YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME			
CG		INDEX NO	

MATHEMATICS 9758/01

Paper 1 **4 September 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiners' Use

Question	1	2	3	4	5	6	7
Marks							

Question	8	9	10	11
Marks				

Total marks	

This paper consists of 20 printed pages.

1 **(i)** Expand $\sin\left(\frac{\pi}{4}-2x\right)$ in ascending powers of x, up to and including the term in x^3. [3]

(ii) The first two non-zero terms found in part **(i)** are equal to the first two non-zero terms in the series expansion of $(a+bx)^{-1}$ in ascending powers of x. Find the exact values of the constants a and b. Hence find the third exact non-zero term of the series expansion of $(a+bx)^{-1}$ for these values of a and b. [3]

2 **(a)** Vectors **a** and **b** are such that $\mathbf{a} \neq 0$ **,** $\mathbf{b} \neq 0$ and $|\mathbf{a}+\mathbf{b}| = |\mathbf{a}-\mathbf{b}|$. Show that **a** and **b** are perpendicular. [2]

(b) Referred to the origin O, points C and D have position vectors $\mathbf{c}$ and $\mathbf{d}$ respectively. Point P lies on OC produced such that $OC:CP = 1:\lambda-1$, where $\lambda > 1$. Point M lies on DP, between D and P, such that $DM:MP = 2:3$. Write down the position vector of M in terms of λ, $\mathbf{c}$ and $\mathbf{d}$. Hence, find the area of triangle OPM in the form $k\lambda|\mathbf{c}\times\mathbf{d}|$, where k is a constant to be found. [4]

3 The function f is defined by

$$\mathrm{f}(x)=\begin{cases}(x-2a)^2 & \text{for } 0\le x<2a,\\ 2ax-4a^2 & \text{for } 2a\le x<4a,\end{cases}$$

where a is a positive real constant and that $\mathrm{f}(x+4a)=\mathrm{f}(x)$ for all real values of x.

(i) Sketch the graph of $y=\mathrm{f}(x)$ for $-3a\le x\le 8a$. [3]

(ii) Hence find the value of $\int_{-2a}^{8a}\mathrm{f}(|x|)\,\mathrm{d}x$ in terms of a. [3]

4 A curve C has parametric equations

$$x = t^2 \ , \ y = \frac{1}{\sqrt{t}} \ , t > 0 \ .$$

(i) The curve $y = \frac{8}{x}$ intersects C at point A. Without using a calculator, find the coordinates of A. [2]

(ii) The tangent at the point $P\left(p^2, \frac{1}{\sqrt{p}}\right)$ on C meets the x-axis at point D and the y-axis at point E. The point F is the midpoint of DE. Find a cartesian equation of the curve traced by F as p varies. [5]

5 The equation of a curve is $2xy + (1+y)^2 = x$.

(i) Find the equations of the two tangents which are parallel to the y-axis. [4]

(ii) The normal to the curve at the point $A(1,\ 0)$ meets the y-axis at the point B. Find the area of the triangle OAB. [3]

6 The sum of the first n terms of a sequence is a cubic polynomial, denoted by S_n. The first term and the second term of the sequence are 2 and 4 respectively. It is known that $S_5 = 90$ and $S_{10} = 830$.

(i) Find S_n in terms of n. [4]

(ii) Find the 54th term of the sequence. [2]

(b) **(i)** Given that $\cos(2n-1)\alpha - \cos(2n+1)\alpha = 2\sin\alpha\sin 2n\alpha$ and α is not an integer multiple of π, show that

$$\sum_{n=1}^{N} \sin 2n\alpha = \frac{1}{2}\cot\alpha - \frac{1}{2}\operatorname{cosec}\alpha\cos(2N+1)\alpha\ .$$ [3]

(ii) Explain whether the series $\sum_{n=1}^{\infty} \sin \frac{2n\pi}{3}$ converges. [1]

7 (a)(i) Find $\int \cos(\ln x)\,\mathrm{d}x$. [3]

(ii) A curve C is defined by the equation $y = \cos(\ln x)$, for $\mathrm{e}^{-\frac{3}{2}\pi} \le x \le \mathrm{e}^{\frac{1}{2}\pi}$. The region R is bounded by C, the lines $x = \mathrm{e}^{-\frac{\pi}{2}}$, $x = \mathrm{e}^{\frac{\pi}{2}}$ and the x-axis. Find the exact area of R. [3]

(b) A curve is defined by the equation $y=\dfrac{\sqrt{e^{\cot x}}}{\sin x}$. The region bounded by this curve, the x-axis, the lines $x=\dfrac{\pi}{6}$ and $x=\dfrac{2\pi}{3}$, is rotated 2π radians about the x-axis to form a solid. Using the substitution $u=\cot x$, find the exact volume of the solid obtained. [4]

8 **(i)** Show that $y=\dfrac{x-x^2-1}{x-2}$ can be expressed as $y=\dfrac{A}{x-2}+B(x+1)$, where A and B are constants to be found. Hence, state a sequence of transformations that will transform the curve with equation $y=\dfrac{1}{3-x}-\dfrac{x}{3}$ onto the curve with equation $y=\dfrac{x-x^2-1}{x-2}$. [3]

(ii) On the same axes, sketch the curves with equations $y=\dfrac{x-x^2-1}{x-2}$ and $y=|2x+1|$, stating the equations of any asymptotes and the coordinates of the points where the curves cross the axes. [4]

Hence, find the exact range of values of x for which $\dfrac{x-x^2-1}{x-2}<|2x+1|$. [4]

9 **(a)** The function f is defined by $\mathrm{f}: x \mapsto 2+\dfrac{3}{x},\ x \in \mathbb{R},\ x>0$.

(i) Sketch the graph of $y=\mathrm{f}(x)$. Hence, show that f has an inverse. [2]

(ii) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(iii) On the same diagram as in part **(i),** sketch the graphs of $y = \mathrm{f}^{-1}(x)$ and $y = \mathrm{f}^{-1}\mathrm{f}(x)$. [2]

(iv) Explain why f^2 exists and find $\mathrm{f}^2(x)$. [2]

(b) The function h is defined by $\mathrm{h} : x \mapsto \dfrac{3-x}{x^2-1}$, $x \in \mathbb{R}$, $x \neq \pm 1$. Find algebraically the range of h, giving your answer in exact form. [4]

10 A tank initially contains 2000 litres of water and 20 kg of dissolved salt. Brine with C kg of salt per 1000 litres is entering the tank at 5 litres per minute and the solution drains out at the same rate of 5 litres per minute. The amount of salt in the tank at time t minutes is x kg. Assume that the solution is always uniformly mixed.

(i) Show that $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{400}(2C - x)$. Hence determine the value of C if the amount of salt in the tank remains constant at 120 kg after certain time has passed. [3]

(ii) Find x in terms of t. [5]

(iii) Sketch the graph of the particular solution, including the coordinates of the point(s) where the graph crosses the axes and the equations of any asymptotes. Find the time t when the amount of salt in the tank is 60 kg, giving your answer to the nearest minute. [3]

(iv) State one assumption for the above model to be valid. [1]

11 A factory produces power banks. The factory produces 1000 power banks in the first week. In each subsequent week, the number of power banks produced is 250 more than the previous week. The factory produces 7500 power banks in the Nth week.

(i) Find the value of N. [2]

(ii) After the Nth week, the factory produces 7500 power banks weekly. Find the total number of power banks that will be produced in the first 60 weeks. [3]

The sales manager predicts that the demand for power banks in this week is $a+bH$ if the demand for power banks in the preceding week is H, where a and b are constants and $b>1$. It is given that the demand for power banks in the first week is 50.

(iii) Show that the demand for power banks in the third week is $a+ba+50b^2$. [2]

(iv) Show that the demand for power banks in the nth week can be written as $a\left(\dfrac{b^{n-1}-1}{b-1}\right)+50b^{n-1}$. [1]

It is now given that $a = 300$ and $b = 1.05$.
In the first week, the number of power banks produced is still 1000. In each subsequent week, the number of power banks produced will be L more than the previous week.

(v) The production manager decides to change the production plan so that the total production can meet the total demand in the first 60 weeks. Find the least value of L. [4]

- End of Paper -

YISHUN INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
Higher 2

CANDIDATE NAME	

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MATHEMATICS 9758/02

Paper 2 **19 SEPT 2019**

3 hours

Candidates answer on the Question Paper.
Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your CG and name on the work you hand in.
Write in dark blue or black pen.
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The total number of marks for this paper is 100.

For Examiners' Use

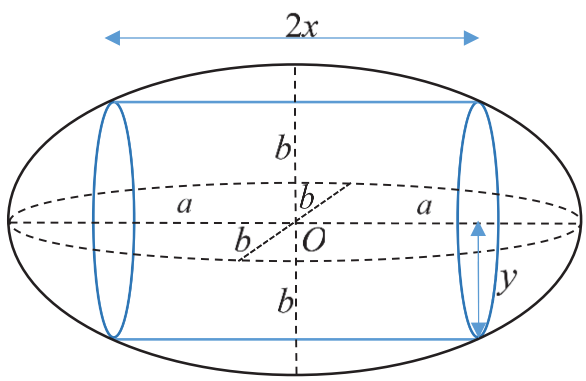
Question	1	2	3	4	5	6
Marks						

Question	7	8	9	10	**Total marks**	
Marks						

This document consists of 22 printed pages and 2 blank pages.

Section A: Pure Mathematics [40 marks]

1



The above diagram shows a hollow ellipsoid with centre O, enclosing a fixed volume of $\frac{4}{3}\pi ab^2$. A solid cylinder of length $2x$ and base radius y is inscribed in the ellipsoid. It is given that $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are positive constants with $a > b$.
Use differentiation to find, in terms of a and b, the maximum volume of the cylinder, proving that it is a maximum. Hence determine the ratio of the maximum volume of the cylinder to that of the volume enclosed by the ellipsoid. [6]

2 **(i)** Find $\int 2\sin(k+1)x \sin kx \; \mathrm{d}x$. [2]

(ii) Hence, determine in terms of k, the value of $\int_0^{\frac{\pi}{2}} \left(\sin(k+1)x - \sin kx\right)^2 \, \mathrm{d}x$, where k is an even integer. [5]

3 The plane p contains the point A with coordinates $(5, -1,\ 2)$ and the line l_1 with equation $\frac{x-3}{2} = y$, $z = 1$.

