Name:		Index Number:		Class:	

DUNMAN HIGH SCHOOL
Preliminary Examination
Year 6

MATHEMATICS (Higher 2) 9740/02

Paper 2

17 September 2012

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Name, Index Number and Class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

At the end of the examination, **attach the question paper to the front of your answer script.**

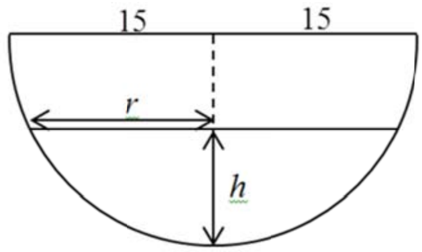
The total number of marks for this paper is 100.

For teachers' use:

Qn	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Total
Score														
Max Score	**6**	**6**	**6**	**10**	**12**	**4**	**4**	**6**	**8**	**8**	**10**	**10**	**10**	**100**

Section A: Pure Mathematics [40 marks]

1



A hemispherical goldfish tank with radius 15 cm (as shown in the figure above) was initially filled with water. The tank has a defect and water is leaking at a constant rate of 20 cm^3 per min. The volume of water in the tank is given by $V=\frac{\pi}{3}\left(45h^2-h^3\right)$, where h is the depth of water at the centre of the tank in cm. Show that r, the radius of the water surface in cm, is given by $r=\sqrt{30h-h^2}$. [1]

Given that the minimum depth of water needed for the goldfish to survive is 5 cm, find, at this instant,

(i) the rate of change of the depth of water, and [2]

(ii) the rate of decrease of the radius of the water surface. [3]

2 A curve C_1 is defined parametrically by

$$x=\frac{2}{t-1} \text{ and } y=\frac{4}{t+1}, \quad t\neq\pm 1.$$

Sketch a clearly labelled diagram of C_1. [2]

Describe a sequence of geometrical transformations which maps C_1 to C_2 defined by

$$x=\frac{1}{1-t} \text{ and } y=\frac{4}{t+1}, \quad t\neq\pm 1.$$ [2]

Sketch C_3 which is the reciprocal function of C_1, stating the equations of any asymptotes and any points of intersection with the axes. [2]

[Turn over

3 In a single Argand diagram, sketch the following loci, labelling each locus clearly.

(i) $|z| = 5$,

(ii) $|z+8| = |z-8\text{i}|$.

The two complex numbers that satisfy the above equations are represented by p and q, where $\arg\left(\dfrac{p}{q}\right) > 0$. Find p and q exactly. [5]

State the exact value of $\arg\dfrac{(5-p)}{(5-q)}$. [1]

4 A finite sequence $\{a_n\}$ has 50 terms and is such that $a_{n+1} = a_n + 0.15$ for $n = 1, 2, 3, \ldots, 49$.

(i) Given that $a_{50} = 99a_1$, show that $a_1 = 0.075$. [2]

(ii) Find, without using a calculator, the value of $\sum_{n=1}^{50} a_n$. [2]

Another infinite sequence $\{b_m\}$ is such that $b_1 = a_{50}$ and $\dfrac{b_m}{b_{m-1}} = 0.98$ for $m \geq 2$.

(iii) Determine the smallest value of k such that $b_k < a_{25}$. [2]

(iv) Find the least value of h such that the sum of the first h terms of $\{b_m\}$ is more than 99% of its sum to infinity. [2]

(v) If $\dfrac{b_m}{b_{m-1}} = -0.98$ instead, find $\sum_{m=0}^{\infty} b_{1+3m}$. [2]

[Turn over

5 The equations of planes p_1 and p_2 are given by

$$p_1 :\ \mathbf{r}\bullet\begin{pmatrix}\alpha\\7\\2\end{pmatrix}=\alpha+20,\ \alpha\in\mathbb{R},$$

$$p_2 :\ \mathbf{r}\bullet\begin{pmatrix}3\\5\\\beta\end{pmatrix}=7,\ \beta\in\mathbb{R}.$$

(i) Verify that the point $A\,(1,2,3)$ lies in p_1 and find the value of β if A lies in p_2 as well. [2]

(ii) Find the equation of the line of intersection l between p_1 and p_2 in terms of α. If l is coincident with another line with equation given by $\mathbf{r}=\begin{pmatrix}1\\2\\3\end{pmatrix}+k\begin{pmatrix}4\\-2\\1\end{pmatrix},\ k\in\mathbb{R},$ show that $\alpha=3$. [3]

(iii) Find the acute angle between p_1 and p_2. [2]

(iv) Another point B has coordinates $(2,-4,8)$. Find the position vector of the foot of the perpendicular from B to l. Hence find the reflection of the line through A and B about l. [5]

Section B: Statistics [60 marks]

6 An office received a total of 810 loan applications on a particular day. The applications were categorised into business, house and study loans. Due to a shortage of staff, the office was only able to process 30 applications on that day.

(i) Describe how you would choose a systematic sample of size 30 from the loan applications. [2]

(ii) Explain why a systematic sample may not ensure that all categories of loan applications were processed within that day. [1]

(iii) Explain why a stratified sample may be preferred over a systematic sample in the context of this question. [1]

[Turn over

7 The number of Green Top taxis and EZCab taxis arriving in a randomly chosen 10-minute period at the airport taxi bay may be assumed to be independent Poisson variables with mean 3 and 5 respectively.

