Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2023

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS

PAPER 2

Thursday	**6th October 2023**	**1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (6 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. [Maximum mark: 6]

(a) Make y the subject of the formula $\dfrac{xy+2y}{x}=\dfrac{3}{5}x$. [2]

$$\frac{xy+2y}{x}=\frac{3x}{5}$$

$$5xy+10y=3x^2$$

$$y(5x+10)=3x^2$$

$$y=\frac{3x^2}{5x+10}$$

Do not leave answers in fraction over a fraction for example $y=\dfrac{\frac{3}{5}x^2}{(x+2)}$

(b) Simplify $\left(\dfrac{1}{a+1}-\dfrac{2a}{a^2-1}\right)\left(\dfrac{1}{a}-1\right)$, expressing your answer as a single fraction in its simplest form. [4]

$$\left[\frac{1}{a+1}-\frac{2a}{(a-1)(a+1)}\right]\left(\frac{1-a}{a}\right)$$

$$=\left[\frac{a-1-2a}{(a-1)(a+1)}\right]\left(\frac{1-a}{a}\right)$$

$$=\left[\frac{-1-a}{(a-1)(a+1)}\right]\left(\frac{-(a-1)}{a}\right)$$

$$=\frac{1+a}{a(a+1)}$$

$$=\frac{1}{a}$$

(c) Evaluate $\frac{\lg[(6.82 \div 1.55 \times 10^{-2})]}{32.8}$, leaving your answer correct to 2 significant figures. [1]

-0.041	Order of calculation should be from left to right: 6.82 divide by 1.55 then multiply by 10^{-2}. Many students divide 6.82 by (1.55×10^{-2}), treating it as standard form.

2. [Maximum mark: 8]

Julian paid \$168 for x tickets to a theme park.

(a) Write down an expression, in terms of x, for the cost of a ticket in dollars. [1]

$$\frac{168}{x}$$

(b) During the peak season, he could buy 7 less tickets with the same amount of money. Write down an expression, in terms of x, for the cost of one ticket during the peak session. [1]

$$\frac{168}{x-7}$$

(c) Given that the increase in the cost of one ticket during the peak season is \$2, form an equation in x and show that it reduces to $x^2 - 7x - 588 = 0$. [3]

$$\frac{168}{x-7} - \frac{168}{x} = 2$$

$$\frac{168x - 168(x-7)}{x(x-7)} = 2$$

$$168x - 168x + 1176 = 2x^2 - 14x$$

$$2x^2 - 14x - 1176 = 0$$

$$x^2 - 7x - 588 = 0$$

(d) How many tickets can he purchase with $200 during the peak season. [3]

$$x^2 - 7x - 588 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(-588)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{2401}}{2}$$

$$x = \frac{7 \pm 49}{2}$$

$$x = 28$$

Price of a ticket during peak season = $\frac{168}{28-7}$

Price of a ticket during peak season = 8

Number of tickets = $\frac{200}{8} = 25$

3. [Maximum mark: 4]

(a) Find the range of values of k for which the expression $x(x+6)-2px+34-6p$ is always positive for all real values of x. [4]

$x^2+6x-2px+34-6p$

$=x^2+(6-2p)x+34-6p$

$D<0$

$(6-2p)^2-4(1)(34-6p)<0$

$36-24p+4p^2-136+24p<0$

$4p^2-100<0$

$p^2-25<0$

$(p-5)(p+5)<0$

$-5<p<5$

Students are still unclear about what the Discriminant should be. When they read always positive, they assume that D > 0.

