1

(a) $\left(n+\frac{1}{2}\right)^4-\left(n-\frac{1}{2}\right)^4$

$$=\left[n^4+\binom{4}{1}n^3\left(\frac{1}{2}\right)+\binom{4}{2}n^2\left(\frac{1}{2}\right)^2+\binom{4}{3}n^1\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4\right]-$$

$$\left[n^4-\binom{4}{1}n^3\left(\frac{1}{2}\right)+\binom{4}{2}n^2\left(\frac{1}{2}\right)^2-\binom{4}{3}n^1\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4\right]$$

$$=2\binom{4}{1}n^3\left(\frac{1}{2}\right)+2\binom{4}{3}n^1\left(\frac{1}{2}\right)^3$$

$$=4n^3+n$$

(b) $\sum_{n=1}^{N}\left[\left(n+\frac{1}{2}\right)^4-\left(n-\frac{1}{2}\right)^4\right]=\sum_{n=1}^{N}(4n^3+n)$

$$\left(\frac{3}{2}\right)^4-\left(\frac{1}{2}\right)^4+$$

$$\left(\frac{5}{2}\right)^4-\left(\frac{3}{2}\right)^4+ \qquad =4\sum_{n=1}^{N}n^3+\frac{N}{2}(N+1)$$

$$\left(\frac{7}{2}\right)^4-\left(\frac{5}{2}\right)^4+$$

$$\vdots$$

$$\left(N+\frac{1}{2}\right)^4-\left(N-\frac{1}{2}\right)^4$$

$$\left(N+\frac{1}{2}\right)^4-\left(\frac{1}{2}\right)^4=4\sum_{n=1}^{N}n^3+\frac{N}{2}(N+1)$$

$$4\sum_{n=1}^{N}n^3=\left(N+\frac{1}{2}\right)^4-\frac{N}{2}(N+1)-\left(\frac{1}{2}\right)^4$$

$$4\sum_{n=1}^{N}n^3=N^4+2N^3+\frac{3}{2}N^2+\frac{1}{2}N+\frac{1}{16}-\frac{1}{2}N^2-\frac{1}{2}N-\frac{1}{16}$$

$$4\sum_{n=1}^{N}n^3=N^4+2N^3+N^2$$

$$\sum_{n=1}^{N}n^3=\frac{1}{4}(N^4+2N^3+N^2)$$

$$\therefore S_N=\frac{1}{4}(N^4+2N^3+N^2)$$

(c) $$N^{-4}S_N = \frac{1}{4}\left(1+\frac{2}{N}+\frac{1}{N^2}\right)$$

As $N \to \infty$, $\frac{2}{N}, \frac{1}{N^2} \to 0$

Thus $N^{-4}S_N \to \frac{1}{4}$. Limit = $\frac{1}{4}$

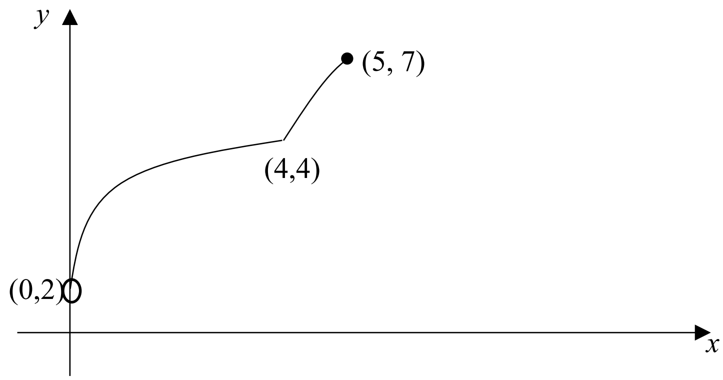
2

<table>
<tr><td>(a)</td><td>

$2x - x^2 - 28 = 8 - (x-6)^2$

Therefore the largest value of $a = 6$

</td></tr>
<tr><td>(b)</td><td>



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<tr><td>(c)</td><td>

For $0 < x < 4$

Let $y = 2 + \sqrt{x}$
$x = (y-2)^2$

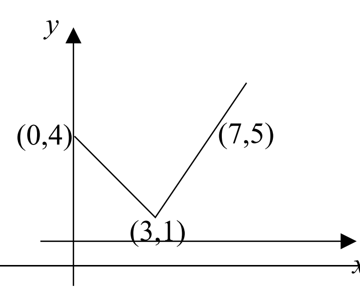
For $4 \le x \le 5$
$y = 8 - (x-6)^2$
$(x-6)^2 = 8 - y$
$x = 6 \pm \sqrt{8-y}$
Since $x \le 5$
$x = 6 - \sqrt{8-y}$, $4 \le y \le 7$

