1. [Maximum mark: 3]

Express $\dfrac{1}{x^2+2x-3}-\dfrac{x^2+1}{2x^4-2}$ as a single fraction in its simplest form.

$$\frac{1}{x^2+2x-3}-\frac{x^2+1}{2x^4-2}$$
$$=\frac{1}{x^2+2x-3}-\frac{x^2+1}{2\left(x^2+1\right)\left(x^2-1\right)}$$
$$=\frac{1}{(x+3)(x-1)}-\frac{1}{2(x+1)(x-1)}$$
$$=\frac{2x+2-(x+3)}{2(x+1)(x+3)(x-1)}$$
$$=\frac{x-1}{2(x+1)(x+3)(x-1)}$$
$$\frac{1}{2(x+1)(x+3)}$$

- Students should not try to combine the fractions without even factorising the denominators.
- x^2+1 should be cancelled out immediately to keep the working simple.
- Some students forgot about the 2 in the denominator when combining the two fractions.
- As the question asked for a fraction in its simplest form, $x-1$ must be cancelled in the final step.

2. [Maximum mark: 4]

Given that $\begin{pmatrix} a & 3a \\ 3 & 4 \end{pmatrix}-2\begin{pmatrix} b & 2b \\ 1 & 2 \end{pmatrix}=\begin{pmatrix} 5 & 13 \\ 1 & 0 \end{pmatrix}$, find the value of a and of b.

$a-2b=5$ ___(1)

$3a-4b=13$ ___(2)

$(2)-(1)\times 2: a=3, b=-1$

- Some students ended up with $3a-2b=13$.
- Careless mistakes were made when solving. These could be rectified by substituting the answers into the original equations for checking.

3. [Maximum mark: 5]

A right cone has base diameter 6 cm and height of 4 cm.

(a) Find, in terms of π, the total surface area of the cone. [3]

(b) A solid hemisphere has the same surface area as the cone. Find the radius of the hemisphere in the form $a\sqrt{2}$ cm, where a is a constant. [2]

(a) Slant height $= \sqrt{3^2+4^2} = 5$ cm

Total surface area of cone $= \pi(3^2) + \pi(3)(5) = 24\pi$

(b) $3\pi(r^2) = 24\pi$

$r = \sqrt{8} = 2\sqrt{2}$ cm

$a = 2$

- The diameter was given in the question, not the radius. Students need to divide by 2 first.
- As this is a solid hemisphere, the base area must be included in the calculations.
- Many students used the wrong formulas for this question, sometimes even mixing up with volume.

4. [Maximum mark: 5]

(a) Solve $27\left(\frac{1}{3}\right)^x = 81^{\frac{5}{4}}$. [2]

(b) Simplify $\log_4 a \times \log_a 64 - \log_{\sqrt{2}} 4$. [3]

(a) $27\left(\frac{1}{3}\right)^x = 81^{\frac{5}{4}}$

$3^3(3^{-x}) = 3^5$

$3 - x = 5$

$x = -2$

- Many misconceptions in the use of indices laws were observed such as: $3^3(3^{-1})^x = 3^{2x}$.
- Another common mistake was: $27\left(\frac{1}{3}\right)^x = 9$.

(b) $\log_4 a \times \log_a 64 - \log_{\sqrt{2}} 4$

$= \frac{\lg a}{\lg 4} \times \frac{\lg 64}{\lg a} - 4$

$= \frac{3\lg 4}{\lg 4} - 4$

$= -1$

- If students want to use shortcuts like $\log_a b = \frac{1}{\log_b a}$ or $\log_a b = \log_{a^2} b^2$, they must be aware of potential pitfalls. For example, $3\log_a 4 \neq \frac{1}{3\log_4 a}$.
- Change of base law must be clearly shown in working. It cannot be taken for granted that $\log_a 4 \times \log_4 a = 1$.

5. [Maximum mark: 6]

(a) Solve the simultaneous equations $xy = 16$ and $y = x^3$. [3]

(b) On the same axes, sketch the graphs of $xy = 16$ and $y = x^3$, labelling the axes-intercept(s) and point(s) of intersection clearly. [3]

(a)

$y = \frac{16}{x}$

Substitute into $y = x^3$.