DO NOT TAKE SQUARE ROOT LEFT SIDE AND RIGHT SIDE WHEN SOLVING QUADRATIC INEQUALITY

(b) Solve the equation $2x-11\sqrt{x}+12=0$ [4]

$2x+12=11\sqrt{x}$

$(2x+12)^2=\left(11\sqrt{x}\right)^2$

$4x^2+48x+144=121x$

$4x^2-73x+144=0$

$$x=\frac{-(-73)\pm\sqrt{(-73)^2-4(4)(144)}}{2(4)}$$

$$x=\frac{73\pm 55}{8}$$

$x=16$ or $x=2.25$

Common mistake includes squaring every single term such as:

$$4x^2+144=121x$$

(c) Given that $x^2 + x = 7 + \sqrt{7}$, find the exact values of x, leaving your answers in simplest form. [4]

$$x^2 - 7 + x - \sqrt{7} = 0$$

$$x^2 - \left(\sqrt{7}\right)^2 + x - \sqrt{7} = 0$$

$$(x - \sqrt{7})(x + \sqrt{7}) + x - \sqrt{7} = 0$$

$$\left(x - \sqrt{7}\right)\left(x + \sqrt{7} + 1\right) = 0$$

$$x - \sqrt{7} = 0 \quad \text{or} \quad x + \sqrt{7} + 1 = 0$$

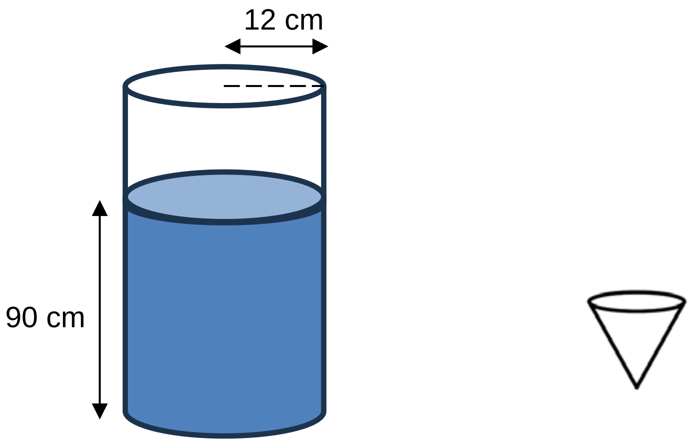
$$x = \sqrt{7} \qquad x = -\sqrt{7} - 1$$

Common mistake includes squaring every single term to get rid of $\sqrt{7}$ such as:

$$x^4 + x^2 = 49 + 7$$

4. [Maximum mark: 6]

During a workshop, water dispenser in the shape of a right circular cylinder of radius 12 cm, is provided for 250 participants.



The initial height of water in the container is 90 cm. If each participant drinks once from the dispenser using a conical cup of diameter 5 cm and water level at 7 cm, determine if the water the dispenser has is enough for all the participants. Show your working clearly.

Volume of water in container $= \pi(12)^2(90)$

$= 40715.0408$

Volume of each cup = $\frac{1}{3} \times \pi \times 5^2 \times 7$

Volume of each cup = 45.8208

Volume of 250 cups = 11455.208

Volume of 250 cups < Volume in container,

Water is sufficient for all participants.

5. [Maximum mark: 9]

(a) Find the equation of the straight line that passes through the points (2, 5) and (0, 1). [2]

Gradient $= \frac{5-1}{2-0}$

Gradient $= 2$

Equation of line:

$$y - 5 = 2(x - 2)$$

$$y - 5 = 2x - 4$$

$$y = 2x + 1$$

(b) A quadratic curve has a maximum point at (2, 5) and it passes through the point (0, 1). Find the equation of the quadratic curve. [2]

Equation of quadratic curve:

$$y = a(x - 2)^2 + 5$$

Let $x = 0$, $y = 1$

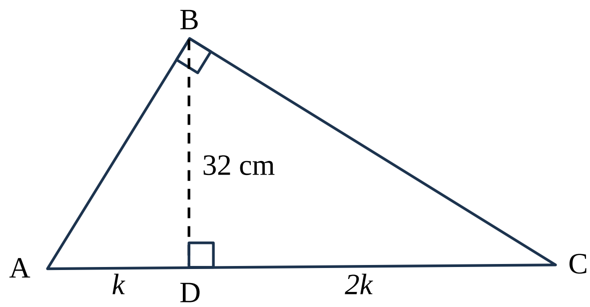
$$1 = a(0 - 2)^2 + 5$$

$$a = -1$$

Equation of quadratic curve:

$$y = -(x - 2)^2 + 5$$

(c) In the diagram, ABC is a right angled triangle and D is a point on AC such that BD is perpendicular to AC. Given that $AD = k$, $DC = 2k$ and $BD = 32$ cm, find the length of BC. [5]