(i) The point B has coordinates $(c,\ 2,\ 2)$. Given that the shortest distance from B to l_1 is $\frac{\sqrt{205}}{5}$, find the possible values of c. [3]

(ii) Find a cartesian equation of p. [3]

The line l_2 has equation $\mathbf{r}=\begin{pmatrix}1\\2\\1\end{pmatrix}+\mu\begin{pmatrix}4\\1\\3\end{pmatrix}$, $\mu\in\mathbb{R}$.

(iii) Find the coordinates of the point at which l_2 intersects p. [3]

The line l_3 has equation $\mathbf{r} = \begin{pmatrix} a \\ 2a \\ a \end{pmatrix} + t\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$, $t \in \mathbb{R}$, where a is a constant.

(iv) Show that p is parallel to l_3 . [1]

(v) Given that l_3 and p have no point in common, what can be said about the value of a? [1]

(vi) It is given instead that $a = 1$, find the distance between l_3 and p, leaving your answer in exact form. [2]

4 **Do not use a calculator in answering this question.**

(a) The equation $2z^3 - 3z^2 + kz + 26 = 0$, where k is a real constant, has a root $z = 1 + a\mathrm{i}$, where a is a positive real constant. Find the other roots of the equation and the values of a and k. [6]

(b) **(i)** Given that $\left(x+\mathrm{i}y\right)^2=15+8\mathrm{i}$, determine the possible values of the real numbers x and y. [3]

(ii) The roots of the equation $z^2-(2+7\mathrm{i})z=15-5\mathrm{i}$ are z_1 and z_2, with $\arg(z_1)<\arg(z_2)$. Find an exact expression for z_2, giving your answer in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$. [3]

(iii) Find the argument of $z_1^{\,2} z_2^{\,*}$ in exact form. [2]

Section B: Probability and Statistics [60 marks]

5 An investigation was carried out to determine the effect of rainfall on crop yield. The table below shows the average monthly rainfall, x mm, and the crop yield, y kg. The data is recorded during different months of a certain year.

x	150	163	172	175	180	187	196
y	48	70	87	92	95	89	80

(i) Draw a scatter diagram for these values. State with a reason, which of the following equations, where a and b are constants, provides the most accurate model of the relationship between x and y.

(A) $y = a\ln(x-100)+b$

(B) $y = a(x-180)^2+b$

(C) $y = \dfrac{a}{x-130}+b$ [3]

(ii) Using the model you chose in part **(i)**, write down the equation for the relationship between x and y, giving the numerical values of the coefficients. State the product moment correlation coefficient for this model. [2]

(iii) Calculate an estimate of the crop yield when the average monthly rainfall is 185 mm. Comment on the reliability of your estimate. [2]

6 A factory produces a large number of packets of cornflakes. On average, two in 7 packets contain a toy. The packets of cornflakes are sold in cartons of 12.

(a) A carton is randomly chosen.

(i) Find the probability that there are fewer than 2 toys. [1]

(ii) Find the probability that there are more than 1 but at most 6 toys. [2]

(b) Find the probability that in five randomly selected cartons, two of them contain exactly 4 toys and three of them contain exactly 2 toys. [2]

(c) A mini-mart ordered 40 cartons of cornflakes from the factory. Find the probability that none of the cartons contains fewer than 2 toys. [2]

The factory also produces a large number of packets of oats. A random sample of n packets of oats is chosen. The number of packets of oats containing a toy in the sample is denoted by A. Assume that A has the distribution $\mathrm{B}(n, p)$, where $p > 0.1$.

Given that $n = 25$ and $\mathrm{P}(A = 2 \text{ or } 3) = 0.25$, write down an equation in terms of p and find p numerically. [2]

7 A box contains five balls numbered 1, 3, 5, 6, 8. Three balls are drawn at random from the box.

(a) Find the probability that the sum of the three numbers drawn is an even number. [2]

(b) The random variable S denotes the smallest of the three numbers drawn.

(i) Determine the probability distribution of S. [2]

(ii) Find $\mathrm{E}(S)$ and $\mathrm{Var}(S)$. [2]

(iii) The mean of a random sample of 55 observations of S is denoted by $\overline{S}$. Find the probability that $\overline{S}$ is within 0.5 of $\mathrm{E}(S)$. [3]

8 **(a)** Events A and B are such that $\mathrm{P}\left(A \cup B\right) = 0.6$ and $\mathrm{P}\left(A \mid B\right) = 0.4$. Given that A and B are independent, find

(i) $\mathrm{P}\left(B\right)$, [2]

(ii) $\mathrm{P}\left(A' \mid B'\right)$. [2]

(b) 12 people are to be seated at 3 different coloured round tables.

(i) Find the number of ways that there are at least 3 people at each table. [4]

(ii) Find the probability that there are 4 people at each table given that there are at least 3 people at each table. [2]

9 A company produces car batteries. The life, in months, of a car battery of the regular type has the distribution $\mathrm{N}\left(\mu,\ \sigma^2\right)$. The mean life of 4 randomly selected car batteries of the regular type is denoted by $\overline{X}$. It is given that $\mathrm{P}\left(\overline{X} < 36.1\right) = \mathrm{P}\left(\overline{X} > 49.1\right) = 0.03355$.

(i) State the value of μ and show that $\sigma \approx 7.10$, correct to 2 decimal places. [4]

(ii) Find the smallest integer value of *k* such that more than 90% of the car batteries of the regular type have a life less than *k* months. [2]

(iii) Past experience shows that 25% of the car batteries of the regular type with lives less than 36 months are due to bad driving habits. A random sample of 100 car batteries of the regular type is selected. Find the expected number of these car batteries which will have lives each less than 36 months due to bad driving habits. [2]

(iv) After research and experimentation, the company produces a premium type of car battery using an improved manufacturing process which is able to increase the life of each car battery by 10%. Find the probability that the total life of 5 randomly chosen car batteries of the premium type is more than the total life of 6 randomly chosen car batteries of the regular type. [4]

10 A previous study revealed that the average time taken to assemble a certain type of electrical component is at least 15 minutes. The manager wants to investigate if the results of the study is valid. A random sample of 40 components is taken and the times taken to assemble the components are summarised in the following table:

Time to assemble a component (min)	9	10	12	13	15	16	17	18
Number of components	1	6	3	8	5	7	8	2

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test at the 5% level of significance whether the results of the study is valid. You should state your hypotheses and define any symbols you use. [5]

(iii) Explain why the manager is able to conduct the test without knowing anything about the distribution of the times taken to assemble the electrical components.[1]

(iv) Explain what is meant by the phrase "5% level of significance" in this context.[1]

The manager claims that the average time taken to assemble another type of electrical component is 30 minutes. A random sample of 50 components of this type is chosen and the time taken to assemble each component is recorded. The mean and standard deviation of the sample are 29.7 minutes and k minutes respectively. Find the range of possible values of k if a test at the 8% significance level shows that there is sufficient evidence that the manager's claim is valid. [4]

~ END OF PAPER ~

Yishun Innova Junior College ♦ Mathematics Department
2019 JC 2 Mathematics H2 9758
Prelim Examination P1
Solutions

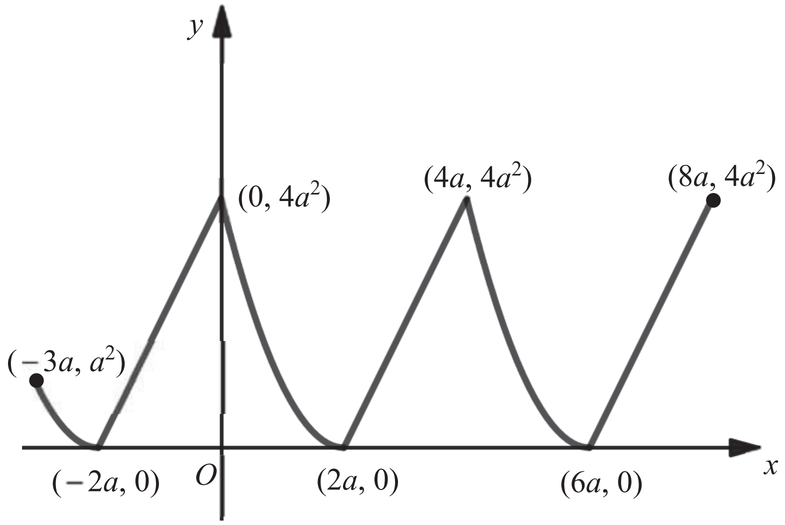
Qn	Solution	Remarks
1(i)	$\sin\left(\frac{\pi}{4}-2x\right)=\sin\frac{\pi}{4}\cos 2x-\cos\frac{\pi}{4}\sin 2x$ ------* $=\frac{1}{\sqrt{2}}(\cos 2x-\sin 2x)$ $=\frac{1}{\sqrt{2}}\left[\left(1-\frac{(2x)^2}{2}+\ldots\right)-\left(2x-\frac{(2x)^3}{3!}+\ldots\right)\right]$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+\ldots\right)$ <u>Alternative Method</u> Let $y=\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=-2\cos\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4\sin\left(\frac{\pi}{4}-2x\right)$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=8\cos\left(\frac{\pi}{4}-2x\right)$ When $x=0$, $y=\frac{1}{\sqrt{2}}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=-\sqrt{2}$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-2\sqrt{2}$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=4\sqrt{2}$ $\sin\left(\frac{\pi}{4}-2x\right)=\frac{1}{\sqrt{2}}-\sqrt{2}x-\frac{2\sqrt{2}}{2}x^2+\frac{4\sqrt{2}}{3!}x^3+\ldots$ $=\frac{1}{\sqrt{2}}-\sqrt{2}x-\sqrt{2}x^2+\frac{2\sqrt{2}}{3}x^3+\ldots$ $=\frac{1}{\sqrt{2}}\left(1-2x-2x^2+\frac{4x^3}{3}+\ldots\right)$	You cannot substitute $\frac{\pi}{4}-2x$ into the standard expansion formula directly. In general, we can apply the standard expansions when x is replaced by g(x), provided g(0) = 0. For instance, $\mathrm{g}(x)=x+x^2$. Be careful with the signs when you doing the higher derivatives.