$\frac{16}{x} = x^3$

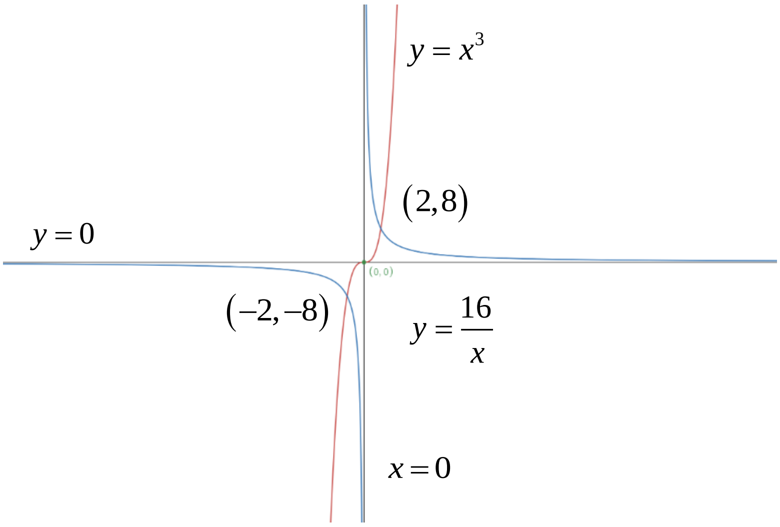
$x^4 = 16$

$x = 2, y = 8$

$x = -2, y = -8$

- Remember to solve for *y* after obtaining the *x* values.
- Many students forgot about $\pm$. This should be rectified once students sketch the graph in the next part and see 2 intersections.

(b)



- Students should take note to indicate the origin, which is both the x and y intercept of $y = x^3$.
- As there are multiple graphs on the same axes, the equation of each graph should be clearly labelled.
- $y = \frac{16}{x}$ must be clearly shown to tend to but not touch the asymptotes. Students should also label the equations of the asymptotes.

6. [Maximum mark: 7]

(a) Simplify $\frac{2}{\sqrt{3}}-\frac{\sqrt{108}}{4}+\frac{5}{1+\sqrt{3}}$, leaving your answer in the form $a+b\sqrt{3}$, where a and b are constants. [4]

(a)

$$\frac{2}{\sqrt{3}}-\frac{\sqrt{108}}{4}+\frac{5}{1+\sqrt{3}}$$
$$=\frac{2\sqrt{3}}{3}-\frac{6\sqrt{3}}{4}+\frac{5\left(1-\sqrt{3}\right)}{1+\sqrt{3}\left(1-\sqrt{3}\right)}$$
$$=\frac{2\sqrt{3}}{3}-\frac{6\sqrt{3}}{4}+\frac{5-5\sqrt{3}}{-2}$$
$$=\frac{8\sqrt{3}-18\sqrt{3}-30+30\sqrt{3}}{12}$$
$$=\frac{-30+20\sqrt{3}}{12}$$
$$=-\frac{5}{2}+\frac{5}{3}\sqrt{3}$$
$$a=-\frac{5}{2},b=\frac{5}{3}$$

- Rationalising the denominators first will make it easier to combine the fractions.
- Final answers should be fully simplified.

(b) Given that $t=\frac{1}{\sqrt{3}}$, express $\frac{t-1}{2t-1}$ the form $p+q\sqrt{3}$, where p and q are constants. [3]

(b)

$$\frac{\frac{1}{\sqrt{3}}-1}{\frac{2}{\sqrt{3}}-1}$$
$$=\frac{\frac{1-\sqrt{3}}{\sqrt{3}}}{\frac{2-\sqrt{3}}{\sqrt{3}}}$$
$$=\frac{1-\sqrt{3}}{2-\sqrt{3}}$$
$$=\frac{1-\sqrt{3}}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$$
$$=\frac{2-\sqrt{3}-3}{4-3}$$
$$=-1-\sqrt{3}$$
$$p=-1,q=-1$$

- A common denominator should be obtained within both the numerator and denominator.
- Students should be extra careful when negative signs are involved in the working.

7. [Maximum mark: 8]

(a) Given that a and b are integers such that $-2 \le a < 5$ and $-3 \le b \le 7$, find the

(i) greatest possible value of $a^2 - \dfrac{b^2}{2}$, [2]

(ii) smallest possible value of $\dfrac{ab}{(a-1)^2}$. [2]

(b) Solve the inequalities $4x - 17 \le 5x - 12 < 3x + 50$. [4]

(a)(i) $4^2 - 0 = 16$

- Take note of the inequality. a cannot be 5.
- 0 is considered an integer.

(b)(ii) $\dfrac{(2)(-3)}{(2-1)^2} = -6$

- a must be the same number in both the numerator and denominator.
- The trick is to make the denominator as small as possible, while keeping the result negative.

