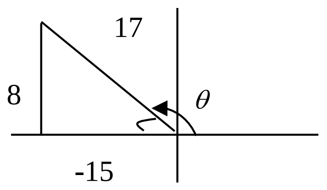
2021 Y3 Express Additional Mathematics

Final Examinations Paper 1 Solutions

Solutions:	
1	$-3xy + 2y^2 = -1$ -- (1) $2x - y = 1$ -- (2) From (2): $y = 2x - 1$ -- (3) Sub (3) into (1): $-3x(2x-1) + 2(2x-1)^2 = -1$ $-6x^2 + 3x + 2(4x^2 - 4x + 1) = -1$ $-6x^2 + 3x + 8x^2 - 8x + 2 = -1$ $2x^2 - 5x + 3 = 0$ $(x-1)(2x-3) = 0$ $x - 1 = 0$ or $2x - 3 = 0$ $x = 1$ or $x = \frac{3}{2}$ $y = 1$ or $y = 2$ **Alternatively,** From (2): $x = \frac{y+1}{2}$ -- (3) Sub (3) into (1): $-3\left(\frac{y+1}{2}\right)y + 2y^2 = -1$ $-3y^2 - 3y + 4y^2 = -2$ $y^2 - 3y + 2 = 0$ $(y-1)(y-2) = 0$ $y = 1$ or $y = 2$ $x = 1$ or $x = \frac{3}{2}$

Solutions:	
2(i)	 $\tan\theta = -\frac{8}{15}$
(ii)	$\cos(-\theta) = \cos\theta$ $= -\frac{15}{17}$
(iii)	$\sec(180° - \theta) = \frac{1}{\cos(180° - \theta)}$ $= \frac{1}{-\cos(\theta)}$ $= \frac{1}{-\left(-\frac{15}{17}\right)}$ $= \frac{17}{15}$

Solutions:	
3	Gradient $=\dfrac{11-1}{8-3}=2$ $1=2(3)+c \quad \therefore c=-5$ or $Y-1=2(X-3)$ $\therefore Y=2X-5$ $\dfrac{1}{y}=2\left(\dfrac{1}{x^2}\right)-5$ $=\dfrac{2-5x^2}{x^2}$ $\therefore y=\dfrac{x^2}{2-5x^2}$

Solutions:

4(a)

$$\frac{3x^2-2x+2}{2x\left(x^2+2\right)}=\frac{A}{2x}+\frac{Bx+C}{x^2+2}$$

$$3x^2-2x+2=A\left(x^2+2\right)+\left(Bx+C\right)2x$$

Let $x=0$, $2=2A \Rightarrow A=1$

Compare coefficients of x^2 :

$3=A+2B=1+2B$

$\therefore B=1$

Compare coefficients of x :

$-2=2C$

$\therefore C=-1$

$$\frac{3x^2-2x+2}{2x\left(x^2+2\right)}=\frac{1}{2x}+\frac{x-1}{x^2+2}$$

(b)

$$\frac{x^2+4x+5}{x^2+4x+3}=1+\frac{2}{x^2+4x+3}$$

$$\begin{array}{r|l} & \quad\quad\quad\quad\;\, 1 \\ x^2+4x+3 & x^2+4x+5 \\ & x^2+4x+3 \\ \hline & \quad\quad\quad\quad\;\, 2 \end{array}$$

$$\frac{2}{x^2+4x+3}=\frac{A}{x+1}+\frac{B}{x+3}$$

$$2=A(x+3)+B(x+1)$$

Let $x=-1$, $2=2A \Rightarrow A=1$

Let $x=-3$, $2=-2B \Rightarrow B=-1$

$$\frac{x^2+4x+5}{x^2+4x+3}=1+\frac{1}{x+1}-\frac{1}{x+3}$$

Solutions:

5 **(i)**

$f(2) = 11$

$a(2)^3 + (2)^2 + b(2) + 15 = 11$

$8a + 4 + 2b + 15 = 11$

$8a + 2b = -8$

$4a + b = -4$ ___(1)

$f(-1) = 14$

$a(-1)^3 + (-1)^2 + b(-1) + 15 = 14$

$-a + 1 - b + 15 = 14$

$a + b = 2$ ___(2)

(1) − (2):

$3a = -6$

$a = -2$

$b = -4(-2) - 4 = 4$

(ii)

$$f\left(\frac{5}{2}\right) = -2\left(\frac{5}{2}\right)^3 + \left(\frac{5}{2}\right)^2 + 4\left(\frac{5}{2}\right) + 15$$

$$= 0$$

$\therefore (5 - 2x)$ is a factor of $f(x)$.