$\therefore \mathrm{f}^{-1} : x \mapsto \begin{cases} (x-2)^2, & 2 < x < 4 \\ 6 - \sqrt{8-x}, & 4 \le x \le 7 \end{cases}$

</td></tr>
<tr><td>(d)</td><td>

Since $\mathrm{R_f} = (2,7] \subset \mathrm{D_g} = \mathbb{R}^+$, $\therefore$ gf exists.

$(0,5] \xrightarrow{\mathrm{f}} (2,7] \xrightarrow{\mathrm{g}} [1,5]$
$\mathrm{R_{gf}} = [1,5]$



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3

(a)	$y=\frac{2\cos 2x}{1+\sin 2x}$ $\approx 2\left(1-\frac{(2x)^2}{2!}\right)(1+2x)^{-1}$ $\approx 2\left(1-2x^2\right)\left(1+(-1)2x+\frac{(-1)(-2)}{2!}(2x)^2\right)$ $=2\left(1-2x^2\right)\left(1-2x+4x^2\right)$ $\approx 2\left(1-2x+4x^2-2x^2\right)=2-4x+4x^2$
(bi)	$y=\ln\left(1+\sin 2x\right)$ $\mathrm{e}^y=1+\sin 2x$ Differentiate wrt x $\mathrm{e}^y\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos 2x$ ---(1) Differentiate wrt x $\mathrm{e}^y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-4\sin 2x$ $\mathrm{e}^y\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)=-4\sin 2x$ ---(2) $h=2,\ k=-4$

	Method 2 (not recommended, most students' working) $y=\ln(1+\sin 2x)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos 2x}{1+\sin 2x}$ Differentiate wrt x $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{(1+\sin 2x)(-4\sin 2x)-(2\cos 2x)(2\cos 2x)}{(1+\sin 2x)^2}$ $=\frac{-4\sin 2x}{1+\sin 2x}-\left(\frac{2\cos 2x}{1+\sin 2x}\right)^2$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-4\sin 2x}{\mathrm{e}^y}-\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ (since $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2\cos 2x}{1+\sin 2x}$ and $\mathrm{e}^y=1+\sin 2x$) $\mathrm{e}^y\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\mathrm{e}^y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=-4\sin 2x$ $\mathrm{e}^y\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)=-4\sin 2x\ ---(2)$ $h=2, k=-4$
(ii)	Differentiate wrt x $\mathrm{e}^y\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)+\mathrm{e}^y\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right)=-8\cos 2x\ ---(3)$ When $x=0,\ y=0$ From (1) : $\mathrm{e}^0\frac{\mathrm{d}y}{\mathrm{d}x}=2\cos 0\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=2$ From (2) : $\mathrm{e}^0\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2)^2\right)=0\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4$ From (3) : $\mathrm{e}^0\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2(2)(-4)\right)+\mathrm{e}^0(2)\left(-4+(2)^2\right)=-8\Rightarrow\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=8$ $y=2x+\frac{(-4)}{2}x^2+\frac{(8)}{3!}x^3+\ldots=2x-2x^2+\frac{4}{3}x^3+\ldots$

