2020 Y3IP Core Mathematics Final Exam P1 Worked Solutions
(a) $\left(\sqrt{2}-\sqrt{10002}\right)\left(\sqrt{10002}+\sqrt{2}\right)$ $=\left(\sqrt{2}-\sqrt{10002}\right)\left(\sqrt{2}+\sqrt{10002}\right)$ $=2-10002$ By a^2-b^2 $=-10000$ (b) $\frac{a-b}{a+b}=2$ $a-b=2a+2b$ $-3b=a$ $\frac{a}{b}=-3$
(a) $\frac{2}{m+4}+\frac{3}{m-2}$ $=\frac{2(m-2)+3(m+4)}{(m+4)(m-2)}$ $=\frac{2m-4+3m+12}{(m+4)(m-2)}$ $=\frac{5m+8}{(m+4)(m-2)}$ (b) $x=\frac{y+z}{y-z}$ $x(y-z)=y+z$ $xy-xz=y+z$ $xy-y=z+xz$ $y(x-1)=z(x+1)$ $y=\frac{z(x+1)}{x-1}$
(a) $\left(\frac{x^2-3x}{4x}\right)\left[\frac{5(x+1)}{x-3}\right]$ $=\left(\frac{x(x-3)}{4x}\right)\left[\frac{5(x+1)}{x-3}\right]$ $=\left(\frac{1}{4}\right)\left[\frac{5(x+1)}{1}\right]$ $=\frac{5}{4}(x+1)$

(b) (i) $0 \le \left(\frac{x^2 - 3x}{4x}\right)\left[\frac{5(x+1)}{x-3}\right] < 5$

$0 \le \frac{5}{4}(x+1) < 5$

$0 \le \frac{5}{4}x + \frac{5}{4} < 5$

$-\frac{5}{4} \le \frac{5}{4}x < \frac{15}{4}$

$-1 \le x < 3$

(ii) 2

$(3-\sqrt{5})(\sqrt{5}+1)$

$= 3\sqrt{5} + 3 - 5 - \sqrt{5}$

$= 2\sqrt{5} - 2$

$h(2\sqrt{5} - 2) = 48 - 16\sqrt{5}$

$h = \frac{24 - 8\sqrt{5}}{\sqrt{5} - 1}$

$h = \frac{4(6 - 2\sqrt{5})(\sqrt{5}+1)}{4}$

$h = 6\sqrt{5} - 10 + 6 - 2\sqrt{5}$

$h = 4(\sqrt{5} - 1)$

Alternatively,

$h = \frac{16(3-\sqrt{5})}{\left(3-\sqrt{5}\right)\left(\sqrt{5}+1\right)}$

$h = 4(\sqrt{5} - 1)$

(a) $-x^2 - 6x + 15$

$= -(x^2 + 6x) + 15$

$= -[(x+3)^2 - 9] + 15$

$= -(x+3)^2 + 24$

(b) $(-3, 24)$

(c) $-x^2 - 6x + 15 = 0$ (Method 1 – Complete Square)

$-(x+3)^2 + 24 = 0$

$(x+3)^2 = 24$

$x = \pm 2\sqrt{6} - 3$

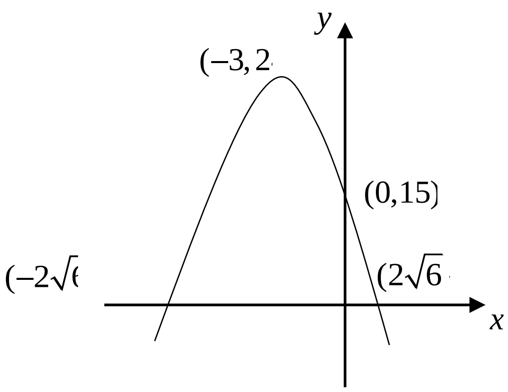
$-x^2 - 6x + 15 = 0$ (Method 2 – Quad. Formula)

$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(15)}}{2(-1)}$

$x = \frac{6 \pm \sqrt{96}}{-2}$

$x = -3 \pm 2\sqrt{6}$

(d)