(ii)	$(a+bx)^{-1}=a^{-1}\left(1+\frac{b}{a}x\right)^{-1}$ $=a^{-1}\left[1+(-1)\left(\frac{b}{a}x\right)+\frac{(-1)(-2)}{2}\left(\frac{b}{a}x\right)^2+...\right]$ $=\frac{1}{a}\left(1-\frac{b}{a}x+\left(\frac{b}{a}\right)^2x^2+...\right)$ $\therefore a=\sqrt{2}$ and $\frac{b}{a}=2 \Rightarrow b=2\sqrt{2}$ Third non-zero term: $\frac{1}{a}\left(\frac{b}{a}\right)^2x^2=\frac{1}{\sqrt{2}}(2)^2x^2=\frac{4}{\sqrt{2}}x^2=2\sqrt{2}x^2$	Note that power is -1, not a positive integer so we need to use the series expansion of $(1+x)^n$ found in MF26 Don't forget to apply the power -1 to a after factorize a out. Third non-zero term and the coefficient of third non-zero term are different.
2(a)	$\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ $\lvert\mathbf{a}+\mathbf{b}\rvert^2=\lvert\mathbf{a}-\mathbf{b}\rvert^2$ $(\mathbf{a}+\mathbf{b})\bullet(\mathbf{a}+\mathbf{b})=(\mathbf{a}-\mathbf{b})\bullet(\mathbf{a}-\mathbf{b})$ $\lvert\mathbf{a}\rvert^2+2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2=\lvert\mathbf{a}\rvert^2-2\mathbf{a}\bullet\mathbf{b}+\lvert\mathbf{b}\rvert^2$ $4\mathbf{a}\bullet\mathbf{b}=0$ $\mathbf{a}\bullet\mathbf{b}=0$ Hence **a** and **b** are perpendicular. <u>Alternative Method:</u> Since $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$, the diagonals of the parallelogram (with sides $\mathbf{a}$ and $\mathbf{b}$) are equal in length and thus the parallelogram must be a rectangle. Therefore, **a** and **b** are perpendicular. 	There are two forms of vector product: dot (scalar) and cross (product). Hence expressions such as $\mathbf{a}^2$ will not make sense, as it will be ambiguous whether it means $\mathbf{a}\cdot\mathbf{a}$ or $\mathbf{a}\times\mathbf{a}$. However $\lvert\mathbf{a}\rvert^2$ is meaningful as $\lvert\mathbf{a}\rvert$ means the length of a vector, so it is a number. Next, as $\lvert\mathbf{a}+\mathbf{b}\rvert$ is the length of the vector $\mathbf{a}+\mathbf{b}$, so $\lvert\mathbf{a}+\mathbf{b}\rvert\neq\lvert\mathbf{a}\rvert+\lvert\mathbf{b}\rvert$, and therefore $\lvert\mathbf{a}+\mathbf{b}\rvert^2\neq\lvert\mathbf{a}\rvert^2+2\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert+\lvert\mathbf{b}\rvert^2$. It is wrong to say that $\lvert\mathbf{a}+\mathbf{b}\rvert=\lvert\mathbf{a}-\mathbf{b}\rvert$ implies $\mathbf{a}+\mathbf{b}=\mathbf{a}-\mathbf{b}$ or $\mathbf{a}+\mathbf{b}=-(\mathbf{a}-\mathbf{b})$. Take for example the vectors $\mathbf{i}$ and $\mathbf{j}$, they are obviously pointing in different directions, so neither $\mathbf{i}=\mathbf{j}$ nor $\mathbf{i}=-\mathbf{j}$, but $\lvert\mathbf{i}\rvert=\lvert\mathbf{j}\rvert=1$.

<table>
<tr>
<td>(b)</td>
<td>$\overrightarrow{OP} = \lambda\mathbf{c}$

$\overrightarrow{OM} = \dfrac{2\overrightarrow{OP}+3\overrightarrow{OD}}{5} = \dfrac{2\lambda\mathbf{c}+3\mathbf{d}}{5}$

Area of triangle $OPM = \dfrac{1}{2}\left|\overrightarrow{OP}\times\overrightarrow{OM}\right|$
$= \dfrac{1}{2}\left|\lambda\mathbf{c}\times\left(\dfrac{2\lambda\mathbf{c}+3\mathbf{d}}{5}\right)\right|$
$= \dfrac{1}{2}\left|\lambda\mathbf{c}\times\dfrac{2\lambda\mathbf{c}}{5}+\lambda\mathbf{c}\times\dfrac{3\mathbf{d}}{5}\right|$
$= \dfrac{1}{2}\left|\lambda\mathbf{c}\times\dfrac{3\mathbf{d}}{5}\right| = \dfrac{3}{10}\lambda\left|\mathbf{c}\times\mathbf{d}\right|$</td>
<td>Read carefully: P lies on OC produced. So P does not lie in between O and C.

Ratio theorem is a very useful and quick way to get $\overrightarrow{OM}$.
$\mathbf{c}\times\mathbf{c}$ is a vector product, so the result should be a vector.
So $\mathbf{c}\times\mathbf{c}\neq 0$, it should be $\mathbf{0}$, the zero vector.
Because of the definition of vector product, $\mathbf{c}\times\mathbf{c}\neq\left|\mathbf{c}\right|^2$, and $\mathbf{c}\times\mathbf{d}\neq\mathbf{d}\times\mathbf{c}$.</td>
</tr>
<tr>
<td>3(i)</td>
<td>
</td>
<td>-label all vertices and end points

-Quadratic curve shape (part) for $0\le x\le 2a$ and straight line segment for $2a\le x\le 4a$</td>
</tr>
<tr>
<td>(ii)</td>
<td>$\int_{-2a}^{8a} \mathrm{f}(|x|)\,\mathrm{d}x = 3\int_{0}^{2a}(x-2a)^2\,\mathrm{d}x + 2\left(\dfrac{1}{2}(2a)(4a^2)\right)$
$= 3\left[\dfrac{1}{3}(x-2a)^3\right]_0^{2a} + 8a^3$
$= 8a^3 + 8a^3$
$= 16a^3$</td>
<td>To draw graph of f(|x|), keep RHS and reflect in the y-axis.

Note that $(x-2a)^2$ is only defined for $0\le x\le 2a$ while $2ax-4a^2$ is only defined for $2a\le x\le 4a$. It is wrong to take for instance: $\int_{4a}^{6a}(x-2a)^2\,\mathrm{d}x$ as the area of the region from $4a\le x\le 6a$.</td>
</tr>
</table>

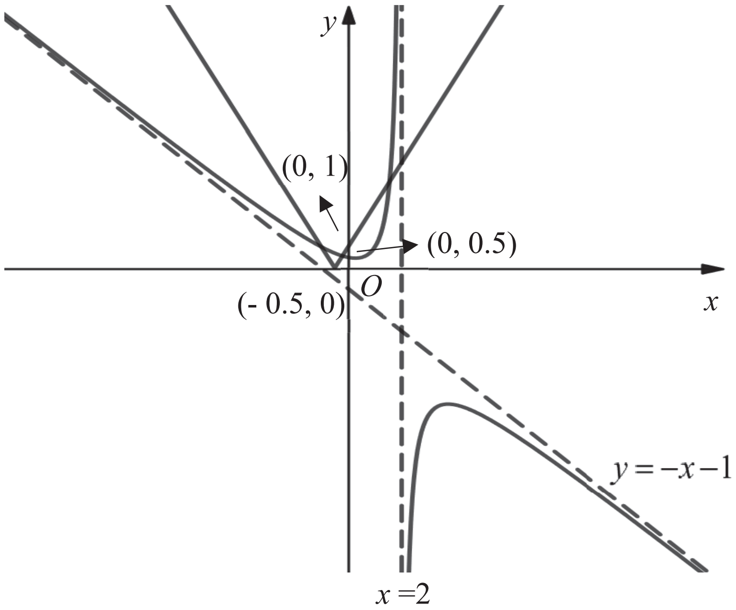
4(i)	Substitute $x=t^2$, $y=\frac{1}{\sqrt{t}}$ into $y=\frac{8}{x}$, $\frac{1}{\sqrt{t}}=\frac{8}{t^2}$ $t^{\frac{3}{2}}=8$ $t=4$ When $t=4$, $x=4^2=16$ $y=\frac{1}{\sqrt{4}}=\frac{1}{2}$ Coordinates of A is $\left(16,\frac{1}{2}\right)$.	Substitute the parametric equation of C into the Cartesian equation Avoid changing the parametric equation to Cartesian form
(ii)	$\frac{dx}{dt}=2t$, $\frac{dy}{dt}=-\frac{1}{2}t^{-\frac{3}{2}}$ $\frac{dy}{dx}=-\frac{1}{2}t^{-\frac{3}{2}}\times\frac{1}{2t}=-\frac{1}{4}t^{-\frac{5}{2}}$ Equation of tangent at point P on curve C, $y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)$ At D , $y=0$ $\therefore 0-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(x-p^2\right)\Rightarrow x=5p^2$ At E , $x=0$ $\therefore y-\frac{1}{\sqrt{p}}=-\frac{1}{4}p^{-\frac{5}{2}}\left(0-p^2\right)\Rightarrow y=\frac{5}{4}p^{-\frac{1}{2}}$ Coordinates of F : $\left(\frac{5p^2}{2},\frac{\frac{5}{4}p^{-\frac{1}{2}}}{2}\right)\Rightarrow\left(\frac{5p^2}{2},\frac{5}{8\sqrt{p}}\right)$ $y=\frac{5}{8\sqrt{p}}\Rightarrow\sqrt{p}=\frac{5}{8y}$, substitute into $x=\frac{5p^2}{2}$ $x=\frac{5}{2}\left(\frac{5}{8y}\right)^4=\frac{3125}{8192y^4}$ Cartesian equation of the curve traced by F is $x=\frac{3125}{8192y^4}$.	Take note that we are finding the gradient of tangent $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ Write down as header what you are trying to find, for example equation of tangent at point P. When t=p, Gradient of tangent at point P is $-\frac{1}{4}p^{-\frac{5}{2}}$ instead of $-\frac{1}{4}t^{-\frac{5}{2}}$ Find the midpoint F which follows the formula $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ From F, let $x=\frac{5p^2}{2}$, $y=\frac{5}{8\sqrt{p}}$ which is in the parametric form. Convert to Cartesian form so that we can trace how the path that point F moves as p varies.