	Method 2 (not recommend) $\ln(1+\sin 2x)=\sin 2x-\frac{(\sin 2x)^2}{2}+\frac{(\sin 2x)^3}{3}+\ldots$ $=2x-\frac{(2x)^3}{3!}-\frac{\left(2x-\frac{(2x)^3}{3!}+\ldots\right)^2}{2}+\frac{\left(2x-\frac{(2x)^3}{3!}+\ldots\right)^3}{3}+\ldots$ $=2x-\frac{(2x)^3}{3!}-\frac{(2x)^2}{2}+\frac{(2x)^3}{3}+\ldots$ $=2x-2x^2+\frac{4}{3}x^3+\ldots$
(c)	$\ln(1+\sin 2x)\approx 2x-2x^2+\frac{4}{3}x^3+\ldots$ differentiating both sides, $\frac{\mathrm{d}}{\mathrm{d}x}\ln(1+\sin 2x)=\frac{\mathrm{d}}{\mathrm{d}x}\left(2x-2x^2+\frac{4}{3}x^3+\ldots\right)$ $\frac{2\cos 2x}{1+\sin 2x}=2-4x+4x^2$

4

(ai)	$\lvert\mathbf{a}\times\mathbf{b}\rvert$ denotes the perpendicular distance of the **point *B*** to the line OA. Or The area of parallelogram with **adjacent** sides OA and OB Or Twice the area of triangle OAB
(ii)	Since **c** is perpendicular to $\mathbf{a}\times\mathbf{b}$, **c** which denotes $\overrightarrow{OC}$ (1) lies on the plane **containing *O*** and (2) **parallel to a** and **b**. Hence **c** can be written as $\mathbf{c}=\mu\mathbf{a}+\lambda\mathbf{b}$. Or Since **c** is perpendicular to $\mathbf{a}\times\mathbf{b}$, it denotes **c** is (1) **parallel to a** and **b** and (2) since **a**, **b** and **c** are position vector containing same point O, (3) hence **c** will lie on the same plane as **a** and **b** **(OR point** O, A, B and C lies on the same plane**)**. Therefore **c** can be written as $\mathbf{c}=\mu\mathbf{a}+\lambda\mathbf{b}$.
(iii)	Since angle $AOC=\frac{\pi}{2}$, we have $\mathbf{c}\cdot\mathbf{a}=0$ $\therefore(\mu\mathbf{a}+\lambda\mathbf{b})\cdot\mathbf{a}=0$ $\Rightarrow\quad \mu\mathbf{a}\cdot\mathbf{a}+\lambda\mathbf{b}\cdot\mathbf{a}=0$ $\Rightarrow\quad \mu\lvert\mathbf{a}\rvert^2+\lambda\lvert\mathbf{b}\rvert\lvert\mathbf{a}\rvert\cos\frac{\pi}{3}=0$ $\Rightarrow\quad \mu(1)^2+\lambda(2)(1)\left(\frac{1}{2}\right)=0$ $\Rightarrow\quad \mu=-\lambda$ Hence $\Rightarrow\quad \mathbf{c}=-\lambda\mathbf{a}+\lambda\mathbf{b}$ $\lvert\mathbf{c}\rvert=\sqrt{3}\quad\Rightarrow\quad\lvert-\lambda\mathbf{a}+\lambda\mathbf{b}\rvert^2=3$ $\Rightarrow\quad \lambda^2\lvert-\mathbf{a}+\mathbf{b}\rvert^2=3$ $\Rightarrow\quad \lambda^2(-\mathbf{a}+\mathbf{b})\cdot(-\mathbf{a}+\mathbf{b})=3$ $\Rightarrow\quad \lambda^2(\mathbf{a}\cdot\mathbf{a}-2\mathbf{a}\cdot\mathbf{b}+\mathbf{b}\cdot\mathbf{b})=3$