(c)

$4x - 17 \le 5x - 12$ and $5x - 12 < 3x + 50$

$x \ge -5$ and $2x < 62$

$-5 \le x < 31$

- Some students were confused when dealing with $-x \le 5$, ending with $x \ge 5$.
- Avoid writing $31 > x \ge 5$. By convention, the smaller number appears on the left of an inequality.

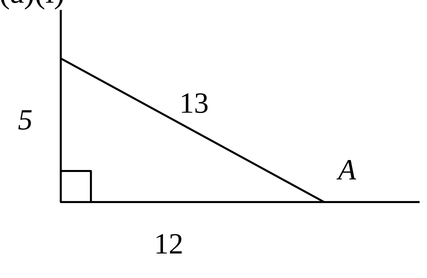
8. [Maximum mark: 10]

(a) If A is an obtuse angle and $\cos A = -\dfrac{12}{13}$, find the value of each of following:

(i) $\tan A$, [2]

(ii) $\sin A + \cos A$. [2]

(a)(i)



$\tan A = -\tan(180° - A)$

$= -\dfrac{5}{12}$

- A simple diagram will make solving this question easier.
- As this is an obtuse angle, students should remember to include the negative sign for $\tan A$.

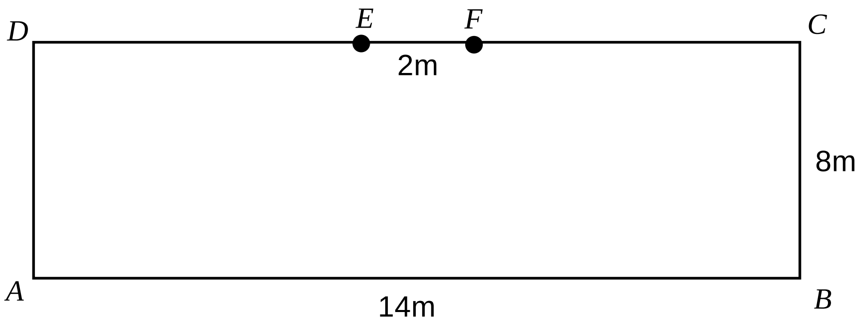
(a)(ii) $\sin A = \dfrac{5}{13}$

$\sin A + \cos A = \dfrac{5}{13} - \dfrac{12}{13}$

$= -\dfrac{7}{13}$

- Since $\sin(180° - \theta) = \sin(\theta)$, $\sin A$ is positive in this case.

(b) The diagram below shows an 8 metres by 14 metres rectangular assembly area where A, B, C and D are points on level ground. Two flagpoles stand at E and F, such that $DE = FC$ and the flagpoles stand 2 metres apart from each other. The height of each flagpole is 7 metres and the top of the flagpole at F is denoted as G.



Find

(i) the length BG, giving your answer in surd form, [3]

(ii) $\tan \angle GBF$, [1]

(iii) $\cos \angle DFB$. [2]

(b)(i)

$$CF = \frac{14-2}{2} = 6$$

$$BF = \sqrt{6^2 + 8^2}$$
$$= 10$$

$$BG = \sqrt{10^2 + 7^2}$$
$$= \sqrt{149}$$

- Students are recommended to add on to the given diagram to aid in visualization. Additional diagrams are also useful once the main diagram gets too cluttered.

(b)(ii) $\tan \angle GBF = \dfrac{7}{10}$

- Students whose answers contain a surd in the fraction, have mistakenly use the wrong side as the opposite/adjacent.

(b)(iii)

$$\cos \angle DFB = -\cos(180° - \angle CFB)$$
$$= -\frac{6}{10}$$
$$= -\frac{3}{5}$$

- Again, this method is more clearly visible if students use the provided diagram wisely.
- It is acceptable, but much more tedious to use cosine rule to solve this question.

9. [Maximum mark: 15]

A company uses the function $P=-n^2+10n-21$ to model its profits, P, in **thousands** of dollars, where n **hundred** units of Product A are produced and sold.

