Award M1 for <u>any 1</u>	ļ
I correct computation	1
I	J
	а.

Amount put into her savings = $35\% \times 5843.75 = \$2045.3125 - --M1$

Amount left after giving parents and paid to savings 5843.75 - 2045.3125 = 3798.4375= \$3798 (nearest dollar) ---A1

Answer \$ [3]

(b) The cash price of a sofa is \$830.Sophie buys this sofa on credit.She pays a deposit of one quarter of the cash price.She then pays 3 monthly payments of \$260.

Calculate the total amount Sophie pays for the sofa.

Total amount for sofa

 $= \left(\frac{1}{4} \times 830\right) + (3 \times 260) \quad --M1 \text{ [both sets of computation to be shown]}$ $= \$987.50 \quad --A1$

8

Answer \$ [2]

(c) Sophie pays a monthly rent of \$3174.20.This is 18% more than her monthly rent last year.

Calculate her monthly rent last year.

$$118\% \rightarrow \$3174.20$$

$$100\% \rightarrow \frac{100}{118} \times 3174.20 ---M1$$

$$= \$2690 ---A1$$

Answer \$ [2]

(d) During her vacation, Sophie visits her friend in Wellington.
Sophie spends NZD 940 in New Zealand using her credit card.
She is charged a 2.6% fee for the currency conversion.
The exchange rate between Singapore dollars (SGD) and New Zealand dollars (NZD) is
SGD 100 = NZD 120.7206.

Calculate the total amount on Sophie's credit card bill, including the fee. Give your answer in Singapore dollars, correct to the nearest cent.

NZD	SGD
120.7206	100
940	$\frac{100}{120.7206} \times 940 = 778.657\ 495 M1$

Total amount on credit card bill

 $= 102.6\% \times 778.657 \ 495 \ ---M1$ = SGD 798.90 (nearest cent) ---A1

9



ABCD is a field on horizontal ground. AD = 163 m, BD = 329 m, CD = 152 m and angle $BDC = 48^{\circ}$. The bearing of *B* from *A* is 151° and the bearing of *D* from *A* is 237°.

(a) Calculate the bearing of *D* from *B*.

 $\angle BAD = 237^{\circ} - 151^{\circ} = 86^{\circ}$ $\frac{\sin \angle ABD}{163} = \frac{\sin 86^{\circ}}{329} - --M1$ $\angle ABD = 29.61924^{\circ}$

Bearing of *D* from *B* $360^{\circ} - \angle ABD - (180^{\circ} - 151^{\circ}) - - - -M1$ $= 360^{\circ} - 29.61924^{\circ} - 29^{\circ}$ $= 301.4^{\circ} - - - - -A1$

(b) Calculate the distance from *B* to *C*.

By cosine rule,

$$BC^{2} = 152^{2} + 329^{2} - 2(152)(329)\cos 48^{\circ} - - -M1$$
$$= \sqrt{64421.23327}$$
$$= 253.81338 = 254m - - -A1$$

Answer m [2]

(c) An aircraft is flying above *D*.Find the angle of elevation of the aircraft from C when it is 245 m vertically above *D*.

Let the angle of elevation be θ .

$$\tan \theta = \frac{245}{152} \quad ---M1$$
$$\theta = 58.2^{\circ} \quad ---A1$$

Answer^o [2]

6 The diagram shows a semicircle, centre *O*, radius 16 cm.



(a) Show that angle AOB = 1.287 radians, correct to 3 decimal places. Answer

Since triangle *OAB* is isosceles, vertical line from *O* to *AB* will be perpendicular.

Let this point be P and OP = 16 - 3.2 = 12.8 cm.

$$\cos \angle AOP = \frac{12.8}{16}$$

 $\angle AOP = 36.86989^\circ = 0.643501 \text{ rad.} ---M1$
 $\angle AOB = 2 \times 0.643501 ---M1$
 $= 1.287 \text{ rad.} (shown)$

(b) Calculate the area of the shaded region.