	$\Rightarrow \quad \lambda^2\left(\lvert\mathbf{a}\rvert^2 - 2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert\cos\frac{\pi}{3} + \lvert\mathbf{b}\rvert^2\right) = 3 \quad ---(*)$ $\Rightarrow \quad \lambda^2\left(1-2+2^2\right)=3$ $\Rightarrow \quad \lambda^2 = 1$ $\Rightarrow \quad \lambda = \pm 1$ **<u>Method 2 (to find λ)</u>** $\lvert\mathbf{c}\rvert = \sqrt{3}$ $\lvert -\lambda\mathbf{a} + \lambda\mathbf{b}\rvert = \sqrt{3}$ $\lvert\lambda\rvert\lvert\mathbf{b}-\mathbf{a}\rvert = \sqrt{3}$ $\lvert\lambda\rvert\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{3}$ <u>Using cosine rule,</u> $\left\lvert\overrightarrow{AB}\right\rvert^2 = \left\lvert\overrightarrow{OA}\right\rvert^2 + \left\lvert\overrightarrow{OB}\right\rvert^2 - 2\left\lvert\overrightarrow{OA}\right\rvert\left\lvert\overrightarrow{OB}\right\rvert\cos\angle AOB$ $= 1 + 2^2 - 2(2)\cos\frac{\pi}{3}$ $= 3$ $\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{3}$ $\therefore \lvert\lambda\rvert\left\lvert\overrightarrow{AB}\right\rvert = \sqrt{3}$ $\lvert\lambda\rvert\sqrt{3} = \sqrt{3}$ $\lvert\lambda\rvert = 1$ $\lambda = \pm 1$
(bi)	Let ***D*** **(0, 2, 0)** be **a point** on plane p_1 (OR let **POSITION VECTOR** of a **point on plane** p_1 be $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$) Distance of A from $p_1 = \dfrac{\left\lvert \overrightarrow{AD} \cdot \begin{pmatrix} \sqrt{11} \\ 2 \\ -1 \end{pmatrix} \right\rvert}{\sqrt{\left(\sqrt{11}\right)^2 + 2^2 + (-1)^2}}$

	$$=\frac{\left\|\begin{pmatrix}2\sqrt{11}-0\\4-(2)\\10-0\end{pmatrix}\cdot\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}\right\|}{\sqrt{\left(\sqrt{11}\right)^2+2^2+(-1)^2}}$$ $$=\frac{22+4-10}{4}=4$$
(ii)	Let $\mathbf{n}=\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}$. Therefore $\hat{\mathbf{n}}=\frac{\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}}{4}$. The equations of p_1 and p_2 are therefore $\mathbf{r}\cdot\hat{\mathbf{n}}=1$ and $\mathbf{r}\cdot\hat{\mathbf{n}}=3$ respectively. Therefore, distance between p_1 and p_2 = 3 -1 = 2 Since distance of A from p_1 is **<u>larger than</u>** the distance between the 2 planes, therefore the point A will not lie between the 2 planes. **<u>Method 2 (for finding distance between 2 planes)</u>** Let E (0, 6, 0) be a point on plane p_2 $\overrightarrow{DE}=\begin{pmatrix}0\\6\\0\end{pmatrix}-\begin{pmatrix}0\\2\\0\end{pmatrix}=\begin{pmatrix}0\\4\\0\end{pmatrix}$ Distance between 2 planes $=\frac{\left\|\overrightarrow{DE}\cdot\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}\right\|}{\sqrt{\left(\sqrt{11}\right)^2+2^2+(-1)^2}}==\frac{\left\|\begin{pmatrix}0\\4\\0\end{pmatrix}\cdot\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}\right\|}{\sqrt{\left(\sqrt{11}\right)^2+2^2+(-1)^2}}=\frac{8}{4}=2$
(iii)	For the line BC to be perpendicular to both p_1 and p_2, C will be the foot of perpendicular of B onto p_2. Since BC is parallel to normal of p_2,

	$\therefore \overrightarrow{BC} = k\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}$ $\Rightarrow \quad \overrightarrow{OC} = \begin{pmatrix}0\\-4\\-12\end{pmatrix} + k\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix}$ Since C is a point on p_2, we have $\begin{pmatrix}\sqrt{11}k\\2k-4\\-k-12\end{pmatrix}\cdot\begin{pmatrix}\sqrt{11}\\2\\-1\end{pmatrix} = 12$ $\Rightarrow \quad 11k + 4k - 8 + k + 12 = 12$ $\Rightarrow \quad k = \frac{1}{2}$ $\therefore \overrightarrow{OC} = \begin{pmatrix}\frac{\sqrt{11}}{2}\\-3\\-\frac{25}{2}\end{pmatrix}$