(a) Explain the meaning of $"-21"$ in the context. [1]

Starting cost of $21000 / Loss of $21000 when no goods were produced yet

The following answers have been accepted due to leniency, but students should take note to be more specific with their phrasing next time:

- Mentioning 'profit' instead of 'cost/loss'
- Not making reference to $n=0$ e.g. starting/initial/no goods produced yet
- Unnecessary assumptions e.g. wages/cost of factory

However, this answer is not accepted:

- Not making reference to the context e.g. *y*-intercept/constant value

(b) The company makes a profit by producing 400 units of Product A. A director suggests doubling the number of units produced. Explain if this is advisable, justifying your answer clearly. [2]

The company makes a profit of $3000 when producing 400 units, but a loss of $5000 when producing 800 units. Hence, this not advisable.

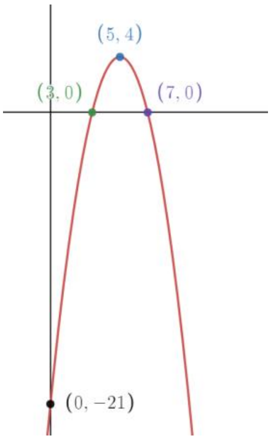
- Clear working must be shown for substituting in 8 to find the profit.
- Sub in 8, not 800.
- An appropriate conclusion must be clearly written.

(c) State the maximum profit and the corresponding number of units to produce. [2]

Maximum profit of $4000 when 500 units produced

- Answers of $4 and/or 5 units are unacceptable.
- Students also tend to mix up the *x* and *y* coordinates, ending up with $5000 and 400 units.

(d) Sketch the graph of $P = -n^2 + 10n - 21$, clearly labelling the coordinates of the axes-intercepts and turning point. [3]



- The vertical and horizontal axes should be denoted by P and n respectively.
- While not penalised this time, students should follow instructions and label all the intercepts in coordinate form.
- Although this is a sketch and not drawn to scale, students should pay attention to rough proportion. The y value of -21 should not be closer to the x-axis than 4.

To meet demands, the company has to increase its production capacity to produce Product B. The costs, in **thousands** of dollars, C, of producing n **hundred** units of Product B is given by $C = kn^2 - 10n + 30$.

(e) If $k = 1.5$, explain why it is not possible to have a production cost of $10,000. [3]

$10 = 1.5n^2 - 10n + 30$

$1.5n^2 - 10n + 20 = 0$

$b^2 - 4ac = 100 - 120 = -20$

Since discriminant is negative, there are no real roots so it is not possible.

- Take not that discriminant is $b^2 - 4ac$, not $\sqrt{b^2 - 4ac}$.
- Remember to substitute in 10 instead of 10 000.
- A conclusion should be clearly written, regardless of the method chosen.
- Some students chose to find the maximum point, which is acceptable but so tedious that it often ends in careless mistakes.

(f) Find the range of values of k, such that the production cost is always more than $20,000. [4]

$kn^2 - 10n + 30 > 20$

$kn^2 - 10n + 10 > 0$

$k > 0$ and $b^2 - 4ac = 100 - 40k < 0$

$k > \frac{5}{2}$

- Some students also chose to find the maximum point, which is acceptable but so tedious that it often ends in careless mistakes.
- Remember to substitute in 20 instead of 20 000.
- Most students forgot to include $k > 0$, which luckily for them did not affect the answer.

10. [Maximum mark: 5]

The roots of the quadratic equation $9x^2 = kx - 1$ are $\frac{1}{\alpha^2}$ and $\frac{1}{\beta^2}$ where $\alpha, \beta > 0$. Show that

$\alpha + \beta = \sqrt{k+6}$.

$$9x^2 - kx + 1 = 0$$
$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$$
$$\frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} = \frac{k}{9}$$
$$\frac{1}{\alpha^2\beta^2} = \frac{1}{9}$$

$$\alpha\beta = 3$$
$$\alpha^2 + \beta^2 = k$$
$$(\alpha+\beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$= k + 6$$
$$\alpha + \beta = \sqrt{k+6}$$

- α and β should not be treated interchangeably with a and b.
- Some signs are wrong when comparing sum and product of roots, usually resulting in $-\frac{k}{9}$.
- Since the final answer is already provided in a show question, each step of the working must be clearly shown so that the student is not perceiving to be 'working backwards'.
- The shown result must not be assumed in the working e.g.