(iii)

$$\begin{array}{r} x^2 + 2x + 3 \\ -2x + 5 \overline{)\, -2x^3 + x^2 + 4x + 15} \\ 2x^3 + 5x^2 \qquad\qquad \\ -4x^2 + 4x \qquad \\ \hline -4x^2 + 10x \qquad \\ -6x + 15 \\ \hline -6x + 15 \\ 0 \end{array}$$

$f(x) = 0$

$(-2x + 5)(x^2 + 2x + 3) = 0$

$-2x + 5 = 0$ or $x^2 + 2x + 3 = 0$

$x = \dfrac{5}{2}$

Method 1: Discriminant

$b^2 - 4ac = 2^2 - 4(1)(3)$

$= -8 < 0$

Method 2:Completing the Square

$x^2 + 2x + 3 = (x + 1)^2 + 2 > 0$

$\therefore x^2 + 2x + 3$ has no real roots and $x = \dfrac{5}{2}$ is the only real root.

Solutions:	
6	**(a)** $\text{cosec}^2 2x = 2$ $\frac{1}{\sin^2 2x} = 2$ $\sin^2 2x = \frac{1}{2}$ $\sin 2x = \pm\frac{1}{\sqrt{2}}$ All quadrants $0 \le x \le 180°$ $0 \le 2x \le 360°$ $\alpha = 45°$ $2x = 45°, 135°, 225°, 315°$ $x = 22.5°, 67.5°, 112.5°, 157.5°$
	(b) (i) Amplitude $= 1-(-5) = 6$ $a = 6$ Period is 8π $b = \frac{2\pi}{8\pi} = \frac{1}{4}$ Axis: $y = -5$ $c = -5$

Solutions:	
7i	$2\log_4 3x = \log_3 27 + \log_2 (x-5)$ $\dfrac{2\log_2 3x}{\log_2 4} = 3 + \log_2 (x-5)$ $\log_2 3x - \log_2 (x-5) = 3$ $\log_2 \dfrac{3x}{x-5} = 3$ $\dfrac{3x}{x-5} = 8$ $3x = 8x - 40$ $5x = 40$ $x = 8$
ii	$\ln(4^x + 2) = x\ln 2 + \ln 3$ $= \ln 2^x + \ln 3$ $= \ln(2^x \cdot 3)$ $4^x + 2 = 2^x \cdot 3$ $2^{2x} - 3\left(2^x\right) + 2 = 0$ Let 2^x be y. $y^2 - 3y + 2 = 0$ $(y-2)(y-1) = 0$ $y = 2$ or $y = 1$ $2^x = 2$ or $2^x = 1$ $x = 1$ or $x = 0$
iii	$81 \times 3^{\lg x} = 9^{1+\lg(x-10)}$ $3^4 \times 3^{\lg x} = 3^{2+2\lg(x-10)}$ $3^{4+\lg x} = 3^{2+2\lg(x-10)}$ $4 + \lg x = 2 + 2\lg(x-10)$ $2\lg(x-10) - \lg x = 2$ $\lg(x-10)^2 - \lg x = 2$ $\lg \dfrac{(x-10)^2}{x} = 2$ $\dfrac{(x-10)^2}{x} = 100$ $x^2 - 20x + 100 = 100x$ $x^2 - 120x + 100 = 0$ $x = 119$ or $x = 0.839$ (rejected $\because x - 10 > 0$)

Solutions:

8

(i) Centre C

$$=\left(\frac{-3+1}{2},\frac{0+2}{2}\right)$$

$$=(-1,1)$$

Radius

$$=\sqrt{\left[1-(-1)^2\right]+(2-1)^2}$$

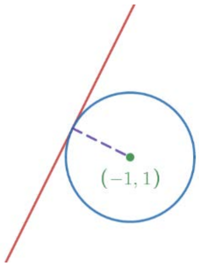
$$=\sqrt{5}$$

(i) Gradient of perpendicular line $=-\dfrac{1}{2}$

$$y-1=-\frac{1}{2}[x-(-1)]$$

$$=-\frac{1}{2}x-\frac{1}{2}$$

$$y=-\frac{1}{2}x+\frac{1}{2}$$



Solve for intersection R.

$$2x+8=-\frac{1}{2}x+\frac{1}{2}$$

$$4x+16=-x+1$$

$$5x=-15$$

$$x=-3 \quad \Rightarrow y=2 \quad \therefore R(-3,2)$$

<u>**Alternatively,**</u>

$$(x+1)^2+(y-1)^2=5$$

$$(x+1)^2+(2x+8-1)^2=5$$

$$x^2+2x+1+4x^2+28x+49=5$$

$$5(x^2+6x+9)=0 \quad \Rightarrow (x+3)^2=0$$

$$x=-3, y=2 \quad \therefore R(-3,2)$$

(iii)

Let centre of C_2 be (x, y).

$$\left(\frac{x-1}{2},\frac{y+1}{2}\right)=(-3,2)$$

$$x=-5, y=3$$

Same radius of 5

Equation of C_2 is $(x+5)^2+(y-3)^2=5$