5

(a)	No. of ways choosing 3 male and 3 female artistes $= {}^7C_3 \times {}^5C_3 = 350$ No of pairing up the chosen = 3! No. of ways of arranging the order of the 3 pairs = 3! Total number of ways = 350 ×3!×3! = 12600
(b)	P(even no.) = $\frac{1}{45}(2+4+6+8) = \frac{20}{45}$ P(odd no.) = $\frac{1}{45}(1+3+5+7+9) = \frac{25}{45}$ P(Pair A loses a game) = P(even & black) + P(odd & black) $= \frac{20}{45}\left(\frac{3}{8}\right) + \frac{25}{45}\left(\frac{8}{12}\right) = \frac{29}{54}$
(c)	P(Pair B wins) $= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right) + \left(\frac{29}{54}\right)^3\left(\frac{25}{54}\right) + \left(\frac{29}{54}\right)^5\left(\frac{25}{54}\right) + \cdots$ $= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right)\left[1 + \left(\frac{29}{54}\right)^2 + \left(\frac{29}{54}\right)^4 + \cdots\right]$ $= \left(\frac{29}{54}\right)\left(\frac{25}{54}\right)\left[\frac{1}{1-\left(\frac{29}{54}\right)^2}\right]$ $= \frac{29}{83}$

6

(a)	Whether or not Charlie answers a question correctly is independent of other questions.
(b)	$C \sim \mathrm{B}(30, 0.25)$ $\mathrm{P}(C \leq 10) = 0.894$
(c)	Let M be the number of marks Charlie can achieve $M = 3C - (30 - C) = 4C - 30$ $\mathrm{E}(M) = \mathrm{E}(4C - 30)$ $= 4\mathrm{E}(C) - 30$ $= 4(30 \times 0.25) - 30$ $= 0$ (shown) $\mathrm{Var}(M) = \mathrm{Var}(4X - 30)$ $= 16\mathrm{Var}(X)$ $= 16(30 \times 0.25 \times 0.75)$ $= 90$
(d)	$\mathrm{P}(M \geq 32)$ $= \mathrm{P}(4C - 30 \geq 32)$ $= \mathrm{P}(C \geq 15.5)$ $= 1 - \mathrm{P}(C \leq 15)$ $= 0.000819$

7

(a)	Let X be the volume of drink dispensed in a cup Sample Mean, $\bar{x} = 196.375$
(b)	Let μ be the population mean volume of drink dispensed per cup. $H_0 : \mu = \beta$ $H_1 : \mu < \beta$
(c)	Test at 5% level of significance Under H_0, the test statistics is $Z = \dfrac{\overline{X} - \beta}{\frac{6.30}{\sqrt{8}}} \sim N(0,1)$ approximately. Critical Region: For H_0 to be rejected, z_{cal} lies inside the critical region $z_{cal} = \dfrac{196.375 - \beta}{\frac{6.30}{\sqrt{8}}} \leq -1.64485$ $\beta \geq 200.039$ $\beta \geq 200 (3s.f.)$
(d)	Since the volume of water after repair **is not known to be normally distributed**, the manager has to take a larger sample (with sample size of at least 30) so that the **sample mean volume of drink** can be approximated to a normal distribution, using Central Limit Theorem.

8

(a)	$\sum P(X = x) = 1$ $3k + 4k + 5k + 5k + 4k + 3k = 1$ $24k = 1 \Rightarrow k = \dfrac{1}{24}$ (shown)
(b)	By symmetry, $E(X) = 5.5$ $E(X^2) = \dfrac{3}{24}(3^2) + \dfrac{4}{24}(4^2) + \dfrac{5}{24}(5^2) + \dfrac{5}{24}(6^2) + \dfrac{4}{24}(7^2) + \dfrac{3}{24}(8^2)$ $= \dfrac{98}{3}$ $\text{Var}(X) = \dfrac{98}{3} - \left(\dfrac{11}{2}\right)^2 = \dfrac{29}{12}$

