Chapter 11

SUPERPOSITION

Content

Stationary waves
Diffraction
Interference
Two-source interference patterns
Diffraction grating

Learning Outcomes

Candidates should be able to:

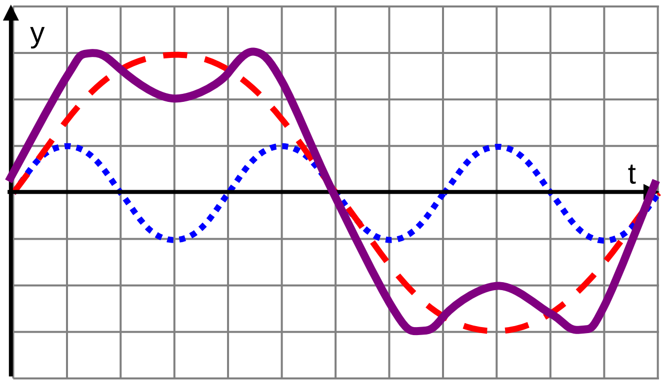
(a) explain and use the principle of superposition in simple applications. ☐ ☐ ☐
(b) show an understanding of the terms interference, coherence, phase difference and path difference. ☐ ☐ ☐
(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns. ☐ ☐ ☐
(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes. ☐ ☐ ☐
(e) explain the meaning of the term diffraction. ☐ ☐ ☐
(f) show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap. ☐ ☐ ☐
(g) show an understanding of experiments which demonstrate two-source interference using water waves, sound waves, light and microwaves. ☐ ☐ ☐
(h) show an understanding of the conditions required if two-source interference fringes are to be observed. ☐ ☐ ☐
(i) recall and use the equation $\lambda = a x/D$ for double-slit interference using light. ☐ ☐ ☐
(j) recall and use the equation $\sin\theta = \lambda / b$ to locate the position of the first minima for single slit diffraction. ☐ ☐ ☐
(k) recall and use the Rayleigh criterion $\theta \approx \lambda / b$ for the resolving power of a single aperture. ☐ ☐ ☐
(l) recall and use the equation $d \sin \theta = n\lambda$ to locate the positions of the principal maxima produced by a diffraction grating ☐ ☐ ☐
(m) describe the use of a diffraction grating to determine the wavelength of light (the structure and use of a spectrometer are not required). ☐ ☐ ☐

AND previously in the WAVES (chapter 10) syllabus:
Candidates should be able to
(n) Determine the wavelength of sound using stationary waves. ☐ ☐ ☐

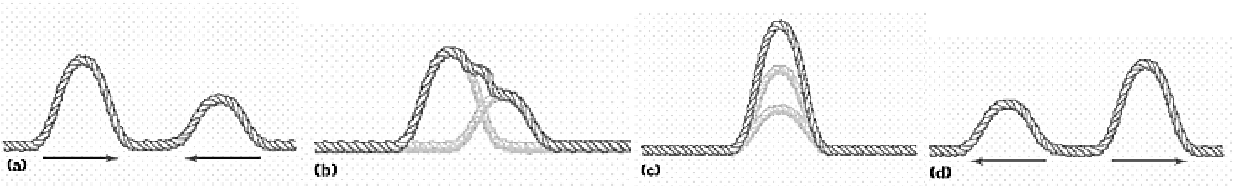
1 PRINCIPLE OF SUPERPOSITION

Previously in Waves, we were introduced to the wave model. In this chapter, we are studying the effect when more than one wave exists simultaneously. Like how each pebble creates its own ripple in the water, what happens when two pebbles are thrown into the pond and the ripples spread out and overlap?

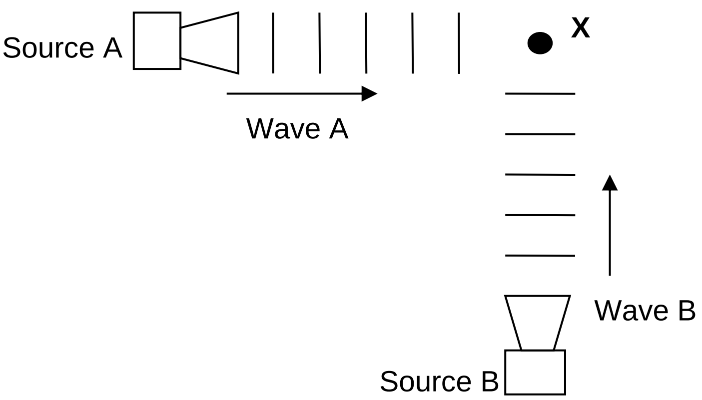
When two waves meet, the resultant <u>displacement</u> is the vector sum of the <u>displacements</u> due to each individual wave.



The figure below shows two wave pulses travelling in opposite directions. When the two waves meet, the resultant displacement of the rope is always equal to the sum of the displacements produced by each pulse. After the waves separate, they behave as if they had never met.

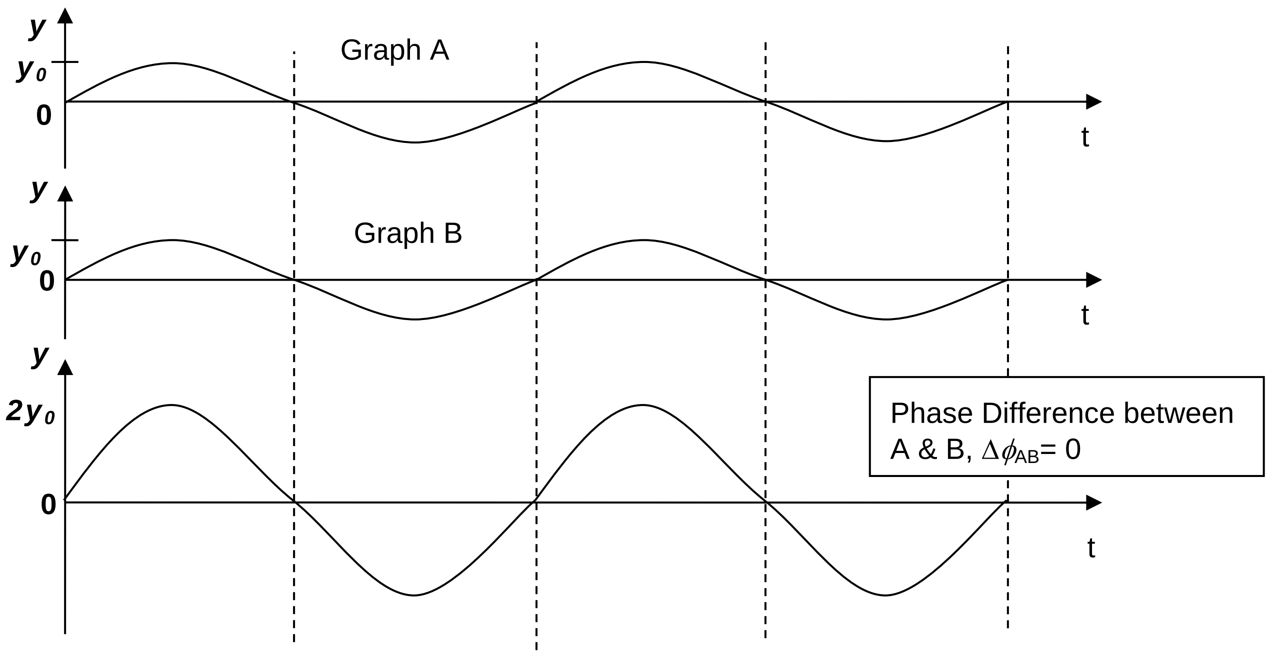


Consider two transverse waves A and B emitted by two sources meeting and interfering at the point X.

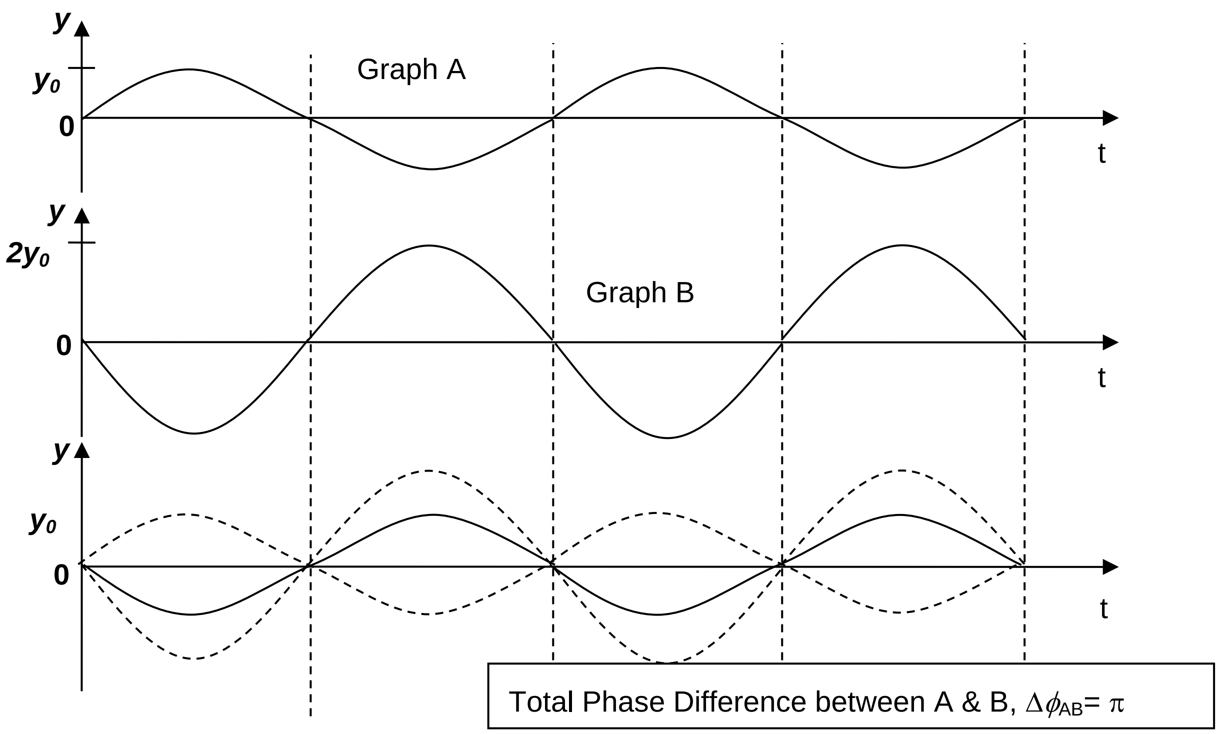


Graph A shows the displacement-time graph of a particle at **X** when **only** source A is switched on. Graph B shows the displacement-time graph of the same particle at **X** when **only** source B is switched on.

