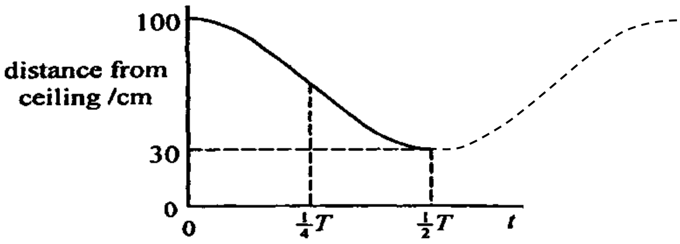
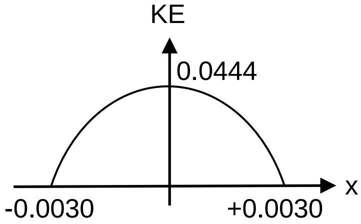
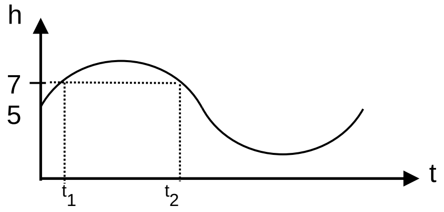
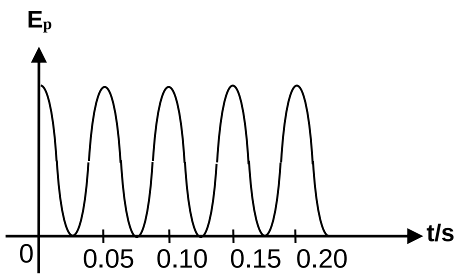
TUTORIAL 8: OSCILLATIONS SOLUTIONS

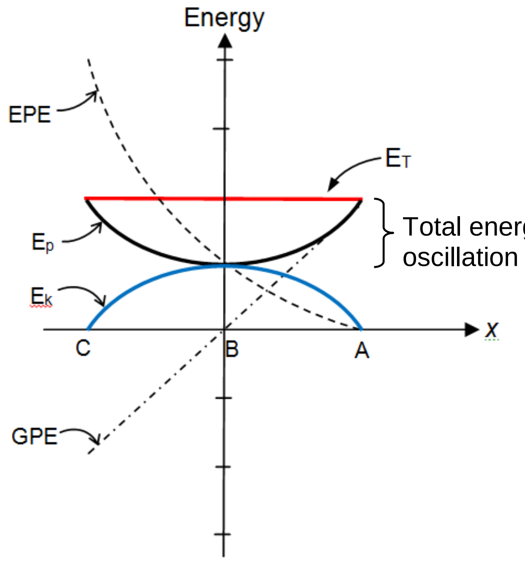
<u>Level 1 Solutions</u>

1	**D**. Refer to the definition of SHM to appreciate the answer.	
2	**D**. Since *F* is proportional to *a*, the *F-r* graph has the same shape as *a-r* graph.	
3	**D**. Speed is always at the maximum at the equilibrium position.  	
4	**C**. The mass is moving away from the equilibrium position. Its acceleration is always directed towards that point. Cannot be A, because a = 0 (since x =0), $\therefore$ no direction to compare with v. Cannot be B, because v=0 (since v = dx/dt=0), $\therefore$ no direction to compare with a. Cannot be D because it is moving towards the equilibrium position.	
5	**A**. Since bob is released at t = 0, $\therefore$ v = 0 when t = 0, $\therefore$ KE = 0 when t = 0 Cannot be C because the frequency of energy variation is doubled.	
6 (a)	(i) 1. amplitude = 0.15 m, 2. period = 1.0 s, 3. frequency = 1.0 Hz, 4. angular frequency = 6.3 rad s^{-1} (ii) amplitude (iii) From the diagram we can see that at t = 0.5 s, that A is at is amplitude while B is at its equilibrium position. $\therefore$ they are $\frac{1}{4}$ of a cycle apart. Since 1 cycle $\rightarrow 2\pi$ $\therefore$ $\frac{1}{4}$ cycle $\rightarrow 2\pi/4 = \frac{1}{2}\pi$ phase difference = 0.5π rad (iv) $\omega = 2\pi f = 2\pi(1) = 2\pi$ $x = -x_o \cos \omega t = -0.15 \cos 2\pi t$ (v) At P, the speed is maximum i.e. $v_B = x_0\omega$ $= (0.1)(2\pi)$ correct amplitude $= 0.63$ m s^{-1}	1 1 1 1 1 1 1 1 1 1 1 1