(c)	Since n = 50 is large, by Central Limit Theorem, $\overline{X} \sim N\left(5.5, \frac{29/12}{50}\right)$ approx ie $\overline{X} \sim N\left(5.5, \frac{29}{600}\right)$ approx $P(\overline{X} < 4.9) = 0.00317$ (3 s.f)
(d)	$\mathrm{E}\left(Y^2\right) = E\left(\left(mX-1\right)^2\right)$ $=\mathrm{E}\left(m^2X^2 - 2mX + 1\right)$ $= m^2E(X^2) - 2mE(X) + 1$ $= \frac{98}{3}m^2 - 2m(5.5) + 1$ $= \frac{98}{3}m^2 - 11m + 1$
(e)	$\mathrm{P}\left(Y \geq a\right) \geq 0.5$ $\mathrm{P}\left(3X - 1 \geq a\right) \geq 0.5$ $\mathrm{P}\left(X \geq \frac{a+1}{3}\right) \geq 0.5$ Since $\mathrm{P}\left(X \geq 5\right) = 0.70833 > 0.5$ $\mathrm{P}\left(X \geq 6\right) = 0.5$ $\therefore \frac{a+1}{3} \leq 6 \Rightarrow a \leq 17$ Largest $a = 17$

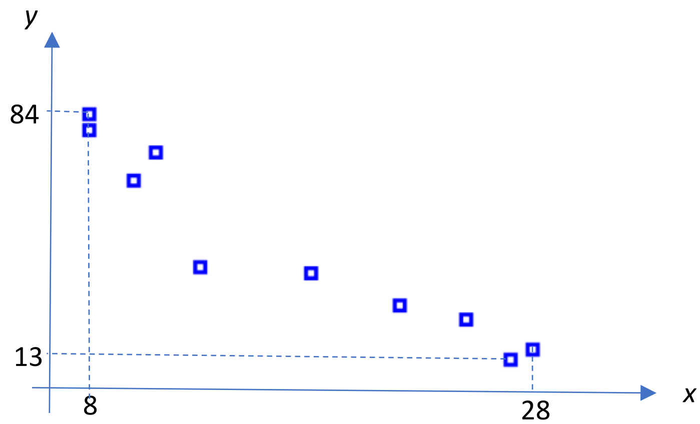
9

(a)	Let C and T be the mass of a randomly chosen chicken and turkey respectively. $C \sim N(2.2, 0.5^2)$ $T \sim N(10.5, 2.1^2)$ $P(1 < C < 3.5) = 0.987$ (3 s.f)

(b)	$P(C > 3.5 \mid 3.2 < C < 3.7)$ $= \dfrac{P(C > 3.5 \cap 3.2 < C < 3.7)}{P(3.2 < C < 3.7)}$ $= \dfrac{P(3.5 < C < 3.7)}{P(3.2 < C < 3.7)}$ $= 0.155 \text{ (3 s.f)}$
(c)	Required probability $= P(C > 3.5)P(C < 3.5)P(T < 14.5) \times 2 + P(C < 3.5)P(C < 3.5)P(T > 14.5) +$ $+\left[P(C < 3.5)\right]^2 P(T < 14.5)$ $= 0.99971 \text{ (5 s.f)}$
(d)	$T - 3C \sim N\left(10.5 - 3(2.2), 2.1^2 + 3^2\left(0.5^2\right)\right)$ $T - 3C \sim N(3.9, 6.66)$ $P\left(\lvert T - 3C \rvert > 0.3\right)$ $= P(T - 3C > 0.3) + P(T - 3C < -0.3)$ $= 1 - P(-0.3 < T - 3C < 0.3)$ $= 0.970$

10

(a)	Using GC, $\bar{x} = 17$ Sub into $y = -3.19x + 100.43$, we have $\bar{y} = 46.2$ $\dfrac{29 + 25 + 16 + 13 + 40 + 65 + 84 + k + 73 + 38}{10} = 46.2$ $k = 79$

(b)	
(c)	r = -0.952 (3 s.f.). As r is close to -1, it indicates a strong negative linear correlation between the yearly COE quota and the average COE price.
(d)	$r_{y,\ln x} = -0.971$ (3 s.f.). This value is closer to -1 than $r_{y,x}$. Hence it will give a better relationship between x and y based on the data.
(e)	Using G,C,. we have $y = 187.91 - 52.00 \ln x$ --- (1) Sub x = 20 into (1), we have $y = 32$ (correct to nearest whole number. Therefore, the estimated average price of COE will be $32000.
(f)	As x = 35 (i.e. yearly quota = 35000) is outside the data range of x, i.e. $[8, 28]$, it will not be appropriate to use either model to do the prediction.