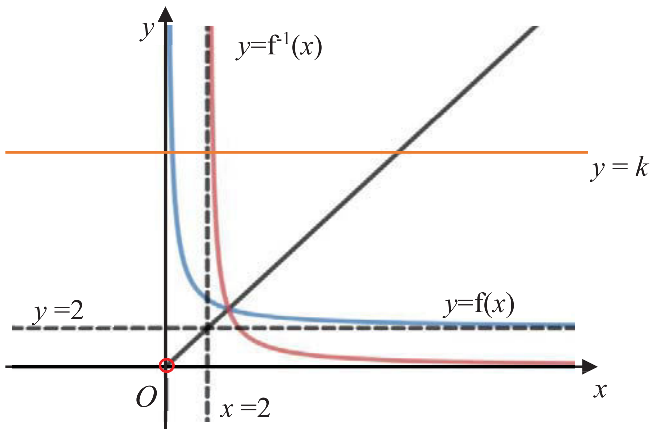
5(i)	$2xy+(1+y)^2=x$ Differentiating wrt x, $2y+2x\frac{\mathrm{d}y}{\mathrm{d}x}+2(1+y)\frac{\mathrm{d}y}{\mathrm{d}x}=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1-2y}{2(x+y+1)}$ When the tangent is parallel to the y-axis, $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. $\therefore 2(x+y+1)=0$ $y=-1-x$, substitute this into the equation of the curve, $2x(-1-x)+(1-1-x)^2=x$ $x^2+3x=0$ $x=0$ or $x=-3$	Do not confuse "tangent is parallel to y-axis", with x-axis: "parallel with y-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. "parallel with x-axis": $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. Vertical lines in the Cartesian grid are of the form "x=…." instead of "y=…"
(ii)	Gradient of normal at $A=\frac{-1}{\frac{1-2(0)}{2(1+0+1)}}=-4$ Equation of normal at A: $y-0=-4(x-1)$ At B, $x=0\Rightarrow y=4$ => Coordinates of B are $(0,4)$ Area of triangle $OAB=\frac{1}{2}\times4\times1=2$ units^2	
6(a)(i)	$S_n=an^3+bn^2+cn+d$ $S_1=a(1)^3+b(1)^2+c(1)+d=2$ $\Rightarrow$ $a+b+c+d=2$ ----(1) $S_2=a(2)^3+b(2)^2+c(2)+d=6$ $\Rightarrow$ $8a+4b+2c+d=6$ ----(2) $S_5=a(5)^3+b(5)^2+c(5)+d=90$ $\Rightarrow$ $125a+25b+5c+d=90$ ----(3)	Don't assume that sequence is an AP/GP. It's neither. It's pointless to just write $S_5=T_1+T_2+...+T_5$ The sum is polynomial in n i.e. dependent on n and generally includes a constant term. Read question carefully. '4' is not S_2.

	$S_{10}=a(10)^3+b(10)^2+c(10)+d=830$ $\Rightarrow\quad 1000a+100b+10c+d=830\quad$ ----(4) Using GC: $a=1,\quad b=-2,\quad c=3,\quad d=0$ $S_n=n^3-2n^2+3n,\quad n\geq 1,\ n\in\mathbb{Z}^+$	
(ii)	54^{th} term of the sequence $u_{54}=S_{54}-S_{53}$ $=(54)^3-2(54)^2+3(54)-(53)^3+2(53)^2-3(53)$ $=8376$	
(b)(i)	Given that $\cos(2n-1)\alpha-\cos(2n+1)\alpha=2\sin\alpha\sin 2n\alpha$ $\sin(2n\alpha)=\dfrac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $\therefore\displaystyle\sum_{n=1}^{N}\sin(2n\alpha)=\sum_{n=1}^{N}\frac{\cos(2n-1)\alpha-\cos(2n+1)\alpha}{2\sin\alpha}$ $=\dfrac{1}{2\sin\alpha}\left[\begin{array}{l}\cos\alpha-\cos 3\alpha\\+\cos 3\alpha-\cos 5\alpha\\+\cos 5\alpha-\cos 7\alpha\\+\cdots\\+\cos(2N-3)\alpha-\cos(2N-1)\alpha\\+\cos(2N-1)\alpha-\cos(2N+1)\alpha\end{array}\right]$ $=\dfrac{\cos\alpha-\cos(2N+1)\alpha}{2\sin\alpha}\quad$ -------(*) $=\dfrac{\cos\alpha}{2\sin\alpha}-\dfrac{\cos(2N+1)\alpha}{2\sin\alpha}$ $=\dfrac{\cot\alpha}{2}-\dfrac{\operatorname{cosec}\alpha\cos(2N+1)\alpha}{2}\quad$ (shown)	Use what was given. Don't waste time to derive it when it's already given. Note that $2\sin\alpha$ (independent of n) is a constant in this case Remember to cancel sufficient number of rows at the beginning and at the end.
(ii)	Let $\alpha=\dfrac{\pi}{3}$, $\displaystyle\sum_{n=1}^{N}\sin\left(\frac{2n\pi}{3}\right)$ $=\dfrac{\cot\frac{\pi}{3}}{2}-\dfrac{\operatorname{cosec}\frac{\pi}{3}\cos(2N+1)\frac{\pi}{3}}{2}$ As $N\to\infty$, $\cos(2N+1)\dfrac{\pi}{3}$ takes values $\dfrac{1}{2}$ or -1. OR cannot converge to a constant number. $\therefore\displaystyle\sum_{n=1}^{\infty}\sin\frac{2n\pi}{3}$ does not converge.	Note that it's the letter N that tends to infinity and not the letter n.

7(a)(i)	$\int \cos(\ln x)\ \mathrm{d}x = x\cos(\ln x) - \int x\left(-\frac{1}{x}\sin(\ln x)\right)\mathrm{d}x$ $= x\cos(\ln x) + \int \sin(\ln x)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int x\left(\frac{1}{x}\cos(\ln x)\right)\ \mathrm{d}x$ $= x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\ \mathrm{d}x$ $2\int \cos(\ln x))\ \mathrm{d}x = x\cos(\ln x) + x\sin(\ln x)$ $\int \cos(\ln x)\ \mathrm{d}x = \frac{1}{2}x\left(\cos(\ln x) + \sin(\ln x)\right) + C$	Use integration by parts directly. $\cos(\ln x)$ is a composite function. It is not a product of $(\cos x)(\ln x)$. Let u = $\cos(\ln x)$ and $\frac{\mathrm{d}v}{\mathrm{d}x} = 1$ apply integration by parts $uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\mathrm{d}x$ twice Bring $-\int \cos(\ln x)\ \mathrm{d}x$ to the LHS to stop the loop. Put + C in the final step
(ii)	Area = $\int_{\mathrm{e}^{-\frac{\pi}{2}}}^{\mathrm{e}^{\frac{\pi}{2}}} \cos(\ln x))\ \mathrm{d}x \quad = \frac{1}{2}\left[\ x\cos(\ln x) + x\sin(\ln x)\ \right]_{\mathrm{e}^{-\frac{\pi}{2}}}^{\mathrm{e}^{\frac{\pi}{2}}}$ $= \frac{1}{2}\mathrm{e}^{\frac{\pi}{2}}\left(\cos\left(\ \ln \mathrm{e}^{\frac{\pi}{2}}\right) + \sin\left(\ \ln \mathrm{e}^{\frac{\pi}{2}}\right)\right) -$ $\frac{1}{2}\mathrm{e}^{-\frac{\pi}{2}}\left(\cos\left(\ \ln \mathrm{e}^{-\frac{\pi}{2}}\right) + \sin\left(\ \ln \mathrm{e}^{-\frac{\pi}{2}}\right)\right)$ $= \frac{1}{2}\mathrm{e}^{\frac{\pi}{2}}\left(\cos\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \frac{1}{2}\mathrm{e}^{-\frac{\pi}{2}}\left(\cos\left(-\frac{\pi}{2}\right) + \sin\left(-\frac{\pi}{2}\right)\right)$ $= \frac{1}{2}\left(\mathrm{e}^{\frac{\pi}{2}} + \mathrm{e}^{-\frac{\pi}{2}}\right)$	Read question carefully. Integrate from $e^{-\frac{\pi}{2}}$ to $e^{\frac{\pi}{2}}$. Use (i) answer. F(Upper limit)– F(lower limit) Note that $\ln \mathrm{e}^{\frac{\pi}{2}} = \frac{\pi}{2}$, $\sin\left(-\frac{\pi}{2}\right) = -1$
(b)	Using $u = \cot x$, $\frac{\mathrm{d}u}{\mathrm{d}x} = -\mathrm{cosec}^2 x = -\frac{1}{\sin^2 x}$ $\Rightarrow \ldots \mathrm{d}u = \ldots -\frac{1}{\sin^2 x}\ \mathrm{d}x$ When $x = \frac{\pi}{6} \Rightarrow u = \frac{1}{\tan x} = \sqrt{3}$ When $x = \frac{2\pi}{3} \Rightarrow u = -\frac{1}{\sqrt{3}}$	Differentiate the given substitution. Memorise the differentiation of the 6 trigo functions. Only differentiation of sec *x* and cosec *x* formula are in MF26 Find $\frac{\mathrm{d}u}{\mathrm{d}x}$ and hence d*x* = .. Need to change limit to u value. Write down the expression of the volume first. Then do substitution. $V = \int_{x_1}^{x_2} y^2\ \mathrm{d}x$

	Required volume is $= \pi \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \left(\frac{e^{\frac{1}{2}\cot x}}{\sin x} \right)^2 \, dx = \pi \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} \frac{e^{\cot x}}{\sin^2 x} \, dx$ $= \pi \int_{\frac{\pi}{6}}^{\frac{2\pi}{3}} e^{\cot x} \frac{1}{\sin^2 x} \, dx$ $= -\pi \int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}} e^u \, du$ or $\pi \int_{-\frac{1}{\sqrt{3}}}^{\sqrt{3}} e^u \, du$ $= \pi \left(e^{\sqrt{3}} - e^{-\frac{1}{\sqrt{3}}} \right)$	Change to $-\pi \int_{\sqrt{3}}^{-\frac{1}{\sqrt{3}}} e^u \, du$ Integrate e^u w.r.t u is e^u
8(i)	$y = \frac{x - x^2 - 1}{x - 2} = \frac{-3}{x-2} - (x+1)$ $y = \frac{1}{3-x} - \frac{x}{3}$ $= \frac{-1}{x-3} - \frac{1}{3}x$ 1) A scaling parallel to the y-axis by a factor of 3. 2) A translation of 1 unit in the negative x-direction. 	Be careful in your algebraic manipulation when trying to express $y = \frac{x - x^2 - 1}{x - 2}$ in the form $y = \frac{A}{x-2} + B(x+1)$. Note that you are required to **describe** the sequence of transformations that will transform the curve with equation $y = \frac{1}{3-x} - \frac{x}{3}$ to $y = \frac{x - x^2 - 1}{x - 2} = \frac{-3}{x-2} - (x+1)$, NOT the reverse. It is **INCORRECT to use** the word **"shift"** to describe translation and the word **"flip/rotate"** to describe reflection. One of the transformations involved is scaling parallel to the y-axis with a factor of 3, it is **NOT with a factor of -3 or 3 units.**