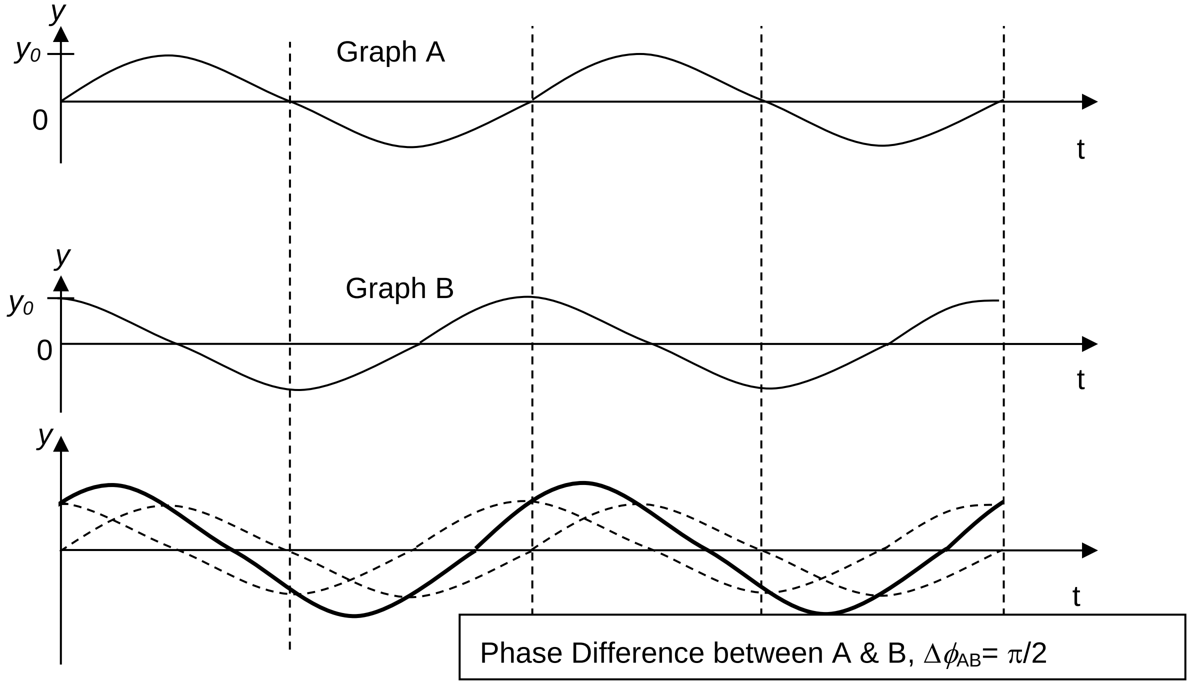
(a) Sketch the displacement-time graph of the particle at **X** when both source A and source B are switched on simultaneously.



(b) The amplitude of the wave from Source B is now **doubled** and Source B is in antiphase to Source A. Sketch the displacement-time graph of the particle at **X** due to both waves.



(c) Source B is now out of phase with Source A. Sketch the displacement-time graph of the particle at **X** due to both waves.

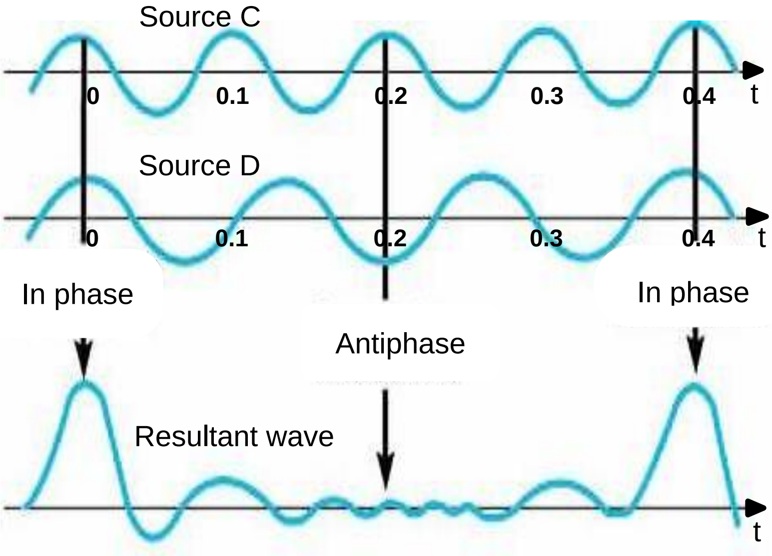


How about superposing two waves of ***different frequencies***?

1.1 COHERENCE

Consider the Sources C and D below with a frequency of 100 Hz and 75 Hz respectively. Would it be meaningful to discuss their phase difference?

Sources are said to be coherent if they have <u>constant phase difference</u>. (i.e. the phase difference of the sources does not change with time)



- This is only possible when the two sources have the same frequency, the same wavelength and the same wave speed.
- In particular for light, it is impossible to achieve coherence if two separate sources are used. Therefore to overcome this problem, we use a single source and a pair of double slits to split the light into two identical sources. Refer to Section 4.3.2 and Appendix for further discussion.

We will study mainly how the principle of superposition is applied to formation of stationary waves, interference of waves and diffraction of light.

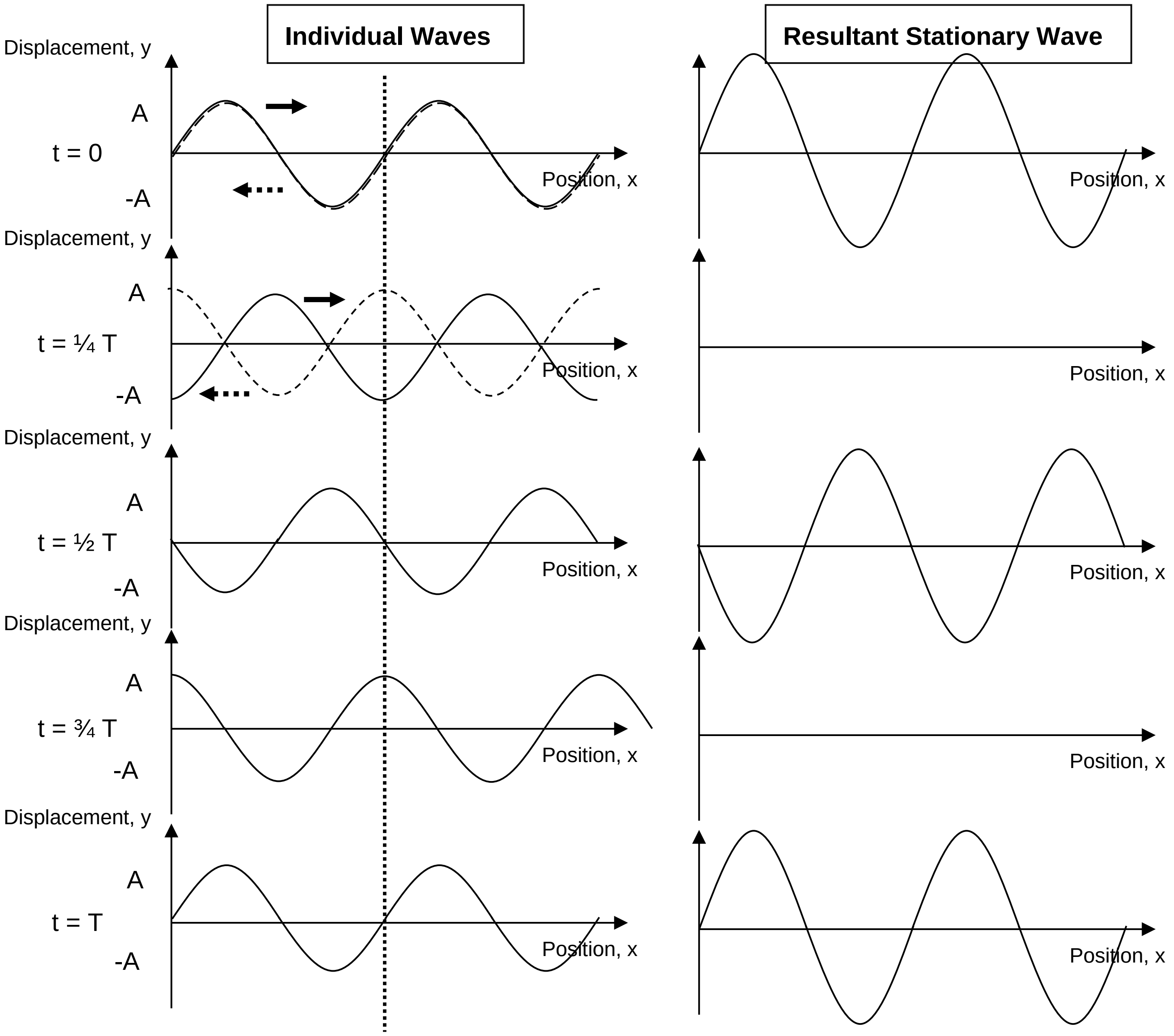
2 STATIONARY WAVES

2.1 FORMATION OF STATIONARY WAVES

explained using graphical method

Stationary waves result from the superposition of two waves of equal amplitude and frequency travelling with the same speed but in opposite directions.

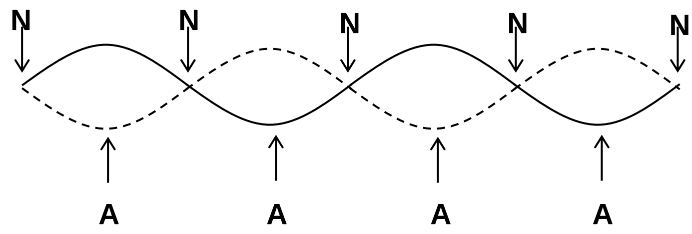
Consider two identical waves moving in opposite directions. The waveform of the rightward-moving wave (solid line) after every $\frac{1}{4}T$ is drawn, do the same for the leftward-moving wave (dotted line). You will see a stationary wave set up after applying the principle of superposition.