Area of sector $OAB = \frac{1}{2} \times 16^2 \times 1.287 = 164.736 \text{ cm}^2 ---M1$ Area of triangle $OAB = \frac{1}{2} \times 16^2 \times \sin 1.287 = 122.87992 \text{ cm}^2 ---M1$ Area of shaded segment = 164.736 - 122.87992 = 41.9 cm² ---A1

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[2]

(c) The semicircle is the cross section of a water trough of length 2.8 m, standing on level ground.

The shaded area represents the water in the trough.

(i) Calculate the volume of water, in cm³, in the trough.Leave your answer in standard form.

Volume of water = **Segment area [Part b]** x length = $41.85608 \times 280 \quad --M2 \quad [M1 \text{ conversion}, M1 (ECF) \text{ expression}]$ = $1.17 \times 10^4 \text{ cm}^3 \quad (\text{standard form}) \quad ---A1$

*Max mark earned from ECF = 1 ONLY

(ii) Calculate the number of litres of water that must be added to fill the trough.

Volume of trough =
$$\left(\frac{1}{2} \times \pi \times 16^2 \times 280\right)$$
 ---M1
= 112594.6807 cm³

Additional water to be added = Vol of trough – Vol of water already present [part ci] = 112594.6807 - 11719.7024 - --M1 [ECF] = $100874.9788 \ cm^3$ = 101l (3 s.f.) ---A1

*Max mark earned from ECF = 1 ONLY

Answer litres [3]

7 A researcher wants to conduct a study to find out if there is a correlation between the analytical skills of adults with age.

The researcher invited 80 adults to solve a number puzzle.

(a)

(b)

The cumulative frequency curve shows the distribution of the time taken.



 $P_{80} = 64^{\text{th}} \text{ term } ---M1$ = 48s ----A1

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Answer $n = \dots$ [2]

(c) The frequency distribution of the data for the 80 adults invited to solve the number puzzle is shown in the table.

Time taken, x (seconds)	Frequency
$0 < x \le 20$	5
$20 < x \le 40$	40
$40 < x \le 60$	29
$60 < x \le 80$	6

Find an estimate of the standard deviation of the time taken by the 80 adults to solve the number puzzle.

Standard deviation

$$=\sqrt{\frac{5(10)^2 + 40(30)^2 + 29(50)^2 + 6(70)^2}{80}} - \left(\frac{5(10) + 40(30) + 29(50) + 6(70)}{80}\right)^2$$
$$=\sqrt{\frac{138400}{80}} - \left(\frac{3120}{80}\right)^2 \quad ---M1$$
$$= 14.4568 = 14.5 \quad ---A1$$

Answer s [2]

(d) The same group of adults were each given a word puzzle to solve.The box-and-whisker plot shows the distribution of the time taken.



Make two comparisons between the performances of the adults in solving the two puzzles.

A	nswer	

	Median	IQR
Word Puzzle (Box Plot)	39 s	25 s
Number Puzzle (From earlier)	38 or 39 s	14 s

- There was **no significant difference in the average performance** of the adults when solving for word and number puzzles. The **median values were about the same** (or equal). ---- **A1**
- Comparing the interquartile range, the adults had a greater spread of timings (i.e. less consistency) when doing word puzzles as the IQR value was higher. ---- A1

**Max ECF mark: 1 ONLY

[2]

8 In the diagram, *XYZ* is a straight line. AY = 20 m, AZ = 12 m and YZ = 16 m.The ratio of *XY* : *YZ* is 1 : 4.



(a) Prove that triangle *AYZ* is right-angled. *Answer*

$$AY^{2} = 20^{2} = 400$$

 $AZ^{2} + YZ^{2} = 12^{2} + 16^{2} = 400 - --M1$ [BOTH correct]

Since $AY^2 = AZ^2 + YZ^2$, by <u>converse of Pythagoras' Theorem</u> triangle *AYZ* is rightangled. [proven] ----- A1

(b) Find the value of $\cos \angle AYX$, giving your answer as a fraction in its lowest terms. $\cos \angle AYX = -\cos \angle AYZ$ $= -\frac{16}{20} = -\frac{4}{5} \quad ---B1$

(c) The area of triangle AXY is $24m^2$.

Lenard says: In another triangle AYW, whereby the length YW = YX and its area is the same as that of triangle AXY, it is possible for angle Y to be acute.