Small right angle triangles:

$$AB^2 = k^2 + 32^2$$

$$BC^2 = (2k)^2 + 32^2$$

Big triangle:

$$AC^2 = AB^2 + BC^2$$

$$BC^2 = AC^2 - AB^2$$

$$(2k)^2 + 32^2 = (3k)^2 - k^2 - 32^2$$

$$4k^2 + 1024 = 9k^2 - k^2 - 1024$$

$$4k^2 = 2048$$

$$k^2 = 512$$

$$k = 22.63$$

6. [Maximum mark: 11]

(a) Solve the equation $7e^x + 10e^{-x} = 37$. [5]

$$7e^x + \frac{10}{e^x} = 37$$

Common mistakes:

$$7e^x + \frac{1}{10e^x} = 37$$

Let $y = e^x$,

$$\ln 7e^x + \ln(10e^{-x}) = \ln 37$$

$$7y + \frac{10}{y} = 37$$

$$7y^2 + 10 = 37y$$

$$7y^2 - 37y + 10 = 0$$

$$y = \frac{-(-37) \pm \sqrt{(-37)^2 - 4(7)(10)}}{2(7)}$$

$y = 5$ or $y = \frac{2}{7}$

$e^x = 5$ or $e^x = \frac{2}{7}$

$x = \ln 5$ or $x = \ln \frac{2}{7}$

$x = 1.61$ or $x = -1.25$

(b) Solve the equation $\log_8(x) + \log_4(x) = \frac{10}{3}$. [5]

$$\log_8 x + \log_4 x = \frac{10}{3}$$

Common mistakes:

$$\frac{1}{\log_x 8} + \frac{1}{\log_x 4} = \frac{10}{3}$$

$$\log_x 8 + \log_x 4 = \frac{3}{10}$$

$$\frac{1}{3\log_x 2} + \frac{1}{2\log_x 2} = \frac{10}{3}$$

Let $y = \log_x 2$,

$$\frac{1}{3y} + \frac{1}{2y} = \frac{10}{3}$$

$$\frac{2+3}{6y} = \frac{10}{3}$$

$$\frac{5}{6y} = \frac{10}{3}$$

$$15 = 60y$$

$$y = \frac{1}{4}$$

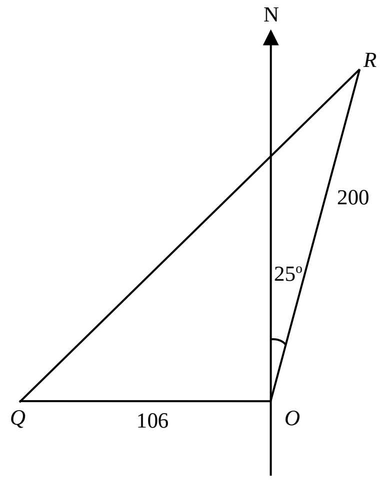
$$\log_x 2 = \frac{1}{4}$$

$$2 = x^{\frac{1}{4}}$$

$$x = 16$$

7. [Maximum mark: 14]

A farmer held his ducks in a triangular pen QOR where Q is due west of O, $OQ = 106$.
R is a point 200 m away from O, and its bearing from O is 025^o.