2.2 CHARACTERISTICS OF STATIONARY WAVES

- As the wave pattern does not appear to move in either direction along the wave and the positions of the crests and troughs of the wave are **fixed** with time, the resultant waveform is known as a stationary or standing wave.

- We represent stationary waves on a string as follows:



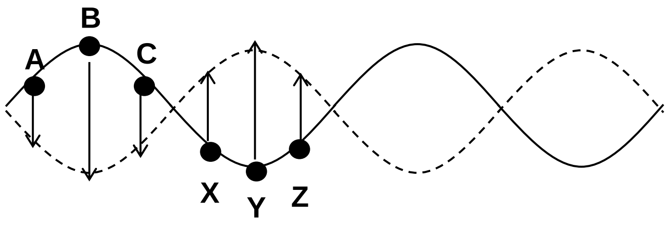
The solid line and the dotted line show the two extreme positions of the string within one period T.

- Points marked by "**N**" in the diagram above are called **displacement nodes**. The particles at these points **do not oscillate** because the component waves travelling in opposite direction superpose destructively (destructive interference) at these points where $\Delta\phi_{total}$ = odd integer multiples of π radians. The positions of nodes do not change with time.

- Points marked by "**A**" in the diagram above are called **displacement antinodes**. The particles at these points **oscillate with maximum amplitude** because the component waves travelling in opposite direction superpose constructively (constructive interference) at these points where $\Delta\phi_{total}$ = zero or even integer multiples of π radians. The positions of antinodes do not change with time.

- The stationary wave is divided by nodes and antinodes into equal segments (or "loops"), and each segment is of the same length. The distance between 2 adjacent nodes, or 2 adjacent antinodes is half a wavelength.

- All particles (except nodes) in the stationary wave are oscillating with simple harmonic motion at the **same frequency, but not at the same amplitude**. Amplitude of vibration is zero at the nodes, maximum at the antinodes.



- All particles within the same loop have a phase difference of zero (ie they oscillate in phase) since all moves up and down together at the same time. Particles in adjacent loops have a phase difference of π radians (i.e. they oscillate in antiphase).

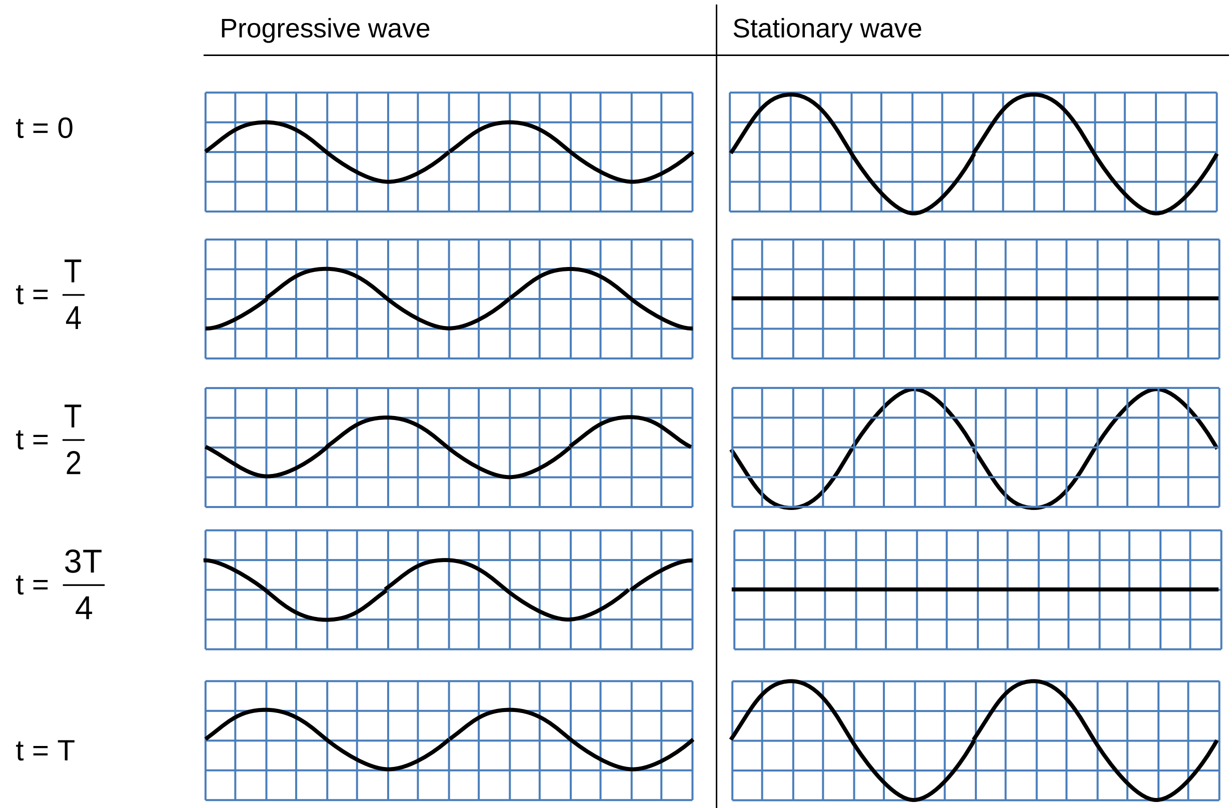
  Note: When the word "node" is used, it typically refers to *displacement* node.

Food for Thought…

How many pairs of particles labelled A, B, C, X, Y and Z are oscillating in anti-phase?

Hint: it is more than 3 pairs

2.3 COMPARING STATIONARY & PROGRESSIVE WAVES



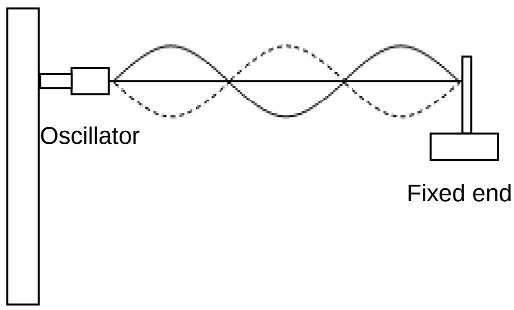
	Progressive wave	**Stationary wave**
Amplitude	It is the same for all particles in the path of the wave.	Varies according to position, from zero at the nodes (permanently at rest), to a maximum of 2A at the antinodes.
Frequency	All particles vibrate in SHM with the same frequency as the wave.	All the particles vibrate in SHM with the same frequency as the component wave (except for those at the nodes which are at rest).
Wavelength	Distance between 2 successive particles which are in phase.	2 x distance between a pair of adjacent nodes or antinodes.
Phase	All particles within one wavelength have different phases.	Phase of all particles between two adjacent nodes is the same. Particles in adjacent segments are in antiphase or have a phase difference of π rad.
Waveform	Advances with the velocity of the wave.	Does not advance.
Energy	Energy translates in the direction of propagation of the wave.	No translation of energy, but there is energy stored locally in the wave.

Food for Thought…

For the diagram on the right, how would you draw the stationary wave at T/8 and 7T/8?

2.4 EXAMPLES OF STATIONARY WAVE EXPERIMENTS

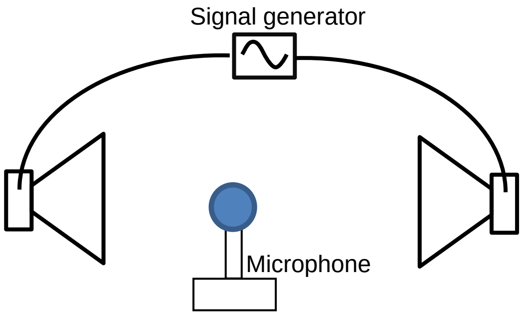
Stationary Mechanical Waves



A oscillator continually generates transverse progressive waves on a string which are reflected at the fixed end of the string.

The reflected waves superpose with the incident waves to form stationary waves on the string.

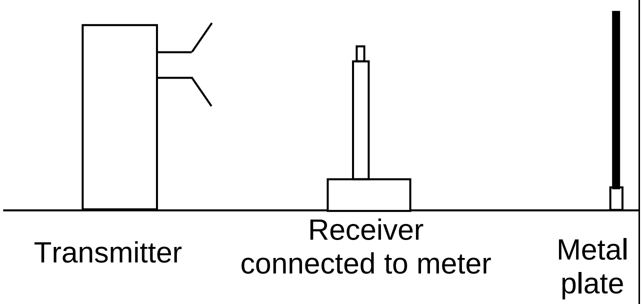
Stationary Sound Waves



Two loudspeakers, facing each other, vibrate to the same signals from the same signal generator and set up a stationary longitudinal wave in the air between the speakers.

A microphone, connected to a CRO, detects the positions where vibrations are maximum (antinodes) and vibrations are minimum (nodes).

Stationary Microwaves

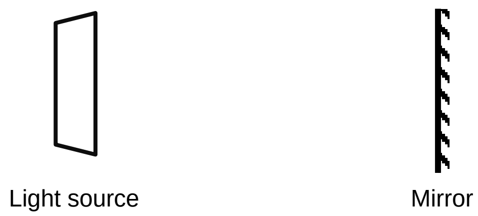


A microwave transmitter continually generates transverse progressive microwaves which are reflected back by a metal sheet.

The reflected waves superpose with the incident waves to form stationary waves in the space between the transmitter and the reflector. A microwave receiver detects the positions where vibrations are maximum (antinodes) and vibrations are minimum (nodes).

Stationary Light Waves

(almost undetectable because of the short wavelength of light)



Parallel light waves are reflected back by a mirror.

The reflected waves superpose with the incident waves to form stationary waves in the space between the source and the mirror.

A thin sheet of photographic film will be able to detect the positions where vibrations are maximum (antinodes) and vibrations are minimum (nodes).

Special Note:

* Stationary waves in air occur between two sound sources facing each other, or a sound source facing a reflector. When a microphone is placed *within that space*, a LOUD SOUND is detected by the microphone if it is placed at a position of a node.