Level 2 Solutions

7(a)	$f = 1/T = 1/0.020 = \underline{50 \text{ Hz}}$	1
(b)	$\omega = 2\pi f = 2\pi(50) = \underline{100\pi \text{ rad s}^{-1}}$	1
(c)	$x = x_o \sin \omega t = \underline{0.0030 \sin(100\pi t)}$ correct amplitude correct ω	1 1
(d)	(i) $\underline{0.9 \text{ m s}^{-1}}$ (ii) For each cycle, the body passes the zero displacement point twice at the same speed but in opposite directions. (iii) For shm, the body always changes directions at the two extreme displacements where its velocity is zero.	1 1 1
(e)	(i) $KE_{max} = ½m\, v_{max}^2 = ½\, m\, (x_o\, \omega)^2 = ½(0.100)\,(0.0030 \times 100\,\pi)^2$ $= \underline{0.0444 \text{ J}}$ (ii)  parabolic shape values at axes	1 1 1
8 (a)	(i) Period = <u>0.6 s</u> (ii) $\omega = 2\pi/T$ $= 2\pi/(0.6) = \underline{10.5 \text{ rad s}^{-1}}$	1 1 1
(b)	(i) <u>0.20 s</u> (ii) $\phi/2\pi = t/T$ $\Rightarrow \phi = 2\pi t/T = 2\pi(0.20)/(0.60) = \underline{2\pi/3 \text{ rad}}$	1 1 1
(c)	(i) Damping is the loss of energy from an oscillating system to the environment due to dissipative forces such as air resistance or friction. (ii) 1. By <u>attaching a card</u> to the mass such that it is <u>perpendicular</u> to the direction of motion of the oscillating system, light damping can be achieved. 2. By immersing the oscillating mass in a viscous fluid such as oil, the degree of damping can be increased. Different degree of damping is achieved by using fluid of different viscosity.	1 2 1
9 (a)	(i) $\theta = \underline{\omega t}$ (ii) $ST = r \sin\theta = r \sin \omega t$	1 1
(b)	The shadow moves in simple harmonic motion.	1
(c)	(i) $v_{max} = x_o\, \omega$ $= 20\,(3.5) = \underline{70 \text{ cm s}^{-1}}$ (ii) $a_{max} = x_o\, \omega^2$ $= 20\,(3.5)^2 = \underline{245 \text{ cm s}^{-2}}$	1 1 1 1

10 (c)	(i) Max. depth = 5.0 + 3.0 = <u>8.0 m</u> (ii) Min. depth = 5.0 – 3.0 = <u>2.0 m</u> (iii) Time between high- and low-water = $\frac{1}{2}$(Period) = $\frac{1}{2}$(45600) = <u>22,800 s</u> (iv) If h = 5.0 m, then $5.0 = 5.0 + 3.0 \sin \frac{2\pi t}{45600}$ $\Rightarrow \quad \sin \frac{2\pi t}{45600} = 0$ $\Rightarrow \quad \frac{2\pi t}{45600} = 0 \text{ or } \pi$ $\Rightarrow \quad t =$ <u>0 s or 22,800 s</u> (v)  If h = 7.0 m, then $7.0 = 5.0 + 3.0 \sin \frac{2\pi t}{45600}$ $2.0 = 3.0 \sin \frac{2\pi t}{45600}$ $\sin \frac{2\pi t}{45600} = 2.0/3.0$ **Caution! Set calculator to radian mode.** $\frac{2\pi t}{45600} = 0.730 \text{ or } 2.41$ t = 5298 s or 17490 s (These are the two instances, t_2 & t_1) $\therefore t_2 - t_1 = 17490 - 5298 =$ <u>12,200 s</u> (to 3 s.f.)	1 1 1 1 1 1 1 2 1
11 (b)	 Graph of $\sin^2 \omega t$ Double the frequency Values shown on t-axis	1 1 1
(c)	From $T = 2\pi \sqrt{(\frac{m}{k})}$, a smaller effective mass means the period is smaller for the subsequent motion.	1
12 (a)	$k = \frac{F}{x} = \frac{4.0}{3.2 \times 10^{-2}} = 125 \text{ N m}^{-1}$	1