(ii)	 	Note that the curve $y=\frac{x-x^2-1}{x-2}$ has asymptotes $y=-x-1$ and $x=2$. You are expected to draw properly and clearly, including the behaviour of the curve $y=\frac{x-x^2-1}{x-2}$ near its asymptotes. Note that the oblique asymptote $y=-x-1$ meets the x-axis and y-axis at $(-1,0)$ and $(0,-1)$ respectively and the curve cuts the y-axis at the point $\left(0,\frac{1}{2}\right)$ which is NOT the minimum point. You are expected to get the <u>correct shape</u> of the graph from your GC. You are required to state the **coordinates of the points where the curves cross the axes.** For the modulus graph $y=\lvert 2x+1\rvert$, the coordinates are $\left(-\frac{1}{2},0\right)$ and $(0,1)$. It has a line of symmetry $x=-\frac{1}{2}$. You should have the sense of the relative positions for the points of intersection with the axes when sketching the two graphs, including the asymptotes on the same axes.
	$\frac{x-x^2-1}{x-2}=-2x-1$ From GC, the first point of intersection has x-coordinate -1 . $\frac{x-x^2-1}{x-2}=2x+1$ $x-x^2-1=(2x+1)(x-2)$ $3x^2-4x-1=0$ $x=\frac{4\pm\sqrt{16-4(3)(-1)}}{2(3)}$ $x=\frac{2\pm\sqrt{7}}{3}$	The question states that "**Hence**, find the **exact range** of …..", so you are required to use graphical method.

<table>
<tr><td></td><td>Thus, the range of values of x is
$x<-1,\ \dfrac{2-\sqrt{7}}{3}<x<\dfrac{2+\sqrt{7}}{3}$ or $x>2$</td><td></td></tr>
<tr><td>9(a)(i)</td><td>

Since any horizontal line $y=k,\ k\in\mathbb{R}$ intersects the graph of f at most once, f is one-one and it has an inverse.</td><td>Must sketch the graph of f for the given domain only ($x>0$) with the asymptotes.
Never use a particular line to explain one-one function and you must state the range of k. Note the 2 possible explanations.

If you use any line $y=k,\ k\in\mathbb{R}$, then the line cuts the graph of f at most once.

If you use any line $y=k,\ k\in R_f$, then the line cuts the graph of f exactly once.

It is important to mention that f is one-one and not just f has an inverse.</td></tr>
<tr><td>(ii)</td><td>Let $y=2+\dfrac{3}{x}$
$\dfrac{3}{x}=y-2$
$x=\dfrac{3}{y-2}$
$\therefore \mathrm{f}^{-1}(x)=\dfrac{3}{x-2}$
$D_{\mathrm{f}^{-1}}=R_{\mathrm{f}}=(2,\infty)$</td><td>To find the rule of f^{-1} :

Let $y = \mathrm{f}(x)$ and then make x the subject</td></tr>
<tr><td>(iii)</td><td>See diagram in (i)</td><td>Must use the same scale for both axes when sketching the graphs of f and f^{-1} on the same diagram.

The graph of f^{-1} is a reflection of the graph of f in the line $y=x$</td></tr>
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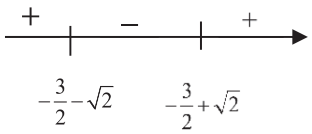
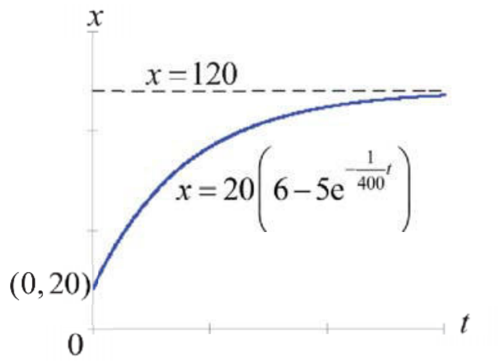
		The graph of $f^{-1}f$ is not simply the line $y = x$. Must sketch for the correct domain and passing through (2, 2). The graphs of f, f^{-1} and $f^{-1}f$ must intersect at the same point.
(iv)	$D_f = (0, \infty)$ and $R_f = (2, \infty)$ Since $R_f \subseteq D_f$, f^2 exists. $f^2(x) = f(f(x))$ $= 2 + \dfrac{3}{2+\dfrac{3}{x}}$ $= 2 + \dfrac{3x}{2x+3}$ $= \dfrac{2(2x+3)+3x}{2x+3}$ $= \dfrac{7x+6}{2x+3}$	It is not sufficient to state $R_f \subseteq D_f$. $D_f = (0, \infty)$, $R_f = (2, \infty)$ must be stated to justify the subset.
(b)	Given $h(x) = \dfrac{3-x}{x^2-1}$ To find the range of g, the graph must intersect the horizontal line $y = k$, therefore, $D \geq 0$ Let $k = \dfrac{3-x}{x^2-1}$ $kx^2 - k = 3 - x$ $kx^2 + x - (k+3) = 0$ $(1)^2 + 4k(k+3) \geq 0$ $1 + 12k + 4k^2 \geq 0$ $4k^2 + 12k + 1 \geq 0$ Consider $4k^2 + 12k + 1 = 0$, $k = \dfrac{-12 \pm \sqrt{12^2 - 4(4)(1)}}{2(4)}$ $= \dfrac{-12 \pm \sqrt{128}}{8}$ $= \dfrac{-3 \pm 2\sqrt{2}}{2} = -\dfrac{3}{2} \pm \sqrt{2}$	This question required an algebraic approach so the GC cannot be used to obtain the graph. It is very tedious to sketch the graph without using a GC as the stationary points would have to be found by differentiation. Students would also need to show the nature of the stationary points before they can sketch the graph on their own. Moreover, it would take some time to find the exact y coordinates of the stationary points in order to obtain the range of h. Thus, students are strongly encouraged to use the discriminant instead (see solution)

	$4k^2+12k+1\geq 0$ $\therefore\ k\leq -\frac{3}{2}-\sqrt{2}\ \ OR\ \ k\geq -\frac{3}{2}+\sqrt{2}$  Hence, the range of h is $\left(-\infty,-\frac{3}{2}-\sqrt{2}\right]\cup\left[-\frac{3}{2}+\sqrt{2},\infty\right)$	
10(i)	Concentration of the brine entering the tank is $\frac{C}{1000}$ kg/L Solution leaving the tank is $\frac{x}{2000}$ kg/L. The rate at which salt enters the tank is $\frac{5C}{1000}=\frac{C}{200}$ kg/min The rate at which leaves the tank is $\frac{5x}{2000}=\frac{x}{400}$ kg/min. $\frac{\mathrm{d}x}{\mathrm{d}t}=\text{inflow rate }-\text{outflow rate}$ $\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{C}{200}-\frac{x}{400}=\frac{1}{400}(2C-x)$ (Shown) When $\frac{\mathrm{d}x}{\mathrm{d}t}=0$, $x=120$ kg and $2C-120=0\Rightarrow C=60$ kg	
(ii)	$\int\frac{1}{120-x}\ \mathrm{d}x=\int\frac{1}{400}\ \mathrm{d}t$ $-\ln\lvert 120-x\rvert=\frac{1}{400}t+B$ where $B=\text{const}$ $120-x=Ae^{-\frac{1}{400}t}$ where $A=\text{const}$ When $t=0$, $x=20$ kg, $Ae^0=A=120-20=100$ $120-x=100e^{-\frac{1}{400}t}$ $x=20\left(6-5e^{-\frac{1}{400}t}\right)$	Remember to include a negative sign and modulus sign after integrating $\frac{1}{120-x}$.

(iii)	 Using GC, $t=204.33=204$ min.	
(iv)	It is assumed that there is no evaporation in the system so that the concentrations of the solution remain as stated/unaffected.	
11(i)	AP : first term = 1000, common difference = 250 $7500=1000+250(N-1)$ $N=27$	It is an AP with first term 1000 and common difference 250. $u_N=7500$. Solve for N.
(ii)	Total number of power banks produced in 60 weeks $=S_{27}+33(7500)$ $=\frac{27}{2}\left(1000+7500\right)+33(7500)$ =362250	For 1st to 27th week, it is an AP with first term 1000 and common difference 250. For 28th to 60th week, the production is at 7500 per week, so $(60-27)\times7500$.
(iii)	Number of power banks on demand on week 1= 50 Number of power banks on demand on week 2 = a+50b Number of power banks on demand on week 3 $= a+b(a+50b)$ $=a+ba+50b^2$	For 2nd week, the demand is $a+b(50)=a+50b$ For 3rd week, demand is $a+b(\text{demand of 2nd week})$ $=a+b(a+50b)$
(iv)	Number of power banks on demand on week 4 $= a+b(a+ba+50b^2)$ $= a+ba+b^2a+50b^3$ Number of power banks on demand on week n $=a+ba+b^2a+b^3a+\ldots+b^{n-2}a+50b^{n-1}$ $=\frac{a(b^{n-1}-1)}{b-1}+50b^{n-1}$	Continue working out for week 4 and deduce the demand of the nth week. Note that the last term of the expression is $b^{n-2}a$ for the GP. The first part of the expression is a summation of GP with first term a, common ratio b and number of terms $n-1$, which can be written as $\frac{a(b^{n-1}-1)}{b-1}$.
(v)	Total number of power banks produced in 60 weeks $=\frac{60}{2}\left(2(1000)+59L\right)$ = 30(2000+59L)	Remember to take summation of the terms from the 1st to 60th week. The total demand can be found using GC (MATH, 0: Summation).