* Stationary waves in pipes and on strings are observed *from outside*.
  An observer will hear LOUD SOUNDS only when stationary waves satisfy the boundary conditions and fit into the pipes and strings.

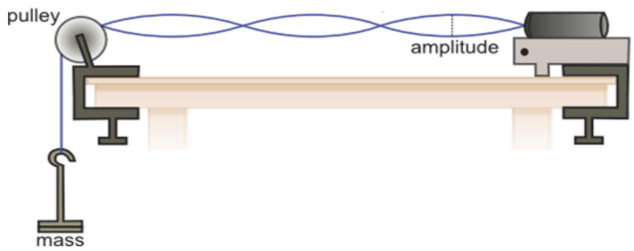
2.4.1 Stationary Wave Vibration Modes on a String

In everyday examples of vibrating strings (e.g. in musical instruments such as a guitar), both ends of the string are fixed and hence cannot vibrate, forming nodes.

HOW STATIONARY WAVES ARE FORMED

A string is made to vibrate using a mechanical oscillator connected to variable frequency signal generator. This causes a transverse wave to travel along the string. The wave then travels outwards along the string and when it reaches the end of the string, it is **reflected**. The incident and reflected waves then **superpose** to produce a stationary wave in the string.

Good to Know:
Speed of wave on the string v depends on tension in the string T and mass per unit length of the string μ: $v = \sqrt{\frac{T}{\mu}}$



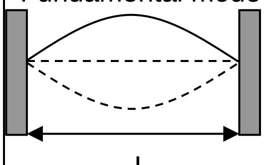
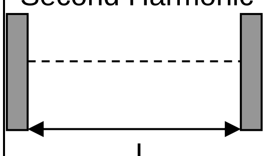
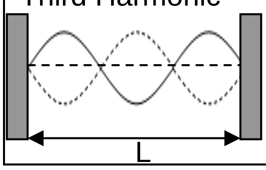
Mechanical oscillator connected to variable frequency signal generator

MODES OF VIBRATION OF A STRING

The frequencies at which stationary waves are produced are known as natural frequencies or resonant frequencies of the string. When the driving frequency matches one of the resonant frequencies of the vibrating string, the amplitude of oscillation is the largest.

A string has many **modes of vibration.** For each mode of vibration, it must meet the boundary conditions at both ends:

- Since the ends are fixed in position, there must be a **displacement node** at each end of the string.

Vibration Mode	L in terms of λ	λ in terms of L	Frequency	Frequency in terms of f_1
First Harmonic/ Fundamental mode  L	$\frac{1}{2}\lambda$	2L	$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$	Fundamental frequency (f_1) / First resonant frequency
Second Harmonic  L	1λ	L	$f_2 = \frac{v}{\lambda} = \frac{v}{L}$	$f_2 = 2f_1$
Third Harmonic  L	$\frac{3}{2}\lambda$	$\frac{2}{3}$L	$f_2 = \frac{v}{\lambda} = \frac{3v}{2L}$	$f_3 = 3f_1$

Resonant frequency of the n^th harmonics, $f_n = \frac{nv}{2L}$ where n = 1, 2, 3…

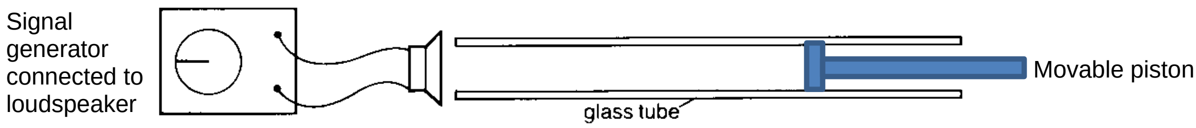
*Note that **all harmonics** are possible for a wave with two closed ends.

2.4.2 Stationary Wave Vibration Modes in Pipes

Unlike a vibrating string, which is a superposition of transverse waves, a closed pipe stationary wave is a **<u>superposition of longitudinal waves,</u>** in particular, sound waves. This is the principle behind how wind instruments create sound.

HOW STATIONARY WAVES ARE FORMED

Stationary waves can be set up in a tube, by using a vibrating tuning fork or a speaker attached to a signal generator as the source of sound waves. When the source is placed at the mouth of the tube, the sound waves travel to the other end of the tube, and are reflected by the closed end of the pipe. The two travelling waves then **<u>superpose</u>** to produce a stationary wave in the tube.



When stationary sound waves 'fit' in a pipe or satisfy the boundary conditions of the pipe, LOUD SOUNDS are heard by observers. RESONANCE occurs when the frequency of the driver (tuning fork or signal generator) matches one of the resonant frequencies of the air column and thus the air in the tube vibrates at maximum amplitude.

We are studying two types of air columns:

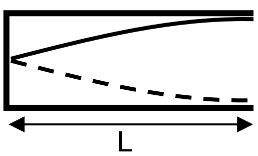
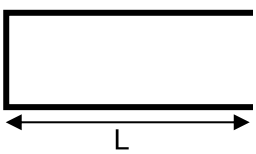
- Closed pipe (closed at one end)
- Open pipe

MODES OF VIBRATION IN AN AIR COLUMN - CLOSED PIPE

The boundary conditions for a pipe are different from that of a vibrating string:

- The **<u>closed end</u>** of the pipe is always a **<u>displacement node</u>** because the layer of air molecules directly in contact with this end cannot vibrate
- The **<u>open end</u>** of the pipe is always a **<u>displacement antinode</u>** since the air molecules at this end are free to vibrate.

Note that the stationary wave is longitudinal (i.e. particles vibrate left and right along the length of the pipe) and not up and down, as the wave representation may *seem* to indicate.

Vibration Mode	**Particle Movement**	**L in terms of λ**	**λ in terms of L**	**Frequency**	**Frequency in terms of f_1**
First Harmonic 		$\frac{1}{4}\lambda$	4L	$f_1 = \frac{v}{\lambda} = \frac{v}{4L}$	Fundamental frequency (f_1) / First resonant frequency
Third Harmonic 		$\frac{3}{4}\lambda$	$\frac{4}{3}L$	$f_3 = \frac{v}{\lambda} = \frac{3v}{4L}$	$f_3 = 3f_1$

Fifth Harmonic 		$\frac{5}{4}\lambda$	$\frac{4}{5}L$	$f_5 = \frac{v}{\lambda} = \frac{5v}{4L}$	$f_5 = 5f_1$

Resonant frequency of the nth harmonics, $f_n = \frac{nv}{4L}$ where n = 1, 3, 5…

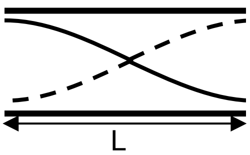
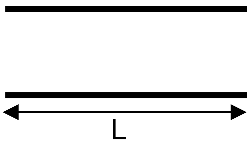
*Note that **only odd harmonics** are possible for a pipe with one closed end.

MODES OF VIBRATION IN AN AIR COLUMN - OPEN PIPE

The boundary condition for an open pipe is such that:

- The **open ends** of the pipe are both **displacement antinodes** since the air molecules at the ends are free to vibrate.

Again, note that the stationary wave is longitudinal (i.e. particles vibrate left and right along the length of the pipe) and not up and down, as the wave representation may seem to indicate.

Vibration Mode	Particle Movement	L in terms of λ	λ in terms of L	Frequency	Frequency in terms of f_1
First Harmonic 		$\frac{1}{2}$	2L	$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$	Fundamental frequency (f_1) / First resonant frequency
Second Harmonic 		1λ	L	$f_2 = \frac{v}{\lambda} = \frac{v}{L}$	$f_2 = 2f_1$
Third Harmonic 		$\frac{3}{2}\lambda$	$\frac{2}{3}L$	$f_2 = \frac{v}{\lambda} = \frac{3v}{2L}$	$f_3 = 3f_1$

Resonant frequency of the nth harmonics, $f_n = \frac{nv}{2L}$ where n = 1, 2, 3…

*Note that **all harmonics** are possible for a pipe with two open ends.

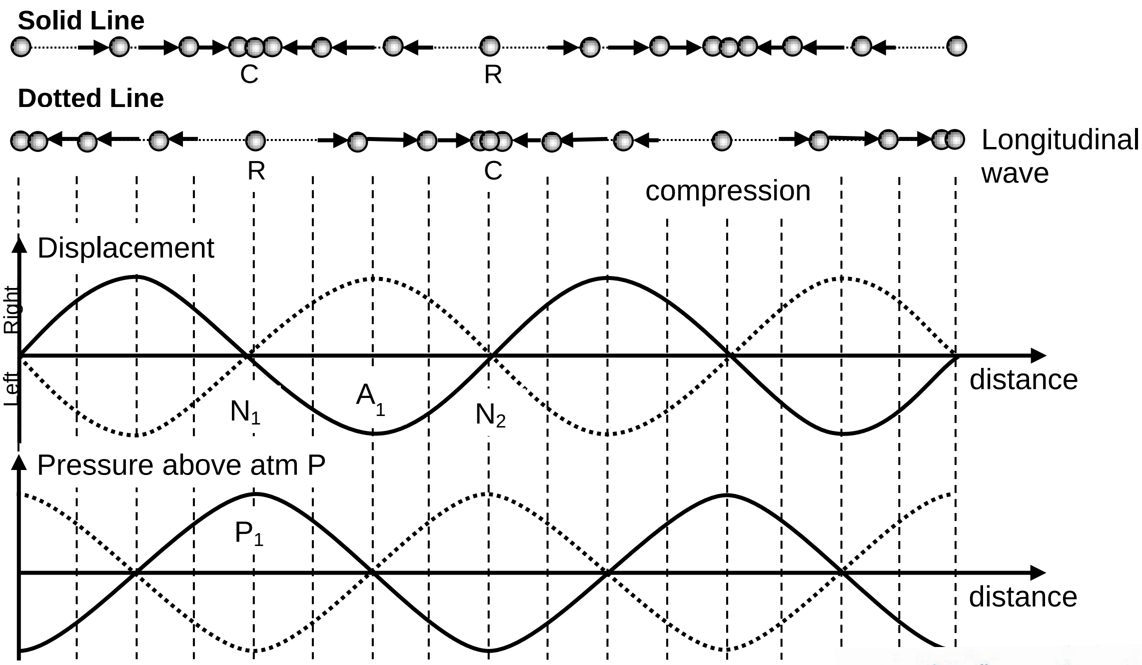
2.4.3 Pressure Variation in Stationary Sound Wave

In Waves, we discussed the pressure variations in a progressive sound wave. Now we will examine how pressure varies in a stationary sound wave.