Explain why Lenard is correct. Use calculations to support your answer.

Answer

$$0.5 \times AZ \times YX = 24$$

$$YX = (24 \times 2) \div 12 = 4$$

Then, $0.5 \times 4 \times 20 \sin \angle AYW = 24$

$$\angle AYW = 36.9^{\circ} \quad (1 \text{ d.p})$$

From above, we see that it is possible to obtain an acute angle for angle Y, thus Lenard is correct.

[Turn Over

[2]

[2]

9 The variables x and y are connected by the equation $y = \frac{x^3}{2} - 5x - 2$.

Some corresponding values of *x* and *y* are given in the table.

x	-3	-2	-1	0	1	2	3	4
у	-0.5	4	2.5	-2	-6.5	k	-3.5	10

(a) Find the value of k.

k = -8 - - - B1

(b) On the grid on page 19, draw the graph of
$$y = \frac{x^3}{2} - 5x - 2$$
 for $-3 \le x \le 4$. [2]

- All 8 points correct ------ B1
- Smooth curve with turning points well-drawn ------ B1

(c) The equation
$$\frac{x^3}{2} - 5x = 7$$
 has only one solution.

Explain how this can be deduced from your graph.

[2]

To solve the equation, we will <u>look for the intersection point of the curve</u> $y = \frac{x^3}{2} - 5x - 2$ and the <u>line</u> y = 5. ----- **B1**

Since the graphs will <u>only intersect once</u>, it means that there is only one solution. ----- B1 *Student answers to <u>contain the keywords in BOLD</u>.

(d) By drawing a tangent, estimate the gradient of the curve at (1, -6.5).

****Actual gradient** = -3.5

Gradient of tangent =
$$\frac{-1.4 + 10.4}{-0.7 - 2.3}$$
$$= -3$$

- Draw acceptable tangent \rightarrow **B1**
- Calculated tangent lies in acceptable range (_____ to ____) → B1

(e) (i) On the same grid, draw the line y = 4 - x for $-1 \le x \le 4$.

x	-1	0	4
у	5	4	0

- Plot at least 2 correct points \rightarrow B1
- Draw smooth line \rightarrow B1

(ii) Write down the *x*-coordinate of the point where this line intersects the curve.

$$x = 3.4 - --B1$$

Answer x = [1]

(iii) This value of x is a solution of the equation $x^3 + Ax + B = 0$. Find the value of A and the value of B.

$$4-x = \frac{x^3}{2} - 5x - 2$$

8-2x = x³ - 10x - 4
x³ - 8x - 12 = 0

A = -8 ----B1 or B = -12 ----B1

Answer $A = \dots, B = \dots$ [2]

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[Turn Over

[2]



10 Javier and Stella are planning their exercise routines.

Javier and Stella have a body mass of 70 kg and 60 kg respectively.

The speed for walking and jogging of the two individuals are represented in the chart.



They research the following information about the benefits of exercise.

HEALTH ADVICE

For optimal health benefits, adults should do at least 150 minutes of moderate-intensity aerobic activity, or at least 75 minutes of vigorous-intensity aerobic activity per week. For additional health benefits, adults should increase their moderate-intensity aerobic activity to 300 minutes per week, or an equivalent combination of moderate-intensity and vigorous-intensity aerobic activity.

1 minute of vigorous-intensity aerobic activity is equivalent to 2 minutes of moderateintensity aerobic activity.

E.g. 10 minutes of jogging = 20 minutes of brisk walking.

Muscle-strengthening activities should be done on 2 or more days per week.

	Body mass			
	60 kg	70 kg	80 kg	90 kg
Walking 5 km/h	105	120	135	150
Walking 6.5 km/h	130	150	170	190
Jogging 8 km/h	240	280	315	350
Jogging 9.5 km/h	300	345	390	435

Approximate calories burned during 30 minutes of aerobic exercise

Non-aerobic muscle-strengthening activities

- 1 hour of yoga burns approximately 3 calories per kilogram of body mass
- 1 hour of weight training burns approximately 4 calories per kilogram of body mass
- (a) In their first week of exercise, they each plan to go for four brisk walks. They will walk the same route each time.