(a) $7^{x+3} = 1^x$

$7^{x+3} = 1$

$x + 3 = 0$

$x = -3$

(b) $e^{x+2} = 2^{x-1}$

$\ln(e^{x+2}) = \ln(2^{x-1})$

$x + 2 = (x-1)\ln 2$

$x + 2 = x\ln 2 - \ln 2$

$x - x\ln 2 = -2 - \ln 2$

$x(1 - \ln 2) = -2 - \ln 2$

$x = \dfrac{-2 - \ln 2}{1 - \ln 2}$

Alternatively,

$x = \dfrac{-\log_a 2 - 2\log_a e}{\log_a e - \log_a 2}$

(c) $3^{x^2} - 18(3^{-x^2}) = 7$

$3^{x^2} - 18\left(\dfrac{1}{3^{x^2}}\right) = 7$

Let $y = 3^{x^2}$

$y^2 - 7y - 18 = 0$

$(y+2)(y-9) = 0$

$y = -2, y = 9$

$3^{x^2} = -2(rej), 3^{x^2} = 9$

$3^{x^2} = 3^2$

$x^2 = 2$

$x = \pm\sqrt{2}$

(a) (i) By Pythagoras' Theorem,

$TB^2 = 5^2 + 12^2$

$TB = \sqrt{169}$

$TB = 13$

(ii) By Pythagoras' Theorem,

$TF^2 = TB^2 + 5^2$

$TF^2 = 194$

$TF = \sqrt{194}$

(b) (i) $\tan\measuredangle FTB = \dfrac{FB}{TB}$

$\tan\measuredangle FTB = \dfrac{5}{13}$

(ii) $\cos \measuredangle FTD$

$= -\cos \measuredangle FTB$

$= -\dfrac{TB}{TF}$

$= -\dfrac{13}{\sqrt{194}}$

(a) $AB = \sqrt{(6-(-2))^2 + (8-2)^2}$

$AB = 10$

Since AD is parallel to vertical axis,

$D = (-2, -8)$

(b) Midpoint of BD = $\left(\dfrac{6+(-2)}{2}, \dfrac{8+(-8)}{2}\right)$

$= (2, 0)$

Gradient of BD = $\dfrac{8-(-8)}{6-(-2)}$

$= 2$

Gradient of $\perp = -\dfrac{1}{2}$

Eqn of $\perp$ Bisector: $y = -\dfrac{1}{2}x + c$

Sub $x = 2, y = 0$

$0 = -\dfrac{1}{2}(2) + c$

$c = 1$

$\therefore y = -\dfrac{1}{2}x + 1$

(c) Since DC is parallel to x axis, sub $y = -8$ into

$y = -\dfrac{1}{2}x + 1$,

$-8 = -\dfrac{1}{2}x + 1$

$-9 = -\dfrac{1}{2}x$

$x = 18$

$\therefore C = (18, -8)$

(d) $\frac{1}{2}\begin{vmatrix} -2 & 6 & 18 & -2 \\ 2 & 8 & -8 & 2 \end{vmatrix}$ $= \frac{1}{2}\lvert(12+144+16)-(-16-48+36)\rvert$ $= \frac{1}{2}\lvert 200 \rvert$ $= 100 units^2$ Alternatively, Area = $\frac{1}{2}(10)(20)$ $= 100 units^2$
(a) $y = x^2 - kx + 3 - k$ Considering the discriminant >0, $(-k)^2 - 4(1)(3-k) > 0$ $k^2 + 4k - 12 > 0$ $(k+6)(k-2) > 0$ $k < -6, k > 2$ (b) $p + q = 4$ $pq = 5$ For the new equation, $kp + kq$ $= k(p+q)$ (sum of roots) $= 4k$ $(kp)(kq)$ $= k^2 pq$ (product of roots) $= 5k^2$ Hence, quadratic equation is $x^2 - 4kx + 5k^2 = 0$
(a) $6\log_3(p-5) = 12$ $\log_3(p-5) = 2$ $p - 5 = 3^2$ $p - 5 = 9$ $p = 14$ (b) $\log_9 x + \log_x 81 = 3$

$$\log_9 x + \frac{\log_9 81}{\log_9 x} = 3$$

$$y + \frac{2}{y} = 3$$

$$y^2 - 3y + 2 = 0$$

$$(y-2)(y-1) = 0$$

$$y = 2, y = 1$$

$$\log_9 x = 2, \log_9 x = 1$$

$$x = 81, x = 9$$

(c) $\log_4 (x-3)^2 + 2\log_2 \sqrt{x+1} = \log_2 (3-x)$

For a solution to exist, $3 - x > 0$

$$\log_4 (x-3)^2 + 2\log_2 \sqrt{x+1} = \log_2 (3-x)$$

$$\log_4 (3-x)^2 + \log_2 (x+1) = \log_2 (3-x)$$

$$\log_2 \left(\frac{(3-x)(x+1)}{3-x} \right) = 0$$

$$\frac{(3-x)(x+1)}{(3-x)} = 1$$

$$x \neq 3,$$

$$x + 1 = 1$$

$$x = 0$$