In the displacement-position graph for a stationary wave, the solid line and the dotted line show the two extreme positions of the wave within one period T. We will take the displacement of the air molecules to the right to be positive.

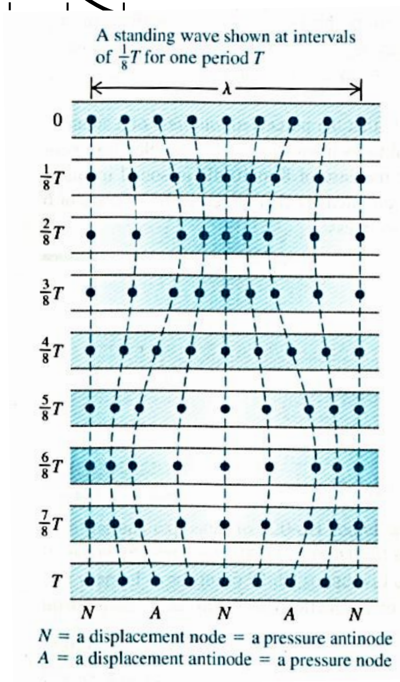
N_1 is a displacement node. Considering only the solid line, air particles to the left of N_1 are displaced to the right and those to the right of N_1 are displaced to the left. Hence a region of **compression** is formed and pressure is **higher** than normal (ie. P_1).

Within the same period, the dotted line shows the other extreme position of the stationary wave. Considering only the dotted line, air particles to the left of N_1 are displaced to the left and those to the right of N_1 are displaced to the right. Hence a region of **rarefaction** is formed and pressure is **lower** than normal (ie. P_1).



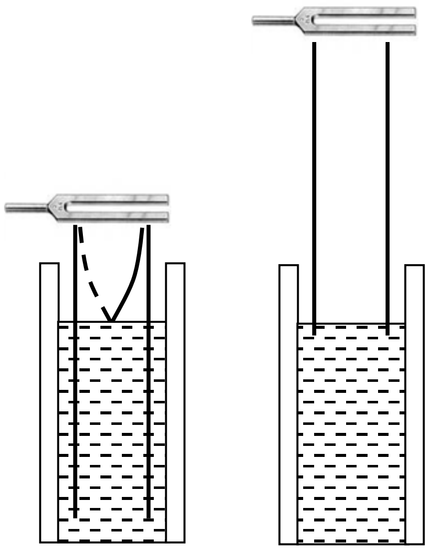
Hence we can conclude that the particles on opposite sides of a displacement node (N) vibrate in antiphase. The air undergoes the maximum amount of compression and rarefaction, and pressure and density varies the most. In other words, **displacement nodes correspond to pressure antinodes**.

In contrast, particles on opposite sides of a displacement antinode (A) vibrate in phase. The distance between the particles is nearly constant, and pressure and density do not vary. In other words, **displacement antinodes correspond to pressure nodes.**



2.4.4 Using Stationary Waves to find Speed/Wavelength of Sound

METHOD I: USING RESONANCE TUBE



- A tube is fully immersed in the water.
- A tuning fork of known frequency is struck and held over the top of the tube.
- The tube is slowly raised vertically out of the water until the loudest note is first heard.
- *A stationary wave now exists in the tube with a displacement node N at the closed end and a displacement antinode A near the open end.* ***The fundamental frequency of the air column is then equal to the frequency of the fork, i.e. there is resonance****.*
- Measure L_1 = length from water level to top of tube
- With the tuning fork still vibrating, the tube is raised further out of the water until the next loud sound is obtained.
- Measure L_2 = length from water level to top of tube for the second resonance position
- To find the wavelength of sound in air:

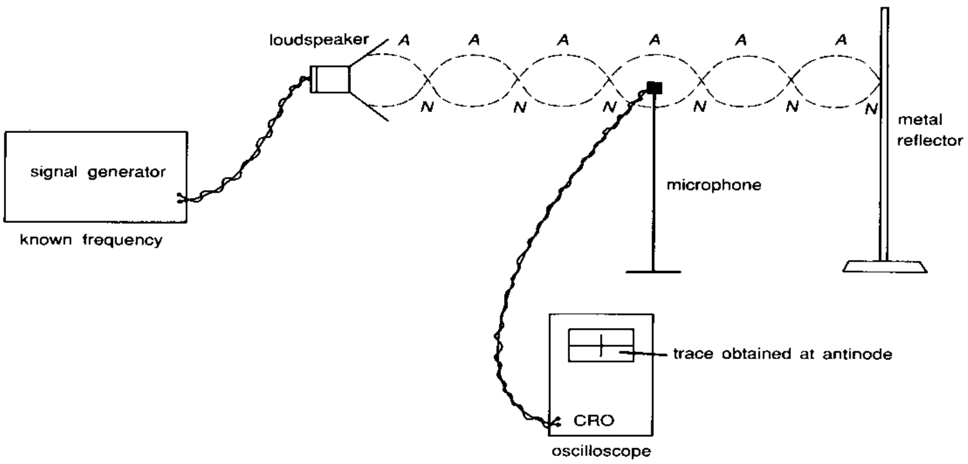
  $L_2 - L_1 = 1/2\ \lambda$

  $\lambda = 2(L_2 - L_1)$

- Applying $v = f\lambda$ to find the speed of sound in air

  $v = f\lambda = 2f(L_2 - L_1)$

METHOD II: USING SIGNAL GENERATOR

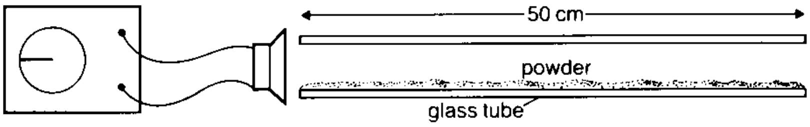


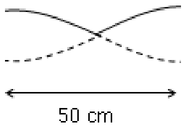
- Sound waves of constant frequency travel from a loudspeaker S towards a metal reflector M. The waves are reflected and superpose with the incident waves to form **a stationary sound wave** between M and S.
- The detection of the nodes (N) and antinodes (A) of the stationary wave can be done by a microphone attached to the CRO.
- By moving the microphone slowly forward and backward along MS, the trace on CRO is seen to vary from minimum to maximum showing the positions of nodes and antinodes respectively.

- Measure the distance between successive antinodes or between successive nodes, L.
  L = 1/2 λ
  λ = 2L
- Knowing the frequency *f* from the signal generator, the speed of sound can be calculated using *v* = f λ = 2fL

Example 1

The experimental setup below demonstrates the formation of stationary waves in air. Given that speed of sound in air is 330 m s^{-1}, suggest three different sound frequencies which will set up visible stationary waves in the tube. For each frequency, sketch what you would expect to see.





Above is the lowest mode of vibration where λ_1 = 100 cm.

Hence f_1 = 330/1.0 = 330 Hz.

The next higher mode is such that λ_2 = 50 cm.

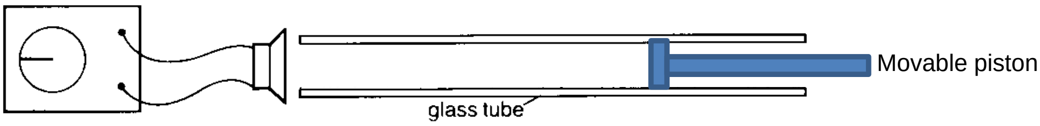
Hence f_2 = 330/0.5 = 660 Hz.

Similarly, f_3 = 990 Hz.

Powder heaps appear at nodes, no powder at antinodes.

Example 2

An air column in a glass tube is open at one end and closed at the other by a movable piston.



With a 384 Hz tuning fork held at the open end, resonance is first heard when the piston is 22.8 cm from the open end and again when it is 68.3 cm from the open end.

(a) What is the speed of sound?

(b) How far from the open end will the piston be when the next resonance is heard?

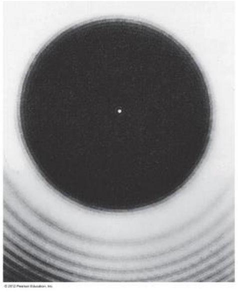
(a) $L_2 - L_1$ = 1/2 λ

λ = 2(68.3-22.8) = 0.910 m

v = f λ = (384)(0.910) = 349 ms^{-1}

(b) Next resonance will be heard at 5λ/4 = 5(0.910)/4 = 1.14 m

3 DIFFRACTION



Light that emerges from apertures do not behave exactly according to the predictions of straight-line ray model of geometric optics. To illustrate this, a beam of light is shone on a steel ball 3 mm in diameter and rings of bright and dark fringes are observed around the ball's supposed shadow. This observed effect is known as diffraction.

WAVE PASSING THROUGH AN APERTURE

The diagrams below show plane wavefronts approaching a narrow gap and a wide gap respectively. Complete the diagram to show what happens after the wave passes through each of the gaps.

