RAFFLES INSTITUTION
MATHEMATICS DEPARTMENT
2024 YEAR 3 RP MATHEMATICS
TOPIC 4: QUADRATIC FUNCTIONS (MATHS 1 & MATHS 2)

SUPPLEMENTARY WORKSHEET

Name: Class: Sec 3 ( ) Date:

1 **2022/Y3RP/M2/T1/Q1**

(i) Find the coordinates of the turning point of the curve $y=\frac{1}{2}(5x+11)(1-x)$. [2]

(ii) Hence sketch the graph of $y=\frac{1}{2}(5x+11)(1-x)$ for $-3\le x\le 1$, showing all the critical points clearly. [3]

[Ans: (i) $\left(-\frac{3}{5},\ 6\frac{2}{5}\right)$*]*

2 **2022/Y3RP/M2/T1/Q2**

The equation of a curve is $y=ax^2+9x+3a-3$, where a is a constant and $a\neq 0$.

(a) Show that the line $y=(3+x)a$ intersects the curve at two distinct points for all real values of a ($a\neq 0$). [5]

(b) Find the range of values of a for which the curve lies completely below the line $y=-\frac{3}{4}$. [5]

[Ans: (b) $a<-2\frac{1}{4}$*]*

3 **2021/Y3RP/M2/T1/Q1**

Find the range of values of a such that $x^2+ax+3a-5$ is always positive. [3]

[Ans: $2<a<10$*]*

4 **2021/Y3RP/M2/T1/Q2**

Find the range of values of k for which the line $y+kx=k$ and the curve $y=kx^2+(2k+1)x+10k$ do not meet. [4]

[Ans: $k<-\frac{1}{9}$ or $k>\frac{1}{3}$*]*

5 **2021/Y3RP/M2/T1/Q3**

Show that $(m-2)x^2+(2m+3)x+(1-3m)=0$ has real and distinct roots for all real values of m such that $m\neq 2$. [4]

6 **2021/Y3RP/M2/T1/Q4**

(i) The equation of a curve is $y=(2x+1)(3-x)$. Find the maximum value of y and the corresponding value of x. [2]

[Ans: (i) Maximum value of $y=\frac{49}{8}$ when $x=\frac{5}{4}$*]*

7 **2020/Y3RP/M2/T1/Q1**

Find the range of values of p for which the curve $y=(3x-p)^2-(4px-1)$ has a positive minimum y value. [4]

[Ans: $-\frac{3}{4}<p<\frac{3}{4}$]

8 **2020/Y3RP/M2/T1/Q2**

Show that the line $y=4x+3(m-1)$ cuts the curve $y=x^2+(2m-1)x+2$ at two distinct points for all real values of m. [5]

9 **2019/Y3RP/M2/T2/Q2**

Find the range of values of m for which the equation $x^2-4x+5=mx+4$ has no real roots. Hence, determine the number of points of intersection between the curve $y=(x-2)^2$ and the line $y=2x+3$. [5]

[Ans: $-6<m<-2$; 2 points of intersection]

10 **2019/Y3RP/M2/T2/Q4**

Find the range of values of p for which $(p-6)x^2+p>-8x$ for all real values of x. [5]

[Ans: $p>8$]

11 **2018/Y3RP/M2/T1/Q1**

Find the range of values of k such that the equation $x^2-2x-4=4k(x+2)$ has no real roots. [4]

[Ans: $-2\frac{1}{2}<k<-\frac{1}{2}$]

12 **2018/Y3RP/M2/T1/Q2**

Show that $3k^2-kx+x^2+2$ is positive for all real values of x. [3]

13 **2018/Y3RP/M2/T1/Q3**

Find the range of values of the constant p for which the line $y=4x-1$ intersects the curve $py-8x=2px^2+1$ at two distinct points. [4]

[Ans: $p<1$ or $p>8$ and $p\neq 0$]

14 **2017/Y3RP/T2/Q1**

Find the range of values of k for which the equation $x^2+kx+x-k^2+1=0$ has real roots. [3]

[Ans: $k\leq -1$ or $k\geq\frac{3}{5}$]

15 **2017/Y3RP/T2/Q2**

(i) Given that ax^2-8x+c is always negative, what conditions must apply to the constants a and c? [3]

(ii) Give an example of values of a and c which satisfy the conditions found in part (i). [1]

[Ans: (i) $a<0$ and $ac>16$]

16 **2017/Y3RP/T2/Q3**

Show that the line $y=2kx+k$ intersects the curve $y=(k+2)x^2+kx-1$ at two distinct points for all real values of k, where $k\neq -2$. [5]

17 **2016/Y3RP/T2/Q1**

Solve the inequality $(x-1)(5-2x)<(x-1)^2$. [2]

[Ans: $x<1$ or $x>2$]

18 **2016/Y3RP/T2/Q3**

Show that the equation $3(x+a)x+(x-2)+a=0$ has real and distinct roots for all real values of a. [4]

19 **2016/Y3RP/T2/Q6 (modified)**

The solution of the inequality $2x^2+px+q<0$, where p and q are constants, is $-3<x<6$.