(a) Find

(i) the bearing of O from R. [2]

$180 - 25 = 155$

$360 - 155 = 205$

(ii) the perimeter of the pen OQR, [4]

$\measuredangle QOR = 90 + 25$

$\measuredangle QOR = 115$

$QR^2 = 106^2 + 200^2 - 2(106)(200)\cos 115$

$QR^2 = 69155.014$

$QR = 262.9734$

Perimeter = $269.9734 + 106 + 200$

Perimeter = 568.9734

Perimeter = 569

(iii) $\angle RQO$ [3]

$$\frac{\sin 115}{262.9734} = \frac{\sin \angle RQO}{200}$$

$$\sin \angle RQO = 0.689277$$

$$\angle RQO = \sin^{-1} 0.689277$$

$$\angle RQO = 43.6$$

(b) A signboard carrying the farm name stands vertically at point O. The top of the signboard, T, is 7.5 m from the ground. Is it possible to find a point, K, on RQ such that the angle of elevation of T from O is 45°. [5]

Let shortest distance from O to QR be h

$$\sin 43.6 = \frac{h}{106}$$

$$h = 106 \sin 43.6$$

$$h = 73.1$$

Maximum angle of elevation occurs when distance is shortest.
Let maximum angle of elevation be θ.

$$\tan \theta = \frac{7.5}{73.1}$$

$$\theta = \tan^{-1} 0.102599$$

$$\theta = 5.86$$

8. [Maximum mark: 14]

Answer the whole of this question on a sheet of graph paper.

A box without lid has a square base of sides x cm. Its volume, y cm^3, is given by $y=\frac{1}{4}(20x-x^3)$. Some corresponding values of x and y are given in the following table:

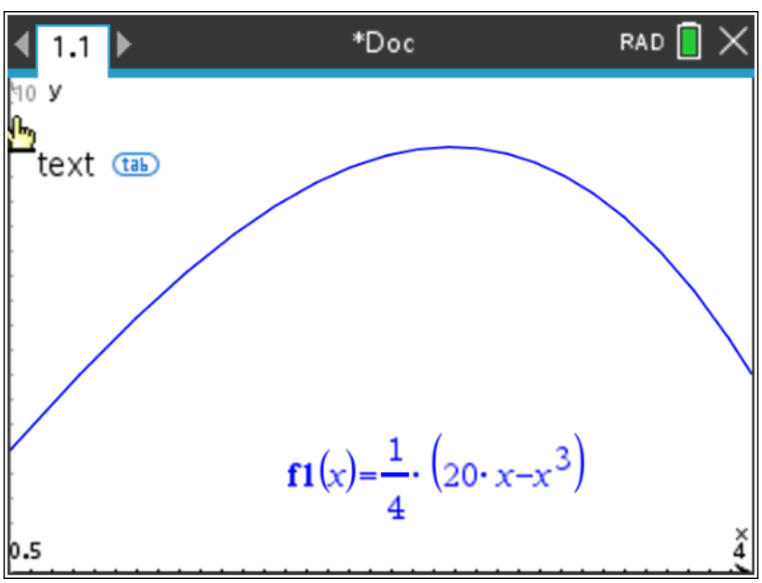
x	0.5	1	1.5	2	2.5	3	3.5	4
y	2.5	4.8	a	8	8.6	b	6.8	4

(a) Find the value of a and of b. [2]

$a = 6.7$

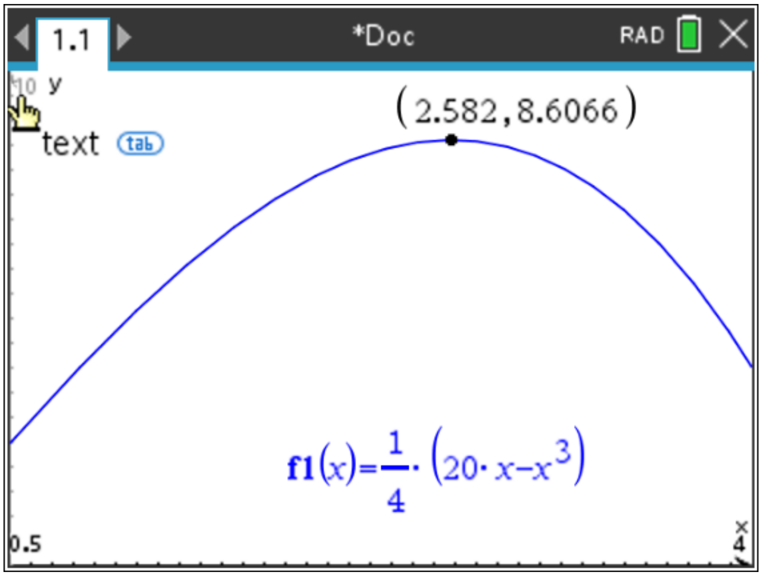
$b = 8.3$

(b) Taking 2 cm to represent 1 unit on the x-axis and 2 cm to represent 1 unit on the y-axis, draw the graph of $y=\frac{1}{4}(20x-x^3)$ for $0.5 \le x \le 4$. [4]