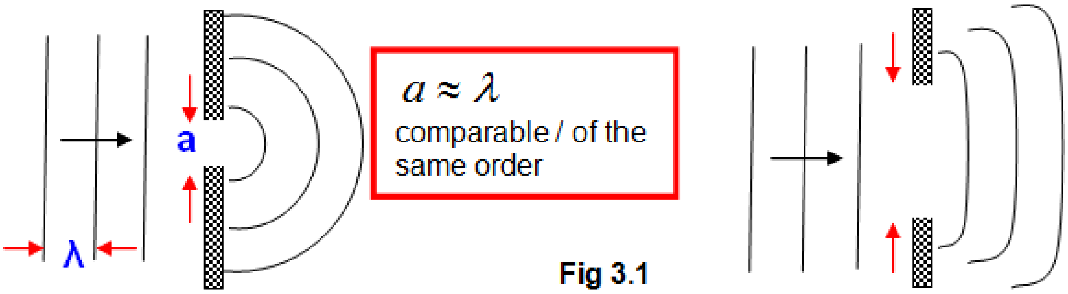


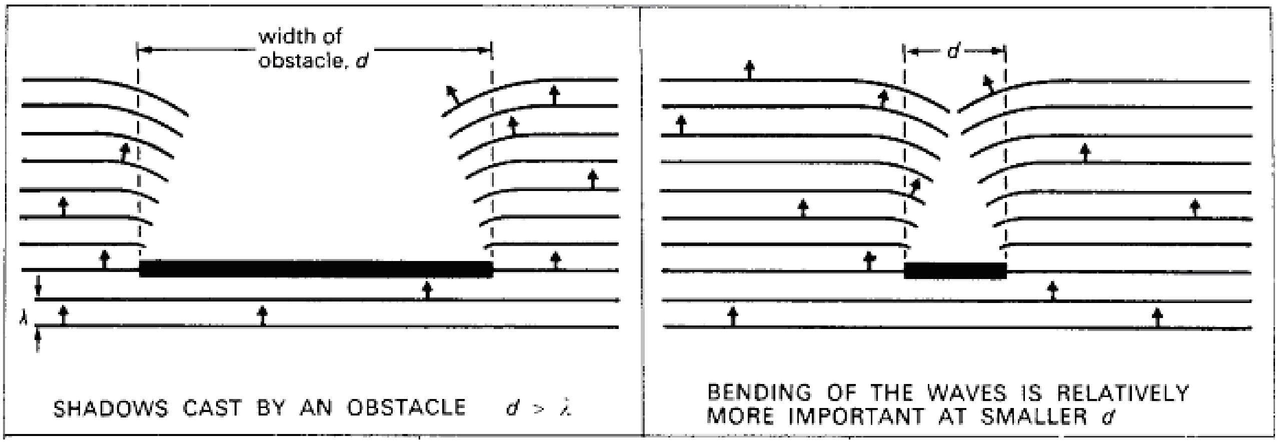
Fig 3.1

When a wave hits an obstacle it does not simply go straight past: it bends round the obstacle. The same type of effect occurs at a hole – the wave spreads out the other side of the hole. This phenomenon is known as **diffraction**.

Diffraction is the spreading of waves when they pass through an aperture or around an obstacle.

- Diffraction is appreciable and hence observable only when the width of the slit is of the **same order of magnitude** as the wavelength of the waves.
- The smaller the slit width, the greater the extent of diffraction.
- The wavelength and frequency of the waves remain unchanged after diffraction.
- **Sound waves** of a particular frequency 256 Hz have a wavelength of about 1.3 m. So sound waves spread round openings such as a doorway or an open window, which have similar dimensions to their wavelengths.
- **Light waves** would not spread round doorways and open windows as they have very small wavelengths of the order 6 x 10^{-7} m (600 nm). However light waves spread around the narrow slits used in Young's double slit experiment.

WAVE PASSING AROUND AN OBSTACLE



We will revisit the diffraction of light through a single slit later in Section 5 (Single Slit Diffraction).

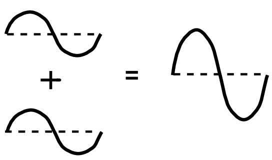
4 INTERFERENCE

Interference is the result of superposition of wavetrains from a finite number of coherent sources.

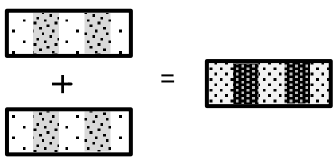
4.1 CONSTRUCTIVE & DESTRUCTIVE INTERFERENCE

Consider two sinusoidal waves of the same wavelength and amplitude meeting at a point. When the waves meet, SUPERPOSITION OCCURS. The effect of this superposition depends on whether the waves meet in phase or in antiphase.

- When two waves meet in phase at a point, they reinforce each other and the resultant amplitude of the wave is a maximum. The waves are said to interfere **constructively.**
  Phase difference between the two waves = 0, 2π, 4π …
  Resultant amplitude = $A_1 + A_2$

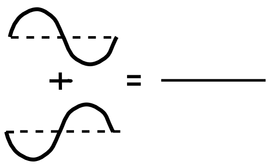


Constructive Interference of Transverse Waves

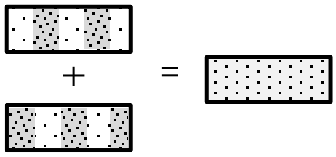


Constructive Interference of Longitudinal Waves

- When two waves meet in antiphase or exactly π radians **out of phase** at a point, they cancel each other and the resultant amplitude of the wave is a minimum. The waves are said to interfere **destructively.**
  Phase difference between the two waves = π, 3π, 5π …
  Resultant amplitude = $A_1 - A_2$ which is zero only if the waves share the same amplitude.



Destructive Interference of Transverse Waves



Destructive Interference of Longitudinal Waves

- This phenomenon of cancellation and reinforcement is called **interference.**
- The resultant displacement depends on the **total phase difference** between the two waves.

4.1.1 Phase Difference due to Sources

- The two sources S_1 and S_2 generating these waves are normally in phase or anti-phase i.e. phase difference $\Delta\phi_{source}$ of 0 or π respectively.

4.1.2 Phase Difference due to Path Difference

- When waves from two sources meet at a point, the length of the paths taken may be different. We refer the extra length that the wave from one source must travel, compared to the wave from the other source, in order to reach the point of observation, as path difference.
- Upon arriving at a point, an additional phase difference between the waves is introduced by the path difference Δx. This is called the **phase difference due to path** $\Delta\phi_{path}$, which is given by:

$$\frac{\Delta\phi_{path}}{2\pi} = \frac{\Delta x}{\lambda}$$

$$\Delta\phi_{path} = \frac{\Delta x}{\lambda} \times 2\pi$$

4.1.3 Total Phase Difference

- When two (or more) waves superpose, to know the resultant displacement-time graph of the particle at the point where they meet, we need to determine the **total phase difference** between the two waves at the point that they meet.

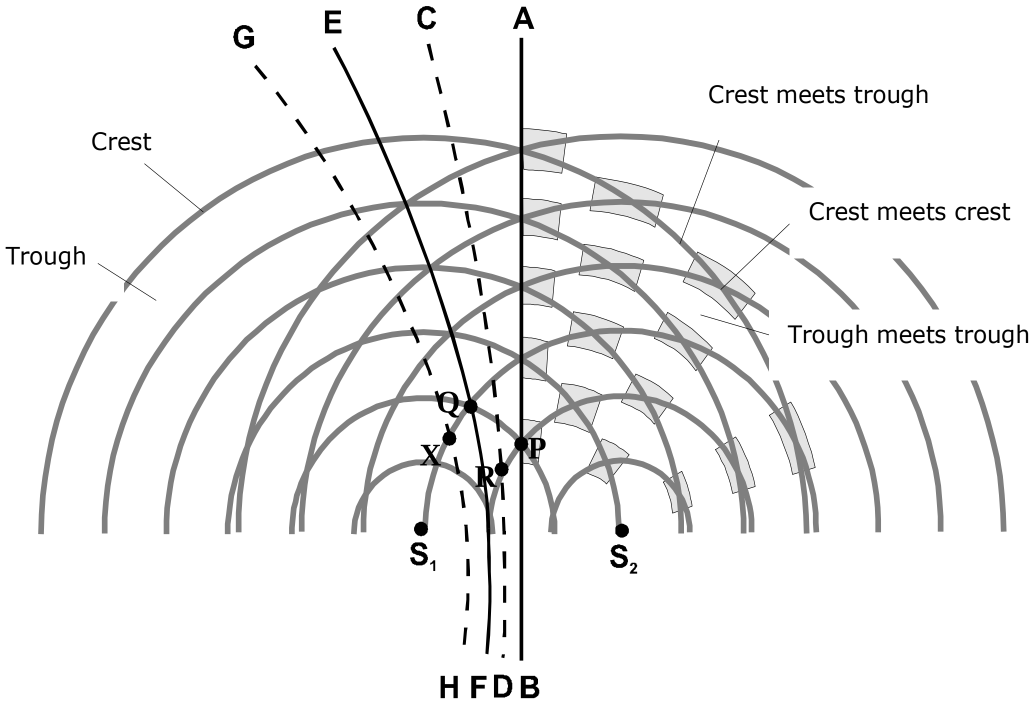
- Hence, the total phase difference $\Delta\phi_{total}$ between the waves at a point P is given by

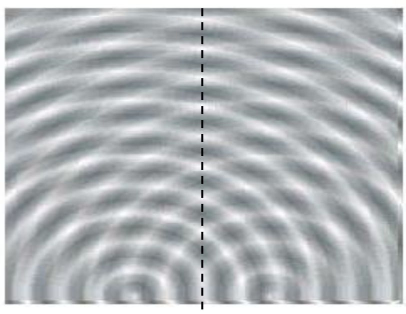
$$\boxed{\Delta\phi_{total} = \Delta\phi_{source} + \Delta\phi_{path}}$$

- When the 2 waves superpose in phase/ interfere constructively, the total phase difference, $\Delta\phi_{total}$ = 0, 2π, 4π, 6π, 8π, etc or even integer multiples of π.
- When the 2 waves superpose in anti-phase/ interfere destructively, the total phase difference, $\Delta\phi_{total}$ = π, 3π, 5π, 7π, etc or odd integer multiples of π.

4.2 INTERFERENCE OF WATER WAVES (RIPPLE TANK EXPERIMENT)

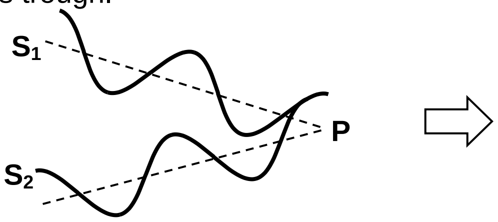
- Two ball-ended dippers, **S_1** and **S_2**, are attached to the vibrator of the ripple tank.
- Each dipper creates a set of circular ripples in the water tank which are in phase. Solid lines represent the crests while the midpoints between two lines represent the troughs.
- These ripples overlap and interference results are as shown below.