	Total number of power banks on demand in 60 weeks $=\sum_{r=1}^{60}\left(\frac{300(1-1.05^{r-1})}{1-1.05}\right)+\sum_{r=1}^{60}\left(1.05^{r-1}(50)\right)$ = 1779181.493 For $30\left(2000+59L\right)\geq 1779181.493$ $L\geq 971.29$ Least L = 972	For total demand to be met, total production $\geq$ total demand for the first 60 weeks, solve for L.

Yishun Innova Junior College
JC2 H2 Mathematics Preliminary Examination
9758 P2 Solutions

Qn	Solution	Remarks
1	From $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $y^2=b^2\left(1-\frac{x^2}{a^2}\right)$ Volume of the cylinder is $V=\pi y^2(2x)=2\pi xy^2$ $V=2\pi b^2x\left(1-\frac{x^2}{a^2}\right)$ $=2\pi b^2x-\frac{2\pi b^2}{a^2}x^3$ $\frac{\mathrm{d}V}{\mathrm{d}x}=2\pi b^2-\frac{6\pi b^2}{a^2}x^2$ $2\pi b^2-\frac{6\pi b^2}{a^2}x^2=0$ $1-\frac{3}{a^2}x^2=0$ $x^2=\frac{a^2}{3}$ $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=-\frac{12\pi b^2x}{a^2}$ which is negative for all $x>0$ i.e. V is maximum for $x=\frac{a}{\sqrt{3}}$ Therefore max. $V=\frac{2\pi ab^2}{\sqrt{3}}-\frac{2\pi ab^2}{3\sqrt{3}}$, i.e. $V_{\max}=\frac{4\pi ab^2}{3\sqrt{3}}$ Required ratio is $\frac{4\pi ab^2}{3\sqrt{3}}:\frac{4\pi ab^2}{3}\Rightarrow 1:\sqrt{3}$	You need to justify clearly why $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}$ is negative. Remember to leave your answer as ratio, not fraction (ie. $\frac{1}{\sqrt{3}}$)
2(i)	$\int 2\sin(k+1)x\sin(kx)\ \mathrm{d}x$ $=\int -\cos(2k+1)x+\cos x\ \mathrm{d}x$ $=-\frac{1}{2k+1}\sin(2k+1)x+\sin x+C$	Use the reverse of Factor formula. From MF26. $\cos P-\cos Q$ $=-2\sin\underbrace{\tfrac{1}{2}(P+Q)}_{A}\sin\underbrace{\tfrac{1}{2}(P-Q)}_{B}$ $P=A+B$ $Q=A-B$ $\sin(k+1)x=\sin((k+1)x)$ Not equal to $x\sin(k+1)$

(ii)	$\int\left(\sin(k+1)x-\sin kx\right)^2\ \mathrm{d}x$ $=\int \sin^2(k+1)x+\sin^2 kx-2\sin(k+1)x\sin kx\ \mathrm{d}x$ $=\int \frac{1}{2}\left(1-\cos 2(k+1)x\right)+\frac{1}{2}\left(1-\cos 2kx\right)-$ $2\sin(k+1)x\sin kx\ \mathrm{d}x$ $=x-\frac{1}{4(k+1)}\sin 2(k+1)x-\frac{1}{4k}\sin 2kx+$ $\frac{1}{2k+1}\sin(2k+1)x-\sin x+C$ $\int_0^{\frac{\pi}{2}}\left(\sin(k+1)x-\sin kx\right)^2\mathrm{d}x$ $=\left[\begin{array}{l}x-\frac{1}{4(k+1)}\sin 2(k+1)x-\frac{1}{4k}\sin 2kx+\\ \frac{1}{2k+1}\sin(2k+1)x-\sin x\end{array}\right]_0^{\frac{\pi}{2}}$ $=\frac{\pi}{2}-\frac{1}{4(k+1)}\sin(k+1)\pi-\frac{1}{4k}\sin k\pi+$ $\frac{1}{2k+1}\sin\frac{2k+1}{2}\pi-\sin\frac{\pi}{2}$ $=\frac{\pi}{2}+\frac{1}{2k+1}-1$ $\because \sin\frac{2k+1}{2}\pi=\sin(\frac{1}{2}\pi+k\pi)=1$ for even k	Expand $(a+b)^2=a^2+2ab+b^2$ Use double angle formula to convert $\sin^2 x$ and $\cos^2 x$ before integrating. $\sin^2 A=\frac{1}{2}\left(1-\cos 2A\right)$ Use (i) for $\int 2\sin(k+1)x\sin kx\ \mathrm{d}x$ Please note $\sin(k\pi)=0,\quad k\in\mathbb{Z}$ $\sin\left(\frac{k\pi}{2}\right)=\pm 1,$ if k is odd $\sin(\frac{1}{2}\pi+k\pi)=1$ if k is even

<table>
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<td>3(i)</td>
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Equation of l_1 : $\mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, $\lambda \in \mathbb{R}$

$$\begin{pmatrix} c \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} c-3 \\ 2 \\ 1 \end{pmatrix}$$

Shortest distance from B to line l_1

$$= \frac{\left| \begin{pmatrix} c-3 \\ 2 \\ 1 \end{pmatrix} \times \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right|}{\left| \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right|}$$

$$= \frac{\left| \begin{pmatrix} -1 \\ 2 \\ c-7 \end{pmatrix} \right|}{\sqrt{5}}$$

$$= \frac{\sqrt{1+4+(c-7)^2}}{\sqrt{5}}$$

$$\frac{\sqrt{1+4+(c-7)^2}}{\sqrt{5}} = \frac{\sqrt{205}}{5}$$

$$\sqrt{1+4+(c-7)^2} = \frac{\sqrt{1025}}{5}$$

$$5+(c-7)^2 = 41$$

$$c-7 = \pm 6$$

$$c = 1 \text{ or } c = 13$$

Alternative method (not recommended):

$l_1 : \mathbf{r} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$

Let F be the foot of perpendicular from B to l_1.

</td>
<td>

Note: The shortest distance form B to line l_1 is different from the shortest distance from B to plane p even though plane p contains the line l_1.

Shortest distance from B to line l

$$= \frac{\left| \overrightarrow{AB} \times \underset{\sim}{m} \right|}{\left| \underset{\sim}{m} \right|}$$

where A is a point on l and $\underset{\sim}{m}$ is the direction vector of line l.

</td>
</tr>
</table>

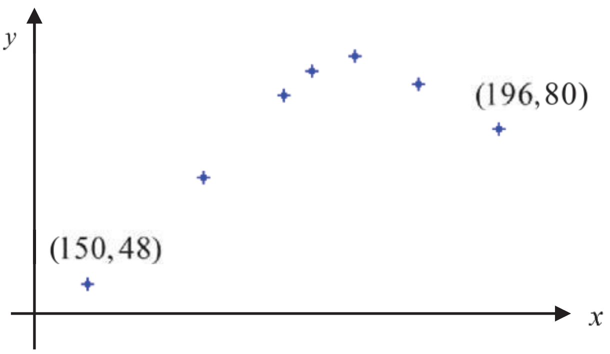
	Then $\overrightarrow{OF} = \begin{pmatrix} 3+2\lambda \\ \lambda \\ 1 \end{pmatrix}$, for some $\lambda \in \mathbb{R}$ $\overrightarrow{BF} = \begin{pmatrix} 3+2\lambda \\ \lambda \\ 1 \end{pmatrix} - \begin{pmatrix} c \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3+2\lambda-c \\ \lambda-2 \\ -1 \end{pmatrix}$ Since $\overrightarrow{BF}$ is perpendicular to l_1 $\overrightarrow{BF} \bullet \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 0$ $\begin{pmatrix} 3+2\lambda-c \\ \lambda-2 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 0$ $6+4\lambda-2c+\lambda-2=0$ $5\lambda = 2c-4$ $\lambda = \dfrac{2c-4}{5}$ $\overrightarrow{BF} = \begin{pmatrix} 3+2\left(\dfrac{2c-4}{5}\right)-c \\ \dfrac{2c-4}{5}-2 \\ -1 \end{pmatrix} = \begin{pmatrix} \dfrac{7}{5}-\dfrac{c}{5} \\ \dfrac{2c-14}{5} \\ -1 \end{pmatrix}$	

	$$\left\|\overrightarrow{BF}\right\| = \frac{\sqrt{205}}{5}$$ $$\left\|\begin{pmatrix} \frac{7}{5}-\frac{c}{5} \\ \frac{2c}{5}-\frac{14}{5} \\ -1 \end{pmatrix}\right\| = \frac{\sqrt{205}}{5}$$ $$\sqrt{\left(\frac{7}{5}-\frac{c}{5}\right)^2+\left(\frac{2c}{5}-\frac{14}{5}\right)^2+1} = \frac{\sqrt{205}}{5}$$ $$\left(\frac{7}{5}-\frac{c}{5}\right)^2+\left(\frac{2c}{5}-\frac{14}{5}\right)^2+1 = \frac{205}{25}$$ $$\frac{49}{25}-\frac{14c}{25}+\frac{c^2}{25}+\frac{4c^2}{25}-\frac{56c}{25}+\frac{196}{25}+1=\frac{205}{25}$$ $$\frac{5c^2}{25}-\frac{70c}{25}+\frac{13}{5}=0$$ Using GC, $c=1$ or $c=13$	
3(ii)	Equation of l_1 : $\mathbf{r}=\begin{pmatrix}3\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}2\\1\\0\end{pmatrix}$, $\lambda\in\mathbb{R}$ $$\begin{pmatrix}5\\-1\\2\end{pmatrix}-\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}2\\-1\\1\end{pmatrix}$$ Normal of p: $\begin{pmatrix}2\\-1\\1\end{pmatrix}\times\begin{pmatrix}2\\1\\0\end{pmatrix}=\begin{pmatrix}-1\\2\\4\end{pmatrix}$ $$p:\mathbf{r}\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}=\begin{pmatrix}3\\0\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}=1$$ Cartesian equation of p is $-x+2y+4z=1$	Note: Point B is not on plane p, so you cannot use it to find the equation of the plane. The question asked for Cartesian equation, giving the equation in dot product form will not give you the full credit.