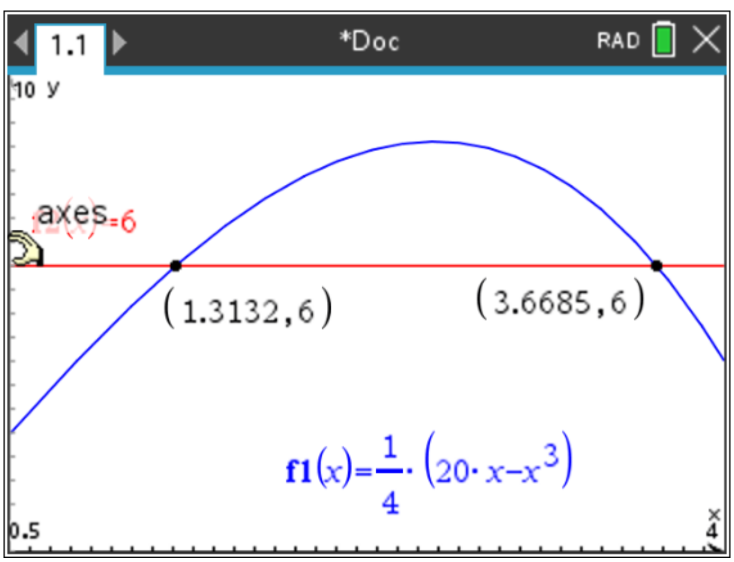


Use your graph to find

(c) the maximum value of y, [1]



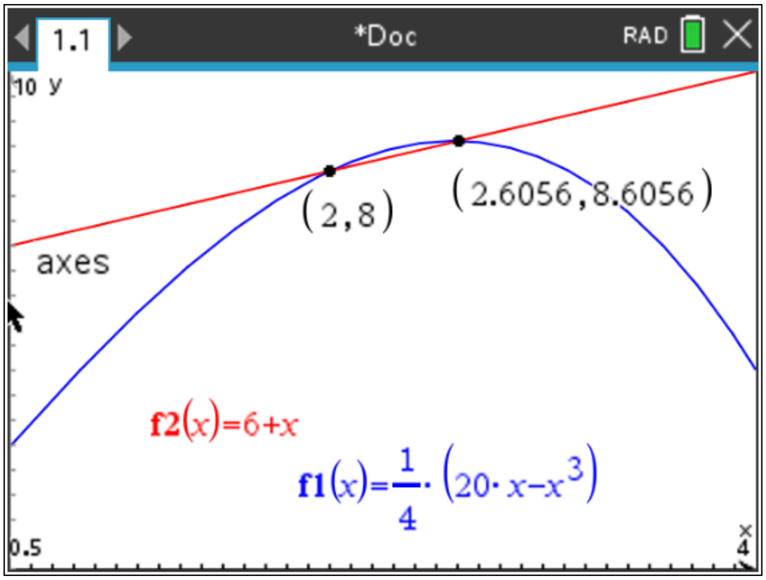
(d) the range of values of x for which $20x - x^3 \geq 24$, [3]



$20x - x^3 \geq 24$

$\frac{1}{4}(20x - x^3) \geq 6$

(e) the solutions of the equation $16x - x^3 - 4 = 0$ by drawing a suitable straight line. [4]



$$16x - x^3 - 24 = 0$$

$$16x - x^3 = 24$$

$$20x - x^3 = 24 + 4x$$

$$\frac{1}{4}(20x - x^3) = 6 + x$$