(i) Find the value of p and of q. [2]

(ii) Find the coordinates of the turning point of the curve $y=2x^2+px+q$. [2]

(iii) If the inequality $2x^2+px+q+c<0$, where c is an integer, has no solution, state the minimum value of c. [1]

[Ans: (i) $p=-6$, $q=-36$ (ii) $\left(\frac{3}{2},-40\frac{1}{2}\right)$ (iii) 41]

20 **2015/Y3RP/T2/Q1**

Solve the inequality $(3+x)(7-5x)<17$. [2]

[Ans: $x<-2$ or $x>\frac{2}{5}$]

21 **2015/Y3RP/T2/Q2**

Calculate the range of values of c for which $3x^2-9x+c>2\frac{1}{4}$ for all real values of x. [3]

[Ans: $c>9$]

22 **2015/Y3RP/T2/Q3**

Show that the line $y=2x-6$ cuts the curve $y=2x^2+2(k-6)x-k$ at two distinct points for all values of k. [5]

23 **2015/Y3RP/T2/Q4**

Find the turning point of the quadratic function $y=(3+2x)(5-x)$. Hence sketch the graph of $y=(3+2x)(5-x)$, showing the intercepts on the axes and the turning point clearly. [5]

[Ans: $\left(1\frac{3}{4},\ 21\frac{1}{8}\right)$]

24 **2014/Y3RP/T3/Q2**

Find the least value of integer b for which $-3x^2+bx-5$ is negative for all real values of x. [3]

[Ans: -7]

25 **2014/Y3RP/T3/Q3**

Find the range of values of k, where $k\neq 0$ for which the equation $kx^2+6(k-2)x+3(k+2)=0$ has real and distinct roots. [4]

[Ans: $k<1$ or $k>6$]

26 **2014/Y3RP/T3/Q4**

(a) By completing the square, express $4x^2-4x-2$ in the form $a(x-h)^2+k$, where a, h and k are constants. [2]

(b) Sketch the graph of $y=4x^2-4x-2$. [3]

[Ans: (a) $4\left(x-\frac{1}{2}\right)^2-3$]

27 **2013/Y3RP/T3/Q3**

Find the range of values of k for which the line $2y = x - 2k$ intersects the curve $y = 2x + \frac{3}{x}$ at two distinct points, leaving your answers in surd form. [4]

[Ans: $k < -3\sqrt{2}$ or $k > 3\sqrt{2}$]

28 **2011/Y3RP/T2/Q2**

Find the range of m for which the line $y = mx + 4$ intersects the curve $y = x^2 - 4x + 5$ at two distinct points. [3]

[Ans: $m < -6$ or $m > -2$]

29 **2011/Y3RP/T2/Q4**

Express $y = 3x^2 + 5x + 7$ in the form $y = a(x+b)^2 + c$ where a, b and c are constants. Sketch the curve, stating clearly on the sketch where the curve cuts the y-axis and where the minimum occurs. [5]

[Ans: $3\left(x + \frac{5}{6}\right)^2 + 4\frac{11}{12}$]

30 **2009/Y3RP/T3/Q2**

Given that q is a constant, show that $(2+qx)^2 + x(2x-q)$ is positive for all real values of x. [4]

31 **2009/Y3RP/T3/Q3**

(a) Sketch the graph of $4y = (x-2)^2 + 12$ for $-2 \le x \le 4$ in the space below, indicating clearly the x- and y-intercepts (if any), the turning point and the end points. [3]

(b) Find the range of values of k for which the line $y = kx + 1$ has two distinct points of intersection with the curve $4y = (x-1)^2 + 12$. [3]

[Ans: (b) $k < -2$ or $k > 1$]

32 **2008/Y3RP/T2/Q2**

(a) Find the range of values of k for which the equation $x^2 + 2kx + 5 = -2k - 2x$ has real roots. [4]

(b) Show that $(3+qx)^2 + 4x(2x-q)$ is positive for all real values of x. [4]

[Ans: (a) $k \le -2$ or $k \ge 2$]

33 **2008/Y3RP/T3/Q1**

The graph of a quadratic function $y = f(x)$ meets the x-axis at the points (–3, 0) and (1, 0). Given that the graph of the function also passes through the point (–1, –20), find the quadratic function. [3]

[Ans: $y = 5(x+3)(x-1)$ or $y = 5x^2 + 10x - 15$]

34 **2008/Y3RP/T3/Q4**

Express $y = -3x^2 + 9x - 2$ in the form $y = a(x-h)^2 + k$ where a, h and k are constants. Sketch the graph of $y = -3x^2 + 9x - 2$ for $0 \le x \le 4$. [5]

[Ans: $-3\left(x - \frac{3}{2}\right)^2 + 4\frac{3}{4}$]

35 **2006/Y3RP/T2/Q4**

Find the range of values of k for which the straight line $y = kx - 12$ does not intersect the curve $y = x^2 - kx - 4$, leaving your answer in surd form if necessary. [4]

[Ans: $-2\sqrt{2} < k < 2\sqrt{2}$]