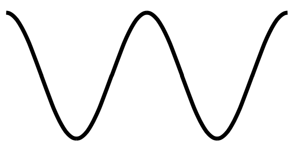


Antinodal lines

- At all the points along lines AB and EF, the crest of wave S_1 meets crest of wave S_2 and trough of wave S_1 meets trough of wave S_2.
- The two waves meet in phase and reinforce each other's effect. The resultant wave has twice the amplitude of the component wave as constructive interference occurs.
- Lines AB and EF are known as antinodal lines.
- Consider point **P** along Line AB, the two waves arrive in phase, that is, crest meets crest or trough meets trough.



Water waves arriving in phase at point **P**



Resultant wave at point **P** with time

The path difference of the 2 sources to P, $\Delta x = S_1P - S_2P = 2\lambda - 2\lambda = 0$

Hence the phase difference due to the path difference, $\Delta\phi_{path} = 0$

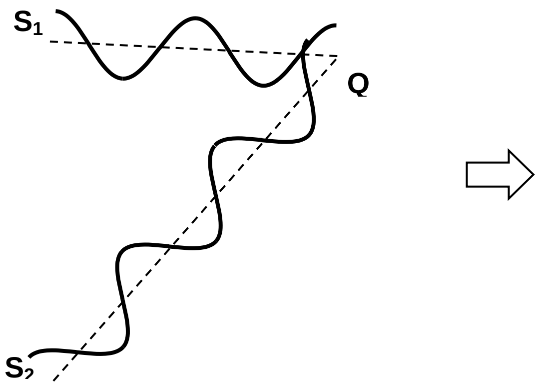
As the sources are in phase, $\Delta\phi_{source} = 0$

Total phase difference, $\Delta\phi_{total} = 0$

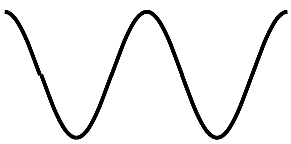
The waves meet **<u>in phase and constructive interference occurs</u>**

Amplitude of resultant wave is a **<u>maximum.</u>**

- Consider at point **Q** along line EF, the two waves arrive in phase with each other, that is, crest meets crest and trough meets trough.



Water waves arriving in phase at point **Q**



Resultant wave at point **Q** with time

The path difference of the 2 sources to Q, $\Delta x = S_2Q - S_1Q = 3\lambda - 2\lambda = 1\lambda$

Hence the phase difference due to the path difference, $\Delta\phi_{path} = 2\pi$

As the sources are in phase, $\Delta\phi_{source} = 0$

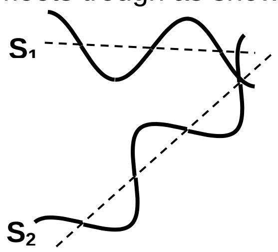
Total phase difference, $\Delta\phi_{total} = 0 + 2\pi = 2\pi$

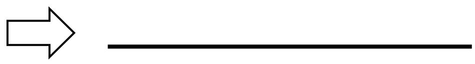
The waves meet **<u>in phase and constructive interference occurs</u>**

Amplitude of resultant wave is a **<u>maximum.</u>**

Nodal lines

- At all the points along lines CD and GH, the crest of wave S_1 meets trough of wave S_2 and the trough of wave S_1 meets crest of wave S_2.
- The two waves meet antiphase or π radians out of phase and cancel out each other's effect. The resultant wave has an amplitude of zero as destructive interference occurs.
- Lines CD and GH are known as nodal lines.
- Consider at point **R** along CD, the two waves arrive antiphase or π radians out of phase, that is, crest meets trough as shown.





Water waves arriving π out of phase at point **R**

Resultant wave at point **R** with time

The path difference of the 2 sources to R, $\Delta x = S_2R - S_1R = 2\lambda - 1\frac{1}{2}\lambda = \frac{1}{2}\lambda$

Hence the phase difference due to the path difference, $\Delta\phi_{path} = \pi$

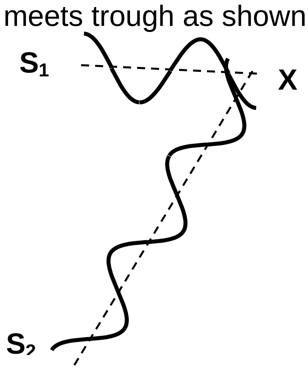
As the sources are in phase, $\Delta\phi_{source} = 0$

Total phase difference, $\Delta\phi_{total} = 0 + \pi = \pi$

The waves meet **<u>in anti-phase and destructive interference occurs</u>**

Amplitude of resultant wave is a **<u>minimum.</u>**

- Consider at point **X** along GH, the two waves arrive antiphase or π radians out of phase, that is, crest meets trough as shown.



Water waves arriving 3π out of phase at point **X**

Resultant wave at point **X** with time

The path difference of the 2 sources to **X**, $\Delta x = S_2X - S_1X = 3\lambda - 1\frac{1}{2}\lambda = 1\frac{1}{2}\lambda$

Hence the phase difference due to the path difference, $\Delta\phi_{path} = 3\pi$

As the sources are in phase, $\Delta\phi_{source} = 0$

Total phase difference, $\Delta\phi_{total} = 0 + 3\pi = 3\pi$

The waves meet **<u>in anti-phase and destructive interference occurs</u>**

Amplitude of resultant wave is a **<u>minimum.</u>**

CONDITIONS FOR CONSTRUCTIVE & DESTRUCTIVE INTERFERENCE

(Assume that sources are in phase)

For Constructive Interference,

Path difference between the two waves at a point, $\Delta x = 0, \lambda, 2\lambda, 3\lambda\ldots = n\lambda$

Total phase difference between the two waves at that point, $\Delta\phi_{total} = 0, 2\pi, 4\pi\ldots = (n)2\pi$

For Destructive Interference,

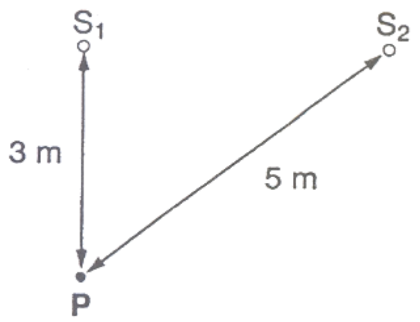
Path difference between the two waves at a point, $\Delta x = ½\lambda, 1½\lambda, 2½\lambda\ldots = (n+½)\lambda$

Total phase difference between the two waves at that point, $\Delta\phi_{total} = \pi, 3\pi, 5\pi\ldots = (n+½)2\pi$

Example 3

Water waves of wavelength 4 m are produced by two generators, S_1 and S_2, as shown below. Each generator, when operated by itself, produces waves which have an amplitude *A* at **P**, which is 3 m from S_1 and 5m from S_2.
When the generators are operated in phase, what is the amplitude of oscillation at **P**?

$\Delta\phi$source = 0
Path diff = 2 m
$\Delta\phi$path = π
$\Delta\phi$total = π
Since waves meet in antiphase, they interfere destructively.
Amplitude of oscillation at P = A – A = 0



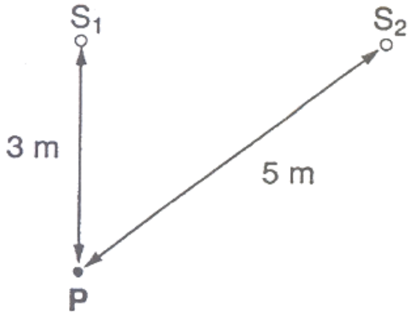
Food for Thought…

If intensity of the wave due to each generator is *I*, what is the intensity of the resultant wave at P when the generators are operating in anti-phase?

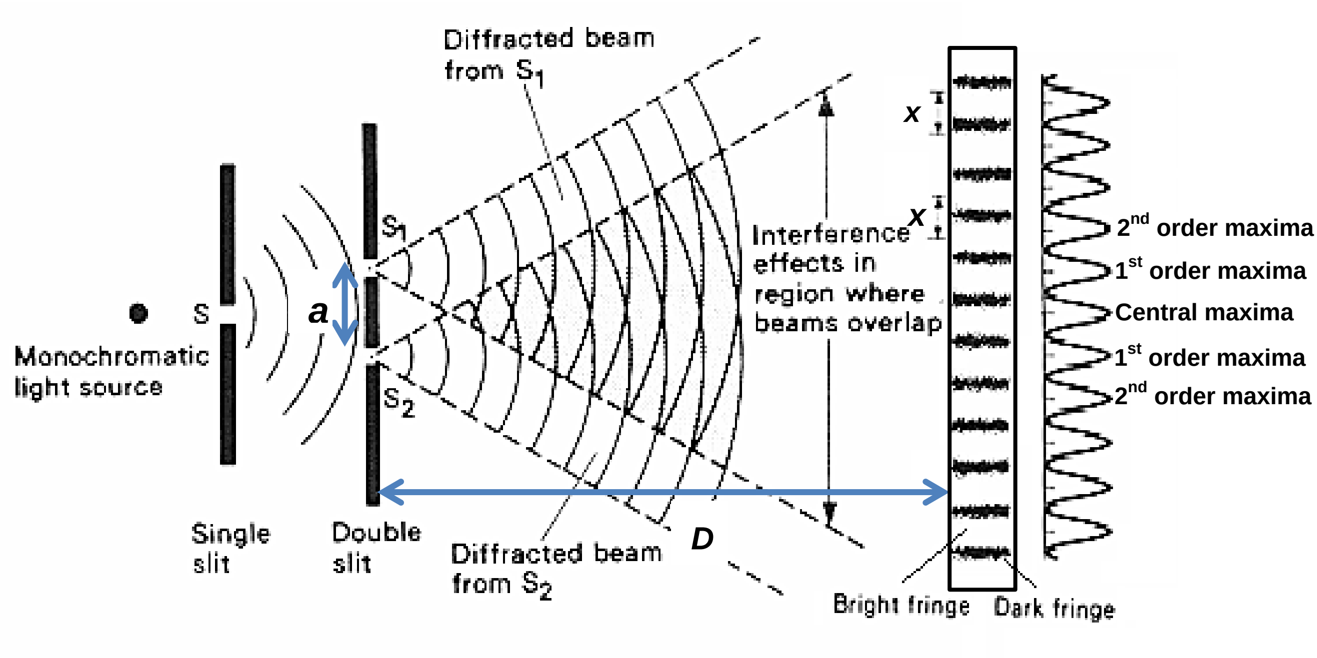
Example 4

The figure shows two sources S_1 and S_2, which are identical and emitted in phase. Calculate two possible values of wavelength for which
(a) constructive interference occurs at point P,
(b) destructive interference occurs at point P.