(b)	(i) Change in GPE = mgh $= 4.0 \times (0.80 \times 10^{-2})$ $= 3.2 \times 10^{-2}$ J (ii) Change in EPE = $\frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$ $= \frac{1}{2}(125)(4.0 \times 10^{-2})^2 - \frac{1}{2}(125)(3.2 \times 10^{-2})^2$ $= 3.6 \times 10^{-2}$ J	1 1 1 1
(c)	Work done = Change in EPE - Change in GPE = PE gained $= 3.6 \times 10^{-2} - 3.2 \times 10^{-2}$ $= 4.0 \times 10^{-3}$ J	1
(d)	(i) Total energy of oscillation = PE gained $= 4.0 \times 10^{-3}$ J Note: Total energy of the oscillation is the amount of energy exchange between KE and PE and thus not the same as the total energy E_T of the system. The total energy of the system is dependent on the reference point of the GPE. Refer to the diagram below:  (ii) 1. KE lost = PE gained $\frac{1}{2} m v_{max}^2 - 0 = 4.0 \times 10^{-3}$ $v_{max} = \sqrt{\frac{2 \times 4.0 \times 10^{-3}}{4.0/9.81}}$ $= 0.14$ m s^{-1} 2. $v_{max} = x_o\omega$ $\omega = \frac{v_{max}}{x_o} = \frac{0.14}{0.80 \times 10^{-2}} = 17.5$ $2\pi f = 17.5$ $f = \frac{17.5}{2\pi} = 2.8$ Hz	1 1 1 1 1 1
(e)	$T = 2\pi \sqrt{(\frac{m}{k})} \quad \Rightarrow f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{(\frac{k}{m})}$ If the spring has some mass, the system will behave as though a greater mass is attached to the spring. Hence, the frequency will decrease.	1 1

<table>
<tr><td>13 (a)</td><td>(i) Loss in gravitational potential energy = (0.400)(9.81)(0.200)
= 0.785 J

(ii) $k = F / e = (0.400 \times 9.81)/0.200 = 19.62\ \text{N m}^{-1}$
$U = \frac{1}{2} ke^2 = \frac{1}{2} (19.62 \times 0.200)^2 = 0.392$ J</td><td>1

1
1</td></tr>
<tr><td>(b)</td><td>The difference is <u>work done against the external force</u> needed to support the mass while lowering it gently. This force is the difference between the mass's weight and the tension in the spring.</td><td>1

1</td></tr>
<tr><td>(c)</td><td>(i) At the lowest point, the total extension of the spring is 0.400 m.
Tension T = k e = 19.62 x 0.400 = 7.848 N
Weight W = m g = 0.400 x 9.81 = 3.924 N
F = T – mg = 7.848 – 3.924 = 3.92 N

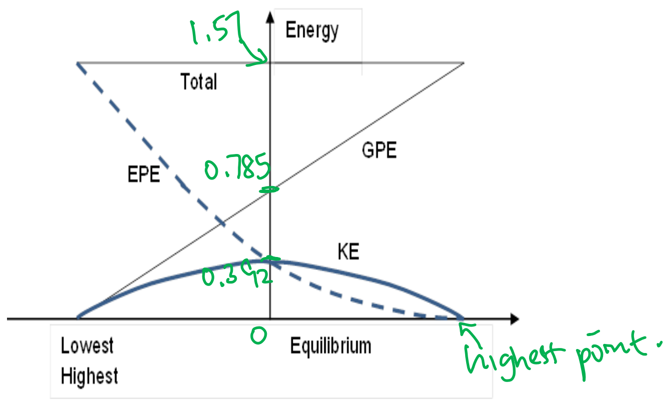
(ii) $\omega = \sqrt{k/m} = \sqrt{19.62/0.400} = 7.00\ \text{rad s}^{-1}$

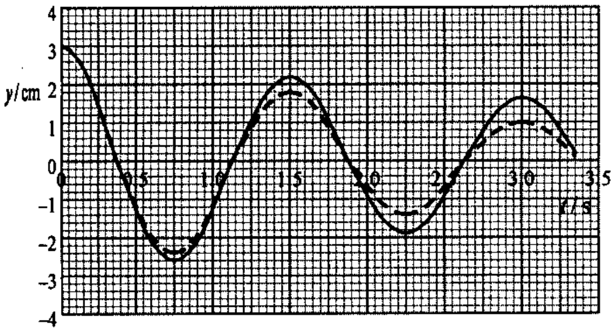
(iii) $v_{max} = \omega x_0 = 7.00 \times 0.200 = 1.40\ \text{m s}^{-1}$</td><td>1

