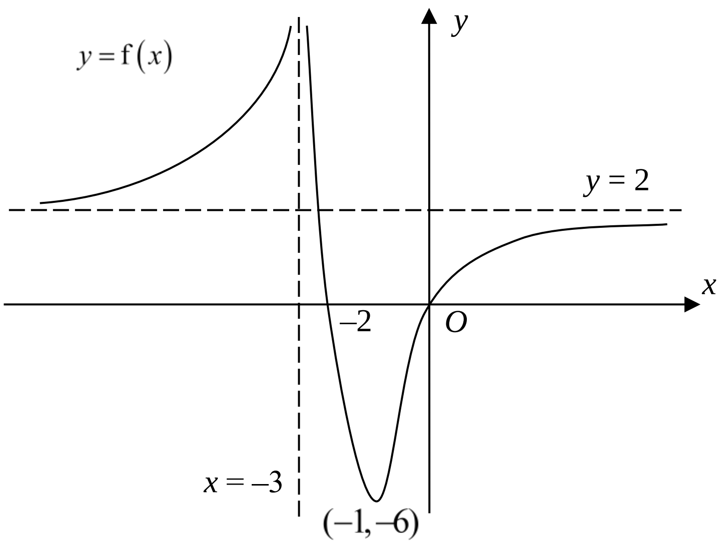
2024 Y5 H2 Math Holiday Assignment 1

1 A curve B has the equation $y^2 = 2y + 8xy - 17$. The tangents to B at points $(2, 1)$ and (a, b) are parallel to each other. Find the values of a and b. [5]

2 The diagram shows the graph of $y = \mathrm{f}(x)$. The curve passes through the origin and the point $(-2, 0)$ and has a turning point at $(-1, -6)$. The equations of the asymptotes are $x = -3$ and $y = 2$.



(a) Find the roots of the equation $\mathrm{f}(1-3x) = 0$. [2]

(b) Determine the number of distinct real roots of the equation $3\mathrm{f}(|x|) - 5 = 0$. Explain your answer clearly using a well-labelled diagram. [2]

(c) Sketch the graph of $y = \dfrac{1}{\mathrm{f}(x)}$, clearly indicating the coordinates of any points of intersection, turning points and equations of asymptotes. [3]

3 A curve D has parametric equations

$$x = t + t^2, \quad y = t^2 + t^3 \text{ where } -2 \le t \le 2 .$$

(a) Sketch D and explain why there is no curve when $x < -\frac{1}{4}$. [3]

(b) Find the gradient of the tangent to D at the point P with parameter $t = p$. [2]

(c) The point A on D has coordinates (2, 2). Show that $t = 1$ at A.
Given that the tangents to D at A and P respectively are perpendicular to each other, find the possible values of p. [4]

4 The function f is defined by $\mathrm{f} : x \mapsto \mathrm{e}^x - 7x, \quad x > 2.$ The function g is obtained by transforming the function f in the following 2 steps.

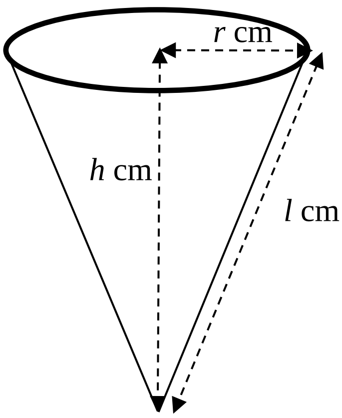
Step 1: Scale parallel to the x-axis by a factor of $\frac{1}{2}$.

Step 2: Reflect in the x-axis.

Find $\mathrm{g}(x)$ and its domain. [3]

5 [Volume and curved surface area of a cone is $\frac{1}{3}\pi r^2 h$ and $\pi r l$ respectively]

A manufacturer makes paper cone cups for water dispensers. He wants the cup to hold a volume of 48π cm^3 of water when full. The cone has a radius r cm, height of h cm and slant height of l cm as shown in the diagram. Assume the thickness of paper used is negligible.



The manufacturer wants to minimise the amount of paper used to make the cup. Using differentiation, find the value of r such that curved surface area of the cone is minimised. You do not need to prove that this value of r minimises the curved surface area. [5]