3(iii)	Substitute equation of l_2 into the equation of p, $\left[\begin{pmatrix}1\\2\\1\end{pmatrix}+\mu\begin{pmatrix}4\\1\\3\end{pmatrix}\right]\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}=1$ $-1-4\mu+4+2\mu+4+12\mu=1$ $10\mu=-6$ $\mu=-\frac{3}{5}$ Position vector : $\begin{pmatrix}1\\2\\1\end{pmatrix}-\frac{3}{5}\begin{pmatrix}4\\1\\3\end{pmatrix}=\begin{pmatrix}-\frac{7}{5}\\\frac{7}{5}\\-\frac{4}{5}\end{pmatrix}$ Coordinates: $\left(-\frac{7}{5},\frac{7}{5},-\frac{4}{5}\right)$	Note: $\begin{pmatrix}-\frac{7}{5}\\\frac{7}{5}\\-\frac{4}{5}\end{pmatrix}\neq\left(-\frac{7}{5},\frac{7}{5},-\frac{4}{5}\right)$ The question ask for coordinates not position vector.
3(iv)	$\begin{pmatrix}4\\0\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}=-4+4=0$ Since the normal of the plane is perpendicular to the direction vector of the line, p is parallel to l_3.	You need to explain clearly why the dot product of normal vector of p and direction vector of l_3 equals to zero implies p is parallel to l_3
3(v)	l_3 and p have no point in common Point on l_3 is not on p $\begin{pmatrix}a\\2a\\a\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}\neq1$ $-a+4a+4a\neq1$ $a\neq\frac{1}{7}$	If l_3 and p have no point in common and they are parallel, then none of the points on l_3 will lie on p, hence, $\begin{pmatrix}a\\2a\\a\end{pmatrix}$ will not satisfy the equation of the plane.
3(vi)	l_3 : $\mathbf{r}=\begin{pmatrix}1\\2\\1\end{pmatrix}+t\begin{pmatrix}4\\0\\1\end{pmatrix}$, $t\in\mathbb{R}$	Distance between l_3 and p is same as the distance from a point (on l_3) to plane p, hence, you should use dot product instead of cross product.

	By observation, $(-1,0,0)$ lies on plane *p*. $\begin{pmatrix}1\\2\\1\end{pmatrix}-\begin{pmatrix}-1\\0\\0\end{pmatrix}=\begin{pmatrix}2\\2\\1\end{pmatrix}$ Distance between l_3 and *p* $=\dfrac{\left\lvert\begin{pmatrix}2\\2\\1\end{pmatrix}\bullet\begin{pmatrix}-1\\2\\4\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}-1\\2\\4\end{pmatrix}\right\rvert}$ $=\dfrac{6}{\sqrt{21}}$ $=\dfrac{2}{7}\sqrt{21}$	
4(a)	$2z^3-3z^2+kz+26=0$ Since the coefficients are all real, $z=1-a\mathrm{i}$ is another root. $(z-(1+ai))(z-(1-ai))(Az+B)$ $=\left[z^2-(1+ai)z-(1-ai)z+(1+ai)(1-ai)\right](Az+B)$ $=\left(z^2-2z+1+a^2\right)(Az+B)$ Comparing the coefficient of z^3, $A=2$ Comparing the coefficient of z^2, $B-2(2)=-3$ $B=1$ Since $2z+1$ is a factor, $2z+1=0$ The third root is $z=-\dfrac{1}{2}$. Comparing the constant term, $1\left(1+a^2\right)=26$ $a^2=25$ $a=5$ or $a=-5$ (rej as a is positive) Comparing the coefficient of z, $-2(1)+2\left(1+5^2\right)=k$ $k=50$	There is a difference between the terms "root" and "factor": If $z=1+a\mathrm{i}$ is a root, then $z-(1+a\mathrm{i})$ is a factor. It is usually much faster to solve such questions by the forming the quadratic factor, compared to doing substitution of the root into the equation given. Be careful in handling $(a\mathrm{i})^2$.

	$\therefore\ a=5,\ k=50$ and the other two roots are $1-5\text{i}$ and $-\dfrac{1}{2}$.	
(bi)	$(x+\text{i}y)^2=15+8\text{i}$ $x^2+2\text{i}xy-y^2=15+8\text{i}$ Comparing real and imaginary parts, $x^2-y^2=15$ and $2xy=8 \Rightarrow y=\dfrac{4}{x}$ $x^2-\left(\dfrac{4}{x}\right)^2=15$ $x^4-15x^2-16=0$ $\left(x^2-16\right)\left(x^2+1\right)=0$ $x^2=16$ or $x^2=-1$ (rejected as x is real) $\therefore\ x=4,\ y=1$ or $x=-4,\ y=-1$	Read carefully, the use of calculator is prohibited for the whole of **Q4**, not just **(a)**. So a calculator, and of course a GC, cannot be used to say, finding the roots of the quartic equation. Similarly, to avoid suspicion that a calculator is used, either the factorisation or quadratic formula working must be shown.
(bii)	$z^2-(2+7\text{i})z=15-5\text{i}$ $z^2-(2+7\text{i})z-(15-5\text{i})=0$ $z=\dfrac{(2+7\text{i})\pm\sqrt{(2+7\text{i})^2-4(1)(-15+5\text{i})}}{2}$ $=\dfrac{(2+7\text{i})\pm\sqrt{4+28\text{i}-49+60-20\text{i}}}{2}$ $=\dfrac{(2+7\text{i})\pm\sqrt{15+8\text{i}}}{2}$ $=\dfrac{(2+7\text{i})\pm(4+\text{i})}{2}$ from (bi) $z=\dfrac{6+8\text{i}}{2}$ or $z=\dfrac{-2+6\text{i}}{2}$ $=3+4\text{i}$ $=-1+3\text{i}$ Since $\arg(3+4\text{i})<\arg(-1+3\text{i})$, $z_1=3+4\text{i}$ and $z_2=-1+3\text{i}$ $\lvert z_2\rvert=\sqrt{1^2+3^2}=\sqrt{10}$ $\arg(z_2)=\pi-\tan^{-1}\left(\dfrac{3}{1}\right)=\pi-\tan^{-1}3$	We cannot compare the real and imaginary parts immediately to get 2 equations to solve for z. This is because z is not guaranteed to be a real number. Just take for example: $w=2+\text{i}$, so $2w-4=2\text{i}$. It will not make sense to "compare real parts" to conclude $2w-4=0$, so $w=2$. Usually such quadratic equations are solved using quadratic formula. Similar to part **(i)**, calculator cannot be used, so all workings must be shown. As the question number is **(ii)**, the working should have some relation with **(i)**: we see that $\pm\sqrt{15+8\text{i}}$ was actually evaluated in **(i)**. Again, the calculator cannot be used, so the argument should be left as $\pi-\tan^{-1}3$, rather than evaluated to 3sf.

	$\therefore\ z_2=\sqrt{10}\mathrm{e}^{\mathrm{i}\left(\pi-\tan^{-1}3\right)}$	
biii)	$\arg\left(z_1^{\,2}z_2^{\,*}\right)=2\arg\left(z_1\right)+\arg\left(z_2^{\,*}\right)$ $=2\arg\left(z_1\right)-\arg\left(z_2\right)$ $=2\tan^{-1}\left(\frac{4}{3}\right)-\pi+\tan^{-1}3$	
5(i)	 Model B is the more accurate model. As x increases, y first increases then decreases, indicating that a quadratic model is more appropriate.	Do a quick check of the GC entries. Use ruler for axes and label axes correctly. Indicate the correct number of data points with '+' or '×'. Spacing of the data points must be appropriate and their relative positions must be correct. Label coordinates of at least two data points instead of putting numbers at the axes. Can use r-value to explain as well, but take note that we want the model which $\lvert r\rvert$ closest to 1, not r closest to -1.
5(ii)	$y=-0.050522\left(x-180\right)^2+91.568$ $=-0.0505\left(x-180\right)^2+91.6$ (3 s.f.) $r=-0.978$ (3 s.f.)	Ensure that GC is appropriately set to display r. Note that $-1\le r\le 1$ Answers should be given to 3 sf correctly.
5(iii)	$y=-0.050522\left(185-180\right)^2+91.568$ $=90.3$ (3 s.f.) The estimate is reliable as $x=185$ is within data range of x and r value is close to -1, indicating a strong negative linear correlation between y and $\left(x-180\right)^2$.	Use the more accurate equation instead of the 3 sf. It is the given x value that is within the range of the given x values, not the estimated value. 2 reasons for concluding estimate is reliable.
6(ai)	Let X be the number of packets with a toy, in a carton of 12. $X\sim \mathrm{B}\left(12,\frac{2}{7}\right)$ $\mathrm{P}(X<2)=\mathrm{P}(X\le 1)$	Fewer than 2 means less than or equal to 1 Hence we convert $\mathrm{P}(X<2)$ to $\mathrm{P}(X\le 1)$ use binomcdf

	$=0.10230$ $=0.102$ (3s.f)	
(aii)	$\mathrm{P}(1<X\le 6)=\mathrm{P}(X\le 6)-\mathrm{P}(X\le 1)$ $=0.868$	$\mathrm{P}(1<X\le 6)$ $=\mathrm{P}(X\le 6)-\mathrm{P}(X\le 1)$ This is a standard conversion to use binomcdf in calculator
(b)	Required probability $=\left(\mathrm{P}(X=4)\right)^2\times(\mathrm{P}(X=2))^3\times\dfrac{5!}{2!3!}$ $=\left(0.223516\right)^2\times\left(0.18626\right)^3\times\dfrac{5!}{2!3!}$ $=0.00323$ (3s.f.)	Since 2 cartons contain 4 toys and 3 cartons contain 2 toys, $\mathrm{P}(X=4)\times\mathrm{P}(X=4)\times\mathrm{P}(X=2)$ $\times\mathrm{P}(X=2)\times\mathrm{P}(X=2)\times\dfrac{5!}{2!3!}$ The $\dfrac{5!}{2!3!}$ or ${}^5C_3$ is the number of cases with 2 cartons containing 4 toys and 3 cartons containing 2 toys.
(c)	Let Y be the number of cartons out of 40 cartons that contains less than 2 toys. $Y\sim\mathrm{B}\left(40,0.1023\right)$ $\mathrm{P}(Y=0)=0.0133$ **Alternative method** Required probability $=\left(1-0.10230\right)^{40}=0.0133$	The probability of success is a carton containing fewer than 2 toys which is the probability in (i). Since there are 40 cartons and we want none to contain fewer than 2 toys, $Y\sim\mathrm{B}\left(40,0.1023\right)$ and we find $\mathrm{P}(Y=0)$
	$\mathrm{P}(A=2\text{ or }3)=0.25$ $\binom{25}{2}p^2(1-p)^{23}+\binom{25}{3}p^3(1-p)^{22}=0.25$ $300\,p^2(1-p)^{23}+2300\,p^3(1-p)^{22}=0.25$ From GC, $p=0.186$ or $p=0.0403$ (reject since $p>0.1$)	$\mathrm{P}(A=2\text{ or }3)$ refers to $\mathrm{P}(A=2)+\mathrm{P}(A=3)$ By formula in MF 26, $\binom{25}{2}p^2(1-p)^{23}+$ $\binom{25}{3}p^3(1-p)^{22}=0.25$
7(a)	By listing all possible outcomes of 3 numbers: 135,136,138,156,158,168,356,358,368,568. Sum: 9, 10, 12, 12, 14 15 14 16 17 19 P(sum of 3 numbers drawn is even) = $\dfrac{6}{10}=\dfrac{3}{5}=0.6$ **Alternative** P(sum of 3 numbers drawn is even) = $\dfrac{{}^3C_2\times 2}{{}^5C_3}=\dfrac{3}{5}=0.6$	Avoid using possibility diagram here unless you are able to depict 3-D table.