1

1

1</td></tr>
<tr><td>(d)</td><td><table>
<tr><th></th><th>Gravitational potential energy / J</th><th>Elastic potential energy / J</th><th>Kinetic energy / J</th><th>Total energy / J</th></tr>
<tr><td>Lowest point</td><td>0</td><td>1.57</td><td>0</td><td>1.57</td></tr>
<tr><td>Equilibrium point</td><td>0.785</td><td>0.392</td><td>0.392</td><td>1.57</td></tr>
<tr><td>Highest point</td><td>1.57</td><td>0</td><td>0</td><td>1.57</td></tr>
</table></td><td>1m for any two correct

total, 5</td></tr>
<tr><td>(e)</td><td>
</td><td>1m for 1 correct or 2m for 2 correct or 3m for 4 correct</td></tr>
<tr><td>14 (a)</td><td>• The amplitude is decreasing with time.
• The curve does not start from zero displacement at t = 0 s.</td><td>1
1</td></tr>
<tr><td>(b)</td><td>(i) $\omega = 2\pi/T$
$= 2\pi/(1.5) = \underline{4.19\ \text{rad s}^{-1}}$
(ii) $a = -\omega^2 x$
$= -(4.19)^2 (2.2) = \underline{-38.6\ \text{cm s}^{-2}}$</td><td>1
1
1
1</td></tr>
</table>

(c)	 • Show that amplitude is decreasing. • Show that the period is the same or larger.	1 1
15 (a)	The amplitude of the block becomes **larger**. This is because, the rate of energy transfer from the waves to the block is greater.	1 1
(b)	The amplitude of the block becomes **smaller**. The waves now have larger wavelength and thus smaller frequency. Hence, the block does not resonate at the new frequency of the waves.	1 1
(c)	The amplitude of the block becomes **smaller**. The mass of the block is now larger and hence its natural frequency becomes smaller (using $f = (1/2\pi)\sqrt{(32/m)}$). Hence, the block no longer resonates at the frequency of the waves.	1 1
16 (a)	(i) Forced oscillation refers to oscillation of a system which is subjected to an input of energy from an external periodic driving force, thus allowing the system to sustain its motion. (ii) Resonance	1 1
(b)	(i) 1. $v_{max} = x_o\,\omega$ $= (1.6 \times 10^{-2})\, 2\pi\, (12)$ correct amplitude and frequency $= 1.21$ m s^{-1} 2. $a_{max} = x_o\,\omega^2$ $= (1.6 \times 10^{-2})\, [2\pi\, (12)]^2$ $= 91.0$ m s^{-2} (ii) Time interval = ¼ (Period) = ¼ (1/f) = ¼ (1/12) = 0.021 s	1 1 1 1 1 1 1
17	• If the sound produced has a frequency equal to the natural frequency of the loudspeaker, resonance will occur. • The amplitude of the speaker cone will be large at or near the resonant frequency. • This causes distortion of the sound produced.	1 1 1

18 (c)	(i) $\left(\frac{\rho Ag}{m}\right)$ is a constant in the equation, $a = -\left(\frac{\rho Ag}{m}\right)x$. Hence, the acceleration of the tube is proportional to displacement x. The negative sign shows that the acceleration is always directed towards the equilibrium position. (ii) $a = -\omega^2 x$ $\Rightarrow \omega^2 = -\left(\frac{\rho Ag}{m}\right)$ $2\pi f = \sqrt{\frac{\rho Ag}{m}}$ $f = \frac{1}{2\pi}\sqrt{\frac{\rho Ag}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.0x10^3(4.2x10^{-4})(9.91)}{32x10^{-3}}} = 1.8\text{ Hz}$	1 1 1 1 1 1
(d)	(i) 1. T = 0.50 s $f = \frac{1}{T} = 2.0\text{ Hz}$ 2. $f = \frac{1}{2\pi}\sqrt{\frac{\rho Ag}{m}}$ $\rho = \frac{(2\pi f)^2 m}{Ag} = \frac{(2\pi 2.0)^2(32x10^{-3})}{4.2x10^{-4}(9.81)}$ ρ = 1230 kg m^{-3} (ii) 1. • The viscous force of the liquid causes damping which dissipates energy of the oscillating tube. • There is no external periodic driving force to replenish the energy lost. 2. Total energy of the oscillating tube is given by $\frac{1}{2}m\omega^2 x_o^2$. $\therefore \Delta E = \frac{1}{2}m\omega^2 x_i^2 - \frac{1}{2}m\omega^2 x_f^2$ $= \frac{1}{2}m\omega^2(x_i^2 - x_f^2)$ $= \frac{1}{2}(32x10^{-3})(2\pi 2.0)^2[(1.5x10^{-2})^2 - (0.85x10^{-2})^2]$ $= 3.86 \times 10^{-4}$ J	1 1 1 1 1 1 1 1 1

- End of Tutorial Solutions -