The four walks together meet the minimum target for the time for optimal health benefits in one week.

(i) Find the distance of one of these walks.

***NOTE:**

- In total, <u>4 walks will take 150 minutes</u> each week for both Javier and Stella.
- Brisk-walking is moderate intensity activity.
- Both Javier and Stella have the same brisk-walking speed (6.5 km/h).

 $60 \text{ min } \to 6.5 \text{ km}$ $150 \text{ min } \to \frac{6.5}{60} \times 150 = 16.25 \text{ km} ---M1$ Distance of 1 walk = $\frac{16.25}{4} = 4\frac{1}{16} = 4.0625 \text{ km} ---A1$

(ii) Find out how many more calories Javier burns in these four walks than Stella.

In every 30 minutes, difference in calories burnt =150-130=20

 $\begin{array}{rcl} 30 \ \text{min} & \rightarrow & 20 \ \text{calories} \\ 150 \ \text{min} & \rightarrow & 100 \ \text{calories} & ---B1 \end{array}$

Answer calories [1]

(b) After one month, they change their routines. Javier wants to gain additional health benefits. He decides to do a 4 km walk 3 times per week and do a 6 km jog 2 times per week. He will also attend a 45-minute yoga class 2 times per week.

Stella wants to maintain optimal health benefits.She decides to do an 8 km jog 2 times per week.She will also attend a 30-minute weight training session 2 times per week.

Javier says:

We will both meet our targets for exercise. However, I will burn about 50% more calories than Stella during our exercise per week.

Is Javier correct?

Justify your decision with clear calculations.

Answer

[7]

From Health Advice	From Health Advice	• Yoga plan: 45 min, 2 times per week
• Brisk-walking is moderate-	• Jogging is vigorous-intensity	
intensity	• Jog plan: 6 km, 2 times per week	
• Walk plan: 4 km, 3 times		
per week		
Total distance per week = 12	Total distance per week = 12 km	From non-aerobic (Pg 21)
km	Jogging speed = 8 km/ h	Total time per week $= 90 \text{ min}$
Walking speed = 6.5 km/ h	$8 \text{ km} \rightarrow 60 \text{ min}$	$60 \text{ min} \rightarrow 3 \text{ cal/ kg}$
$6.5 \text{ km} \rightarrow 60 \text{ min}$	$12 \text{ km} \rightarrow 90 \text{ min}$	90 min \rightarrow 4.5 cal/ kg
$12 \text{ km} \rightarrow 110 \frac{10}{12} \text{ min}$	<u>Ratio (convert to moderate-</u>	From 90 min yoga per week, total calories
13	<u>intensity)</u>	burnt for 70 kg Javier,
<u>Award J1</u> for Javier's	1 min jog \rightarrow 2 min walk	$4.5 \times 70 = 315 \text{ cal}^*$
walking/jogging	90 min jog \rightarrow 180 min walk	Award JC1*for calories
	Calories burned	counted correctly once!
See calories table	See calories table	Overall calories burned
$30 \min \rightarrow 150 \text{ cal}$	$30 \min \rightarrow 280 \text{ cal}$	$553\frac{11}{1} + 840 + 315 - 1709$ cal (nearest int)
$110\frac{10}{10} \text{ min } \rightarrow 553\frac{11}{10} \text{ cal}^*$	90 min \rightarrow 840 cal*	13 13 1707 car (nearest int.)
13 13 13		
Total moderate-intensi	ity time (e.g brisk-walk) = $180 + 110\frac{1}{1}$	$\frac{0}{3} = 290\frac{10}{13}$ min (< 300 min)
		Award JT 1 for

JAVIER (gain *additional* health benefits)



STELLA (gain optimal health benefits)

Comparing % of calories burned for Javier to Stella,

- $=\frac{Javier}{Stella} \times 100\% = \frac{1709 1251}{1251} \times 100\%$ Award M1 (ECF) for comparing calories in percentage =37% (nearest int.) (< 50%)
- From above, we see that **Javier will only burn 37% more**, not 50% more than Stella.
- Comparing the timings, we see that only Stella will achieve her target to maintain optimal health benefits. Thus, Javier is incorrect.



- END OF PAPER -