(bi)	S = smallest of the 3 numbers drawn <table><tr><td>s</td><td>1</td><td>3</td><td>5</td></tr><tr><td>$P(S=s)$</td><td>0.6</td><td>0.3</td><td>0.1</td></tr></table>	Read question carefully. It's not about sum of the three numbers but the minimum of them. Check that the sum of all the probabilities is ONE.
(bii)	Using g.c., E(S) = 2, Var(S) =1.8 or $E(S)=\sum_s sP(S=s)$ $=1(0.6)+3(0.3)+5(0.1)$ $=2$ $\text{Var}(S)=E(S^2)-[E(S)]^2$ $=\sum_s s^2P(S=s)-(2)^2$ $=1(0.6)+9(0.3)+25(0.1)-4$ $=1.8$	
(biii)	Since sample size = 55 is large, by Central Limit Theorem, $\overline{S}=\frac{S_1+S_2+S_3+...+S_{55}}{55}\sim N\left(2,\frac{1.8}{55}\right)$ approx $P\left(\left\lvert\overline{S}-E(S)\right\rvert\le 0.5\right)=P\left(-0.5\le\overline{S}-2\le 0.5\right)$ $=P\left(1.5\le\overline{S}\le 2.5\right)$ $=0.99429$ $=0.994$ (3 sig. fig.)	The rv S does not follow a normal distribution. It's $\overline{S}$ that follows normal distribution approximately when the the sample size if sufficiently large.
8(ai)	Since A and B are independent, $P(A)=P(A\mid B)=0.4$ and $P(A\cap B)=0.4P(B)$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $0.6=0.4+P(B)-0.4P(B)$ $0.6P(B)=0.2$ $P(B)=\frac{1}{3}$	For any events A and B, if the probability of occurrence of one of them is <u>not</u> influenced by the occurrence of the other, then events A and B are <u>**independent**</u> and (i)$P(A)=P(A\mid B)$, (ii)$P(B\mid A)=P(B)$, (iii)$P(A\cap B)=P(A)\times P(B)$. Note that if A and B are <u>**mutually exclusive**</u>, then $P(A\cap B)=0$.
(aii)	Since A and B are independent, A' and B' are also independent, hence $P(A'\mid B')=P(A')=1-P(A)=0.6$ <u>Alternatively</u>	If events A and B are independent, then {A' and B}, {A and B'}, {A' and B'} are also independent.

	$P(A' \mid B') = \frac{P(A' \cap B')}{P(B')}$ $= \frac{1-P(A \cup B)}{1-P(B)}$ $= \frac{1-0.6}{1-\frac{1}{3}} = 0.6$	
(bi)	Case 1: 3, 3, 6 people no. of ways = ${}^{12}C_3 \times {}^{9}C_3 \times (3-1)! \times (3-1)! \times (6-1)! \times 3 = 26611200$ Case 2: 3, 4, 5 people no. of ways = ${}^{12}C_3 \times {}^{9}C_4 \times (3-1)! \times (4-1)! \times (5-1)! \times 3! = 47900160$ Case 3: 4, 4, 4 people no. of ways = ${}^{12}C_4 \times {}^{8}C_4 \times (4-1)! \times (4-1)! \times (4-1)! = 7484400$ Total no of ways =26611200+47900160+7484400 = 81995760	You need to consider circular permutations at each round table after you have considered number of selections to form the three groups of people in each case.
(bii)	Required Probability $P(4 \text{ people at each table} \mid \geq 3\text{people at each table})$ $= \frac{P(4 \text{ people at each table} \cap \geq 3\text{people at each table})}{P(\geq 3\text{people at each table})}$ $= \frac{P(4 \text{ people at each table})}{P(\geq 3\text{people at each table})}$ $= \frac{7484400}{81995760} = \frac{45}{493} = 0.0913 (3 \text{ s.f.})$	Note that $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$. The event A:"4 people at each table" is a subset of the event B:"$\geq$ 3people at each table" Hence, $A \subseteq B \Rightarrow A \cap B = A$
9(i)	$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{4}\right)$ $\mu = \frac{36.1+49.1}{2} = 42.6$ $P(\bar{X} < 36.1) = 0.03355$ $P\left(Z < \frac{36.1-42.6}{\sigma/2}\right) = 0.03355$ $-\frac{13}{\sigma} = -1.8310$ $\sigma = 7.0999$ ≈ 7.10 (2 d.p.) (shown)	You need to state the Distribution of $\bar{X}$. There seems to be confusion in writing out the standardization correctly, with many writing σ instead of $\frac{\sigma}{2}$. Standardising the sample mean should be $Z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ in this case.
9(ii)	Let X be the life, in months, of a car battery of the regular type. $X \sim N(42.6, 7.10^2)$ $P(X < k) > 0.9$ $k > 51.699$ $\therefore$ Smallest integer value of $k = 52$	While most of you are able to understand the probability that the question needs, however errors usually occur during calculation and/or lack of working to support your answers.

	Alternative $P(X<k)>0.9$ $k=51: P(X<k)=0.882<0.9$ $k=52: P(X<k)=0.907>0.9$ $\therefore$ Smallest integer value of $k=52$	
9(iii)	$P(X<36)=0.17629$ Expected number of car batteries which are due to bad driving habits $=0.25(100)P(X<36)$ $=4.41$ (3 s.f.)	In the question, 25% refers to the drivers with bad driving habits, not to the car batteries with lives less than 36 months. Need to consider the P(car battery with life less than 36 months), i.e $P(X<36)$
9(iv)	Let Y be the life, in months, of a car battery of the premium type. $Y=1.1X$ $Y\sim N(1.1(42.6),\ 1.1^2\ (7.10^2))$ $Y\sim N(46.86,\ 60.9961)$ Let $A=Y_1+Y_2+\ldots+Y_5-(X_1+X_2+\ldots+X_6)$ $A\sim N(5(46.86)-6(42.6),\ 5(60.9961)+6(7.1)^2)$ $A\sim N(-21.3,\ 607.4405)$ $P(A>0)=0.194$	There were errors in the calculation of variance (eg should be $(1.1^2(7.1^2))$ instead of $(1.1^2(7.1))$.
10(i)	An unbiased estimate of μ is $\bar{x}=14.2$ An unbiased estimate of σ^2 is $s^2=7.0871$ $=7.09$	Use GC.
10ii	μ is the population mean time taken to assemble an electrical component. H_0 : $\mu=15$ (results of study) H_1 : $\mu<15$ Under H_0, the test statistic is $Z=\dfrac{\overline{X}-\mu}{S/\sqrt{n}}\sim N(0,1)$ approximately (by CLT), where $\mu=15$, $s^2=7.0871$, $n=40$, $\bar{x}=14.2$. From GC, p-value = 0.0287	Question requires you to define any symbols used. You need to define μ. The result of study is "at least 15 mins" ie $\mu\geq 15$. Hence, we need to test whether there is sufficient evidence that $\mu<15$ (H_1).

	Since the p-value = 0.0287 $\leq$ 0.05, we reject H_0 and conclude that at the 5% level, there is sufficient evidence that the results of the study is not valid.	
10iii	Since sample size is large, by Central Limit Theorem, the sample mean time taken to assemble an electrical component will be approximately normal.	Need to mention sample size is large and CLT. Note that CLT results in the **sample mean time** ($\overline{X}$)approximately Normal. It does not result in the distribution of the time taken to assemble an electrical component (X) Normal.
10iv	"5% level of significance" means that there is a probability of 0.05 of **concluding** that the results of the study is not valid when the mean time taken to assemble an electrical component is indeed 15 minutes.	α % level of significance =P (reject H_0 | H_0 is true) Need to conclude in terms of H_1 (reject H_0)
	H_0 : $\mu = 30$ (claim) H_1 : $\mu \neq 30$ Under H_0 , the test statistic is $Z = \dfrac{\overline{X} - \mu}{S/\sqrt{n}} \sim N(0, 1)$ approximately (by CLT), where $\mu = 30$, $s^2 = \dfrac{50}{49}k^2$, $n = 50$, $\overline{x} = 29.7$. Critical Regions: $z \geq 1.750686$ or $z \leq -1.750686$ Given that H_0 is not rejected $-1.750686 < \dfrac{29.7 - 30}{k/7} < 1.750686$ $-0.250098k < -0.3$ and $-0.3 < 0.250098k$ $k > 1.19953$ $\quad -1.19953 < k$ $\therefore k > 1.20$ (3 s.f)	The manager claims that the mean is 30 so H_1 is $\mu \neq 30$. It doesn't mean that when sample mean is 29.7 <30 means H_1 is always left-tailed (<). Read the question carefully. sample S.D is given. It is neither unbiased estimate of population variance (s^2) nor population variance (σ^2). $s^2 = \dfrac{n}{n-1} \times$ sample variance sufficient evidence that the manager's claim is valid $\Rightarrow$ H_0 is not rejected $\Rightarrow \dfrac{29.7 - 30}{k/7}$ is outside the critical regions. Note that it is "and" not "or". $k > 1.19953$ and $k > -1.19953$ $\Rightarrow k > 1.19953$