(i) Show that the probability that at least 7 taxis arrive at the taxi bay in a randomly chosen 10-minute period is 0.687. [2]

(ii) In a randomly chosen 10-minute period, at least 7 taxis arrived. Find the probability that all these taxis were EZCab taxis. [2]

8 How many six-digit numbers can be formed from the digits 1, 2, 3, 4, 5, 6 and 7 if

(i) repetition of digits is not allowed? [1]

(ii) repetition of digits is allowed? [1]

(iii) every digit in the number appears at least twice? [4]

9 Data is gathered on how exam scores, y, vary with the number of hours, x, students spent studying for Mathematics each week. The data for 7 students is shown below:

Study hours, x	1	2	3	6	12	24	36
Exam score, y	41	61	71	86	92	93	95

(i) Sketch a scatter diagram for the data and determine the linear product-moment correlation coefficient. Comment on whether a linear model would be appropriate here. [3]

(ii) The following are two possible models for the data above:

Model A: $y = a + \dfrac{b}{x^2}$

Model B: $y = c + \dfrac{d}{\sqrt{x}}$, where a, b, c, and d are real constants.

State, with a reason, which of the above is a better model. [2]

(iii) For your choice of the model in **(ii)**, find the corresponding equation of the least squares regression line. Use your equation to estimate the value of y when $x = 65$. Comment on the reliability of your answer. [3]

[Turn over

10 The waiting time for a patient to see a doctor in a clinic is T minutes. The waiting times of a randomly chosen sample of 120 patients are summarised by

$$\sum(t-15)=123, \text{ and } \sum(t-15)^2=2504.$$

(i) Find an unbiased estimate of the population mean and show that the unbiased estimate of the population variance is 19.983. [2]

The management of the clinic claims that the mean waiting time is 15 minutes.

(ii) Carry out a test at the 5% significance level to determine whether the mean waiting time differs from 15 minutes. [3]

(iii) In another sample, the waiting times of n patients (where $n \geq 50$) were recorded and a one-tail test was conducted. Given that the sample mean was $\overline{t}=15.5$ minutes and the null hypothesis was not rejected at the 6% level of significance, find the range of values of n. [3]

11 **(a)** A pair of siblings, Abbey and Betty, went to a book fair with six other friends. The eight of them stood in a queue in random order to get the autograph of a well-known writer. Find the probability that

(i) Abbey is first and Betty is sixth in the queue, [1]

(ii) either Abbey is first or Betty is second in the queue (or both). [2]

(b) A man owns a wooden chest with three drawers. One drawer contains two gold coins, another contains one gold coin and one silver coin, and the last drawer contains two silver coins. A drawer is selected randomly and a coin is then selected randomly from that drawer.

(i) Given that a gold coin is selected, find the probability that the chosen drawer is the one that contains two gold coins. [3]

(ii) The man's two sons take turns to select a coin each from a drawer in the same manner, starting with the older son. What is the probability that the older son picks a gold coin, and the younger one picks a silver coin? [4]

[Turn over

12 On a Friday evening, the class chairperson emails his 20 classmates to propose an outing to watch a newly released action thriller. The random variable X is the number of classmates who respond positively to his email that weekend.

It is assumed that X has the distribution $\mathrm{B}(20, p)$ such that $0 < p \leq 0.5$.

(i) Given that $\mathrm{P}(X = 0 \text{ or } 19) = 0.1$, formulate an equation for p, and find its numerical value. [2]

On each weekday morning, the class chairperson draws a name from a bag containing the names of all the students in the class. The chairperson notes the name and then puts it back into the bag. The person whose name is drawn will be responsible for keeping the homeroom tidy for that day. The class consists of 12 girls and 9 boys. The random variable Y is the number of days, out of a 5-day week, in which a girl is picked to be on duty.

(ii) Explain in context clearly why the random variable Y is more suited to be modelled by a binomial distribution than X. [2]

(iii) Find the probability that in a week, a girl is on duty more often than a boy. [3]

(iv) Using a suitable approximation, find the probability that in a semester of 20 weeks, there are less than 11 weeks in which a girl is on duty more often than a boy in the week. [3]

[Turn over

13 Students seeking admission to Kingdom University and Island University are required to apply using their national examination scores. The scores of the prospective applicants for each university can be modelled by independent normal distributions with means and standard deviations as shown in the table.

	Mean Score	Standard Deviation
Kingdom University	500	100
Island University	480	60

(i) Kingdom University only admits the top 30% of its prospective applicants. Find the minimum score that a student must achieve in order to gain admission into this university. [2]

A random sample of 5 prospective applicants of Kingdom University and a random sample of 5 prospective applicants of Island University are chosen.

Find the probability that

(ii) the difference in mean scores between the prospective students of the two universities exceeds 50, [3]

(iii) the minimum score of these 10 prospective students is at least 500, [2]

(iv) the total score of the 5 prospective students of Island University is more than 5 times the score of a randomly chosen prospective student of Kingdom University. [3]

END OF PAPER