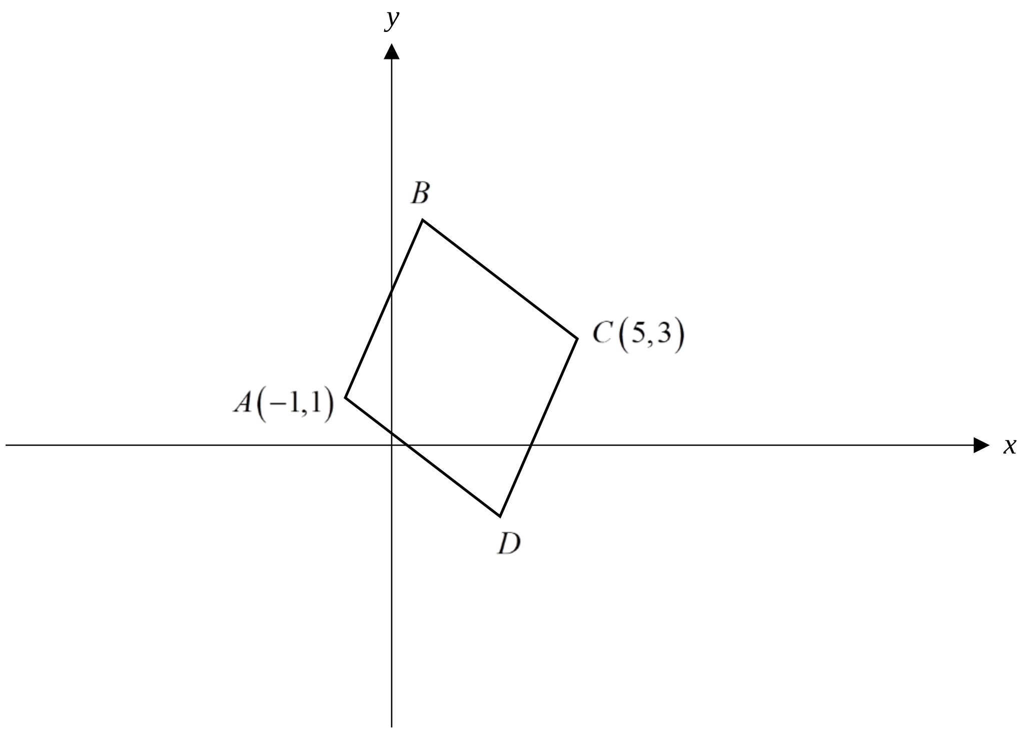
  $\alpha + \beta = \sqrt{k+6}$

  $(\alpha + \beta)^2 = k + 6$

11. [Maximum mark: 12]

Solutions to this question by accurate drawing will not be accepted.

In the diagram below, $A(-1,1)$ and $C(5,3)$ are two vertices of a parallelogram $ABCD$. AB has a gradient of 2 and the perpendicular bisector of AB passes through the center of $ABCD$.



Find

(a) the equation of AB, [2]

$y-1=2(x+1)$
$y=2x+3$

(b) the equation of the perpendicular bisector of AB, [2]

Midpoint of $AC=\left(\dfrac{-1+5}{2},\dfrac{1+3}{2}\right)=(2,2)$

Perpendicular gradient $-\dfrac{1}{2}$

$y-2=-\dfrac{1}{2}(x-2)$

$y=-\dfrac{1}{2}x+3$

- Some students remembered midpoint formula wrongly as $\left(\dfrac{x_1-y_1}{2},\dfrac{x_2-y_2}{2}\right)$.
- The perpendicular gradient should be $-\dfrac{1}{2}$ not $\dfrac{1}{2}$.

(c) the coordinates of B, [3]

$-\dfrac{1}{2}x+3=2x+3$

$x=0$

$y=3$

$\left(\dfrac{-1+b_x}{2},\dfrac{1+b_y}{2}\right)=(0,3)$

$B(1,5)$

- Since the only given information is that *ABCD* is a parallelogram, students should not make assumptions like $AB\perp BC$.
- Another viable method was to find equation of *BD* since it passes through the center and $AC\perp BD$.

- Students who wrote down *B* and *D* without providing full working will only be awarded partial marks.

(d) the coordinates of D, [2]

$\left(\dfrac{1+d_x}{2},\dfrac{5+d_y}{2}\right)=(2,2)$

$D(3,-1)$

- Any other methods such as gradient/intersection are acceptable but midpoint is the fastest method.
- Students should learn to present their working for reverse midpoint formula properly if they want method marks.

(e) the area of *ABCD*. [3]

$$\frac{1}{2}\begin{vmatrix} -1 & 1 & 5 & 3 & -1 \\ 1 & 5 & 3 & -1 & 1 \end{vmatrix}$$

$=\dfrac{1}{2}|-5+3-5+3-1-25-9-1|$

$=20\text{ units}^2$

- Some students forgot to repeat the first point.