

Anglo - Chinese School

(Independent)



FINAL EXAMINATION 2019

YEAR THREE EXPRESS

ADDITIONAL MATHEMATICS

PAPER 1

Friday

4 October 2019

1 hour 30 minutes

Candidates answer on the Question Paper.

No additional materials are required.

READ THESE INSTRUCTIONS FIRST

Write your index number on boxes above. Write in dark blue or black pen. You may use a soft pencil for any diagrams or graphs. Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all questions.

Write your answers in the spaces provided under each question. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of a scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question. The total number of marks for this paper is 60.





Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and
$$\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)...(n-r+1)}{r!}$$

2. TRIGONOMETRY

Identities

$$\sin^{2} A + \cos^{2} A = 1$$
$$\sec^{2} A = 1 + \tan^{2} A$$
$$\cos ec^{2} A = 1 + \cot^{2} A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^{2} = b^{2} + c^{2} - 2bc \cos A$$
$$\Delta = \frac{1}{2}ab \sin C$$

Answer all the questions.

1 Find the range of values of x for which $3x^2 - 4x + 6 \le 7x$. [2]

2 Given that the line y = 2x + k does not meet the curve $y = \frac{2}{1-x}$, find the least integer value of k. [4]

[Turn Over

3 On the same axes, sketch the graphs of $y = 3\cos 2\theta$ and $y = \sin \theta$ for $0 \le \theta \le 2\pi$, showing clearly the point(s) of intersection. [3]

4 Simplify

(i)
$$\frac{2 - \log_k 1 + \log_m \frac{1}{\sqrt{m}}}{\log_m m^2 + 3}$$
 [2]

(ii)
$$\frac{\log_m 4 \times \log_2 m^2}{\log_{25} \sqrt{m}} \times \log_5 m.$$

[3]

[Turn Over

5 (a) When the graph of xy is plotted against y, the resulting straight line has a gradient

of -2 and an intercept of 0.5 on the *xy*-axis. Find the gradient and vertical intercept of the graph if $\frac{1}{xy}$ is plotted against $\frac{1}{x}$. [4]

5 (b) The variables x and y are related in such a way that when $\ln y$ is plotted against x^2 ,

a straight line is obtained which passes through (1,-2) and (4,7).

(i) Express y in terms of x.

[3]

(ii) State the value y would take as x approaches zero, leaving your answer in terms of e.

[1]

[Turn Over

(ii) On the same diagram, sketch the graphs of y = |x| and y = 3 - |x-1|. [2]

6 (iii) Solve the equation |x| + |x-1| = 3.

[3]

(iv) Hence, find the range of values of x such that $|x| + |x-1| \le 3$. [1]

- 7 The points P and Q have coordinates (3, 5) and (9, 11).
 - (i) Calculate the length of PQ. Express your answer in the form $a\sqrt{b}$. [2]

(ii) Find the equation of the circle with PQ as the diameter. [3]

[Question 7 continues next page]

(iv) Given that the tangent at Q meets the x-axis at the point R, find the area of triangle PQR. [2]

[Turn Over

[3]

8 Solve

(i)
$$\log_2 x + \log_2 (x+2) = 3$$
,

(ii)
$$\frac{x}{\log_4 2} + 6 = \frac{12}{\log_{16} 2}$$
,

[3]

[3]

8 (iii) $\log_3 4x - 2\log_9(2x-5) - \log_3(x+1) = 1$.

[Turn Over

[4]

9 (a) The roots of the quadratic equation $2x^2 + 4x + 5 = 0$ are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. Find the quadratic

equation with roots lpha and eta .

- (b) The equation $kx^2 2k^2x + 9 = 2k x$ has roots which are reciprocal of each other.
 - (i) Find the value of k.

[2]

[5]

9

(b)

•

(ii) Without solving the equation, show that if α is a root, it would satisfy $9\alpha^3 = 280\alpha - 51$.

ANSWER KEY





(iii) x = 2 or x = -1 (iv) $-1 \le x \le 2$ 7. (i) $6\sqrt{2}$ (ii) $(x-6)^2 + (y-8)^2 = 18$ (iii) y + x = 20 (iv) 66 units² 8. (i) x = 2 (ii) x = 21 (iii) x = 39. (a) $5x^2 + 4x + 2 = 0$ (b)(i) k = 3