Path difference = 10 – 8 = 2 m
For constructive interference to occur, path diff must be $n\lambda$
$1\lambda = 2\text{ m} \Rightarrow \lambda = 2\text{ m}$
$2\lambda = 2\text{ m} \Rightarrow \lambda = 1\text{ m}$
$4\lambda = 2\text{ m} \Rightarrow \lambda = 0.5\text{ m}$
For destructive interference to occur, path diff must be $(n+½)\lambda$
$½\lambda = 2\text{ m} \Rightarrow \lambda = 4\text{ m}$
$2½\lambda = 2\text{ m} \Rightarrow \lambda = 0.8\text{ m}$



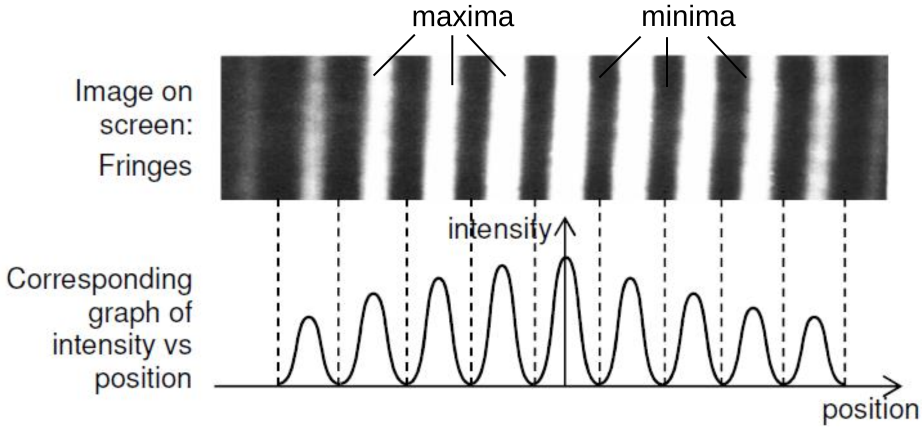
4.3 INTERFERENCE OF LIGHT: (YOUNG'S DOUBLE SLIT EXPERIMENT)



1. Monochromatic light from a small filament lamp diffracts as it passes through narrow slit S.
2. Two narrow slits S_1 and S_2, about 0.5 mm apart (very close together) act as two coherent sources and diffraction causes the light waves to spread into the regions beyond the slits.
3. The light coming from S_1 and S_2 is viewed in the eyepiece of a travelling microscope 1 metre away. Alternate bright and dark equally-spaced vertical bands (interference fringes) can be observed.

4.3.1 Fringe Separation

- The bright stripes are called **maxima** or bright fringes and they are regions of constructive interference. The dark stripes are called **minima** or dark fringes and they correspond to regions of destructive interference. This interference pattern is also referred to as a **fringe pattern**.



- The ideal intensity pattern shows a series of maxima of equal height, and is based on the assumption that each slit (alone) would illuminate the screen uniformly. This is never quite true due to diffraction effects which will result in the strongest central maximum, with each succeeding maximum on each side less strong.

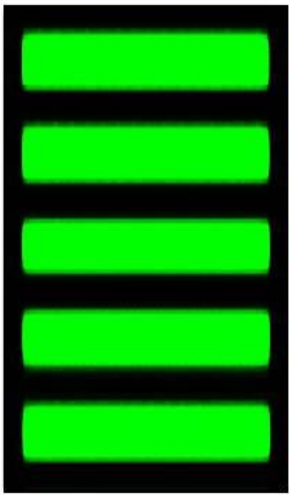
- The distance between two maxima or bright fringes, also known as fringe separation **x**, is given by

$$\boxed{x = \frac{\lambda D}{a}}$$

where λ is the wavelength
D is the distance between the screen and the double slit
a is the distance separating the two slits (slit separation)

Note: The equation holds only when $D >> a$. The derivation of this relationship is discussed in Appendix.

CONDITIONS FOR MAXIMA & MINIMA



Interference pattern on screen

4.3.2 Conditions for Observable Interference Pattern

1. Sources must be coherent. i.e. constant phase difference.
   - If the sources are coherent, the points of constructive and destructive interference remain unchanged. Thus the interference pattern is **fixed** with time.
   - If the sources are not coherent, the points of constructive and destructive interference change. This means that the interference pattern is **changing** in its position and hence will be not be observable by human eye as it changes too rapidly.

   To ensure this, a single source is normally used, coupled with a pair of double slits to split the wave into two sources.

2. The amplitude of both waves must be approximately the same.
   This is to ensure that there will be total cancellation at regions of destructive interference to produce the dark fringes.

3. Waves must either be polarised in the same plane or unpolarized.
   Otherwise, the vector sum of displacement will not give zero resultant amplitude.

Note: These conditions are for distinct (high contrast) and observable fringe pattern.
If the conditions are not met, interference still occurs as the waves do overlap. However, the fringe pattern is not distinctive or permanent enough for the observer to see.

Example 5

In a Young's double slits experiment, the separation between the first and fifth bright fringe is 2.5 mm when the wavelength used is 6.2×10^{-7} m. The distance from the slits to the screen is 0.80 m. Calculate the separation of the two slits.

The fringe separation, x = 0.0025/4

Using x = λD/a

$0.0025 / 4 = 6.2 \times 10^{-7} \times 0.80/a$

$\therefore a = 7.94 \times 10^{-4}$ m

Example 6

Fringes of separation *y* are observed in a plane 1.00 m from a Young's Double slit arrangement illuminated by yellow light of wavelength 600nm.

At what distance from the slits would fringes of the same separation *y* be observed when using blue light of wavelength 400 nm?

Using x = λD/a

$$D \ \alpha \ \frac{1}{\lambda}$$

$$\frac{D_1}{D_2} = \frac{\lambda_2}{\lambda_1}$$

$$\frac{1}{D_2} = \frac{400}{600}$$

$$D_2 = 1.50 \ m$$

Example 7

An interference pattern is formed on a screen when red light is passed through two narrow slits which are placed close together. State the changes, if any, of the interference pattern when

(a) the separation of the slits is increased;

(b) blue light is used instead of red light;

(c) the screen is moved further away from the slits;

(d) the source of the red light is moved closer to the slits.

(e) the slits are made narrower but their separation is unchanged.

(a) a decreases, fringe separation x increases

(b) λ decreases, fringe separation x increases

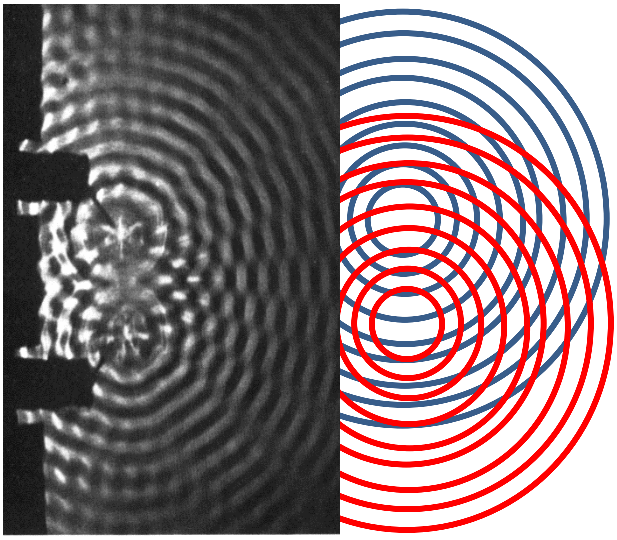
(c) D increases, fringe separation x decreases

(d) fringe separation x remains unchanged

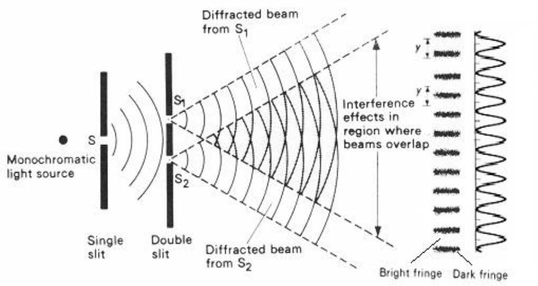
(e) fringe separation x remains unchanged but more fringes are observed

4.4 EXAMPLES OF TWO-SOURCE INTERFERENCE EXPERIMENTS

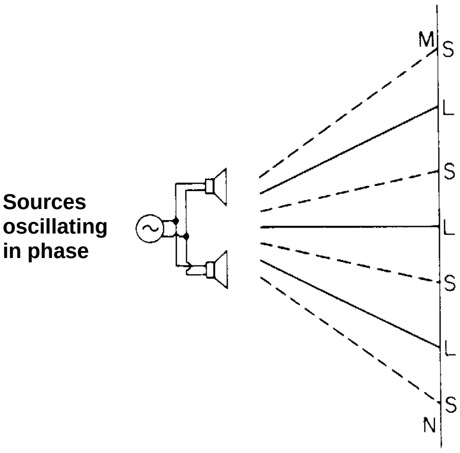
Two-source Interference of Water Waves in a Ripple Tank



Two-source Interference of Light Waves YOUNG'S DOUBLE SLIT EXPERIMENT



Two-source Interference of Sound Waves



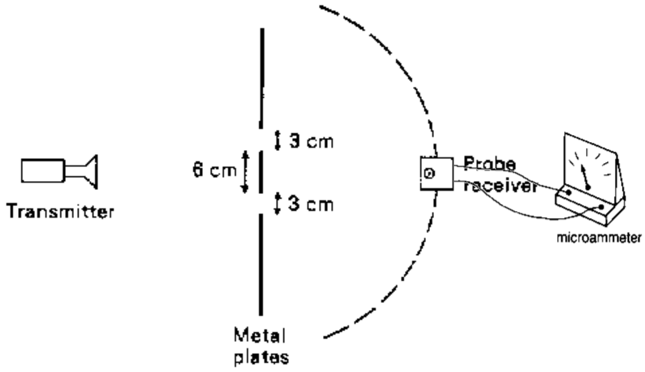
Two loudspeakers are connected in parallel to an audio-frequency oscillator.

As the ear