2022 PHSS Prelim AMATH Paper 1 Solutions

Question	Solution
1	Sum of length of the 2 parallel sides $=\dfrac{2\left(12+5\sqrt{2}\right)}{32-18\sqrt{2}}$ $=\dfrac{24+10\sqrt{2}}{32-18\sqrt{2}}\times\dfrac{32+18\sqrt{2}}{32+18\sqrt{2}}$ $=\dfrac{768+432\sqrt{2}+320\sqrt{2}+360}{1024-648}$ $=\dfrac{1128+752\sqrt{2}}{376}$ $=\left(3+2\sqrt{2}\right)$ cm
2	$3y-x+1=0$ ---------- (1) $x+1=\dfrac{1}{y}$ ---------- (2) From (1), $x=3y+1$ ---------- (3) Subst. (3) into (2), $3y+1+1=\dfrac{1}{y}$ $3y^2+2y-1=0$ $y=\dfrac{1}{3}$ or -1 $\therefore$ *y*-coordinates of *A* and *B* are $\dfrac{1}{3}$ and -1.
3i	$-\dfrac{1}{4}x^2-2x+5$ $=-\dfrac{1}{4}\left(x^2+8x\right)+5$ $=-\dfrac{1}{4}\left(x^2+8x+(4)^2-(4)^2\right)+5$ $=-\dfrac{1}{4}(x+4)^2+9$

Qn	Solution
3ii	stationary value of $y = 9$ corresponding value of $x = -4$
4	$\int\left(\frac{9}{(3x-2)^4}+\frac{4x}{4x^2-5x}\right)dx$ $=\int\left(9(3x-2)^{-4}+\frac{4}{4x-5}\right)dx$ $=\frac{9(3x-2)^{-3}}{(3)(-3)}+\frac{4\ln(4x-5)}{4}+c$ $=-\frac{1}{(3x-2)^3}+\ln(4x-5)+c$
5	$\frac{15x^3+19x^2+116x-6}{(5x-1)(x^2+9)}$ $=3+\frac{22x^2-19x+21}{(5x-1)(x^2+9)}$ $\frac{22x^2-19x+21}{(5x-1)(x^2+9)}=\frac{A}{5x-1}+\frac{Bx+C}{x^2+9}$ $22x^2-19x+21=A(x^2+9)+(Bx+C)(5x-1)$ Subst $x=\frac{1}{5}$, $\frac{452}{25}=\frac{226}{25}A$ $A = 2$ Subst $x = 0$, $21 = 18-C$ $C = -3$ Subst. $x = 1$, $24 = 20 + (B-3)(4)$ $B = 4$ $\frac{15x^3+19x^2+116x-6}{(5x-1)(x^2+9)}=3+\frac{2}{5x-1}+\frac{4x-3}{x^2+9}$

Qn	Solution
6a	By remainder thm, subst. $x = -2$, $2(-2)^3 + 3(-2)^2 - 2m + 30 = 36$ $m = -5$
6b	$2x^3 + 3x^2 - 29x + 30 = (x^2 + 3x + q)(Ax + B)$ Compare coeff. of x^3: $A = 2$ Compare coeff. of x^2: $B = -3$ Compare coeff. of x: $q = -10$ $2x^3 + 3x^2 - 29x + 30 = (x^2 + 3x - 10)(2x - 3)$ $= (x+5)(x-2)(2x-3)$ **Alternative Solution:** $2x - 3$ $x^2 + 3x + q \;\big)\; 2x^3 + 3x^2 - 29x + 30$ $-2x^3 - 6x^2 - 2qx$ $-3x^2 - 29x - 2qx + 30$ $+3x^2 + 9x + 3q$ $-20x - 2qx + 3q + 30$ Since $x^2 + 3x + q$ is a factor of $2x^3 + 3x^2 - 29x + 30$, $-20x - 2qx + 3q + 30 = 0$ $-20 - 2q = 0$ and $3q + 30 = 0$ $\therefore\ q = -10$ $2x^3 + 3x^2 - 29x + 30 = (x^2 + 3x - 10)(2x - 3)$ $= (x+5)(x-2)(2x-3)$

Qn	Solution
7a	Period $= \frac{10\pi - 2\pi}{2}$ $= 4\pi$ p $= \frac{3}{2} \times 4\pi + 2\pi$ $= 8\pi$ **Alternative Solution:** Midpoint of *AC* $= \left(\frac{2\pi + 10\pi}{2},\ 9.5 \right)$ $= (6\pi,\ 9.5)$ P $= \frac{6\pi + 10\pi}{2}$ $= 8\pi$
7b	amplitude $= \frac{9.5 - 0.5}{2}$ $= 4.5$ $2\pi b = 4\pi$ $b = 2$ c $= \frac{9.5 + 0.5}{2}$ $= 5$ $\therefore$ equation of the curve is $y = -4.5\cos\frac{x}{2} + 5$.
8a	Since ΔCQR is similar to ΔCAB, $\frac{QR}{50} = \frac{80 - x^2}{80}$ $QR = \left(50 - \frac{5}{8}x^2 \right)$ m $A = x^2 \left(50 - \frac{5}{8}x^2 \right)$ $= \left(50x^2 - \frac{5}{8}x^4 \right)$ m^2 (shown)

Qn	Solution
8b	$\frac{dA}{dx}=100x-\frac{5}{2}x^3$ For stationary value, $\frac{dA}{dx}=0$ $100x-\frac{5}{2}x^3=0$ $\frac{5}{2}x\left(40-x^2\right)=0$ $x=0$ (rej.), 6.3245 or -6.3245 (rej.) $\therefore$ stationary value of A = 1000 m^2 (3 sf) $\frac{d^2A}{dx^2}=100-\frac{15}{2}x^2$ When $x=6.3245$, $\frac{d^2A}{dx^2}=-199.99<0$ $\therefore$ the stationary value of A is a maximum.
9i	gradient of PQ = tan 135° = –1 (shown)
9ii	Subst. $x = 2$, $y = 0$ into $y = -x + c$ $c = 2$ $\therefore$ coordinates of P is (0, 2). Let coordinate of Q be (a, b). $\left(\frac{a}{2},\ \frac{b+2}{2}\right)=(2,\ 0)$ $\frac{a}{2}=2$ and $\frac{b+2}{2}=0$ $a = 4$, $b = -2$ $\therefore$ coordinates of Q is (4, –2).

Qn	Solution
9iii	Since $PM \perp MR$, gradient of $MR = 1$ $\frac{r-0}{-3-2}=1$ $r = -5$
9iv	area of ΔPQR $=\frac{1}{2}\times\begin{vmatrix}0 & -3 & 4 & 0\\ 2 & -5 & -2 & 2\end{vmatrix}$ $=\frac{1}{2}(6+8+6+20)$ $= 20$ units^2
10a	LHS $=\frac{\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}+1}+\frac{\cos\theta}{\sin\theta}$ $=\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}$ $=\frac{\sin^2\theta+\cos\theta+\cos^2\theta}{\sin\theta(1+\cos\theta)}$ $=\frac{1+\cos\theta}{\sin\theta(1+\cos\theta)}$ $=\frac{1}{\sin\theta}$ $= \text{cosec }\theta$ =RHS (proved)
10b	$\frac{\tan 2\theta}{\sec 2\theta+1}+\cot 2\theta=-4$ cosec $2\theta = -4$ $\sin 2\theta = -\frac{1}{4}$ basic $\angle = 14.477°$ $2\theta = 194.477°, 345.523°$ $\theta = 97.2°, 172.8°$ (1 d.p)

Qn	Solution
11a	LHS $= \angle QRT$ $= \angle SQP$ ($\angle$s in alt. segment) $= \angle QSR$ (alt. $\angle$s; PQ // SR) = RHS (shown)
11b	$RTS = \angle PTQ$ (vert. opp. $\angle$s) $\angle RST = \angle PQT$ (alt. $\angle$s; PQ // SR) $\therefore$ ΔRTS is similar to ΔPTQ (AA similarity test) $\frac{PQ}{RS} = \frac{PT}{RT}$ $\frac{PQ}{RS} = \frac{PT}{\frac{1}{3}PR}$ $\frac{PQ}{RS} = \frac{3PT}{PR}$ $PQ \times PR = RS \times 3\ PT$
12a	$4^{3x} \times 9^{x} = 2^{4x+3} \times 3^{x-2}$ $2^{6x} \times 3^{2x} = 2^{4x} \times 2^{3} \times \frac{3^{x}}{3^{2}}$ $2^{2x} \times 3^{x} = \frac{8}{9}$ $12^{x} = \frac{8}{9}$ $x \ln 12 = \ln \frac{8}{9}$ $x = -0.0474$ (3sf)
12b	$\log_3(6-3x) - \log_{\sqrt{3}}(2-x) = 0$ $\log_3(6-3x) - \frac{\log_3(2-x)}{\log_3 \sqrt{3}} = 0$ $\log_3(6-3x) = \log_3(2-x)^2$ $6-3x = (2-x)^2$ $6-3x = 4-4x+x^2$ $x^2 - x - 2 = 0$ $x = 2$ (rej.) or -1
13ai	$\pi r^2 = 270$ $r = 9.2705$ $= 9.27$ cm (3 sf)

Qn	Solution
13aii	$A = \pi r^2$ $\frac{dA}{dr} = 2\pi r$ $10 = 2\pi(9.2705)\times\frac{dr}{dt}$ $\frac{dr}{dt} = 0.172$ cm/s (3sf) $\therefore$ rate of change of radius is 0.172 cm/s.
13b	Subst. $t = 2$, $M = 8$, $k = 56$ $M = \frac{56}{3t+1}$ $\frac{dM}{dt} = 56(-1)(3t+1)^{-2}(3)$ $= -\frac{168}{(3t+1)^2}$ when $t = 5$, $\frac{dM}{dt} = -\frac{168}{(3(5)+1)^2}$ $= -\frac{21}{32}$ g/min $\therefore$ rate of decrease of M is $\frac{21}{32}$ g/min.

Qn	Solution
14a	$y=\sqrt{1+4x}$ $\frac{dy}{dx}=\frac{1}{2}(1+4x)^{-\frac{1}{2}}(4)$ $=\frac{2}{\sqrt{1+4x}}$ gradient of tangent at A = $\frac{2}{3}$ $\frac{2}{\sqrt{1+4x}}=\frac{2}{3}$ $1+4x=9$ x = 2 When x = 2, y = 3 Coordinates of A is (2, 3). subst.x = 2, y = 3 into $y=-\frac{3}{2}x+c$ c = 6 $\therefore$ coordinates of B is (0, 6).
14b	Shaded area $=\frac{1}{2}\times(6+3)\times 2-\int_0^2(1+4x)^{\frac{1}{2}}\,dx$ $=9-\left[\frac{1}{6}(1+4x)^{\frac{3}{2}}\right]_0^2$ $=9-\left[\frac{1}{6}(1+4(2))^{\frac{3}{2}}-\frac{1}{6}(1+4(1))^{\frac{3}{2}}\right]$ = $\frac{14}{3}$ units^2 **Alternative solution:** Shaded area $=\int_1^3\left(\frac{y^2-1}{4}\right)dy+\frac{1}{2}\times(6-3)\times 2$ $=\left[\frac{1}{4}\left(\frac{1}{3}y^3-y\right)\right]_1^3+3$ $=\frac{1}{4}\left(\frac{1}{3}(3)^3-3-\frac{1}{3}+1\right)+3$ = $\frac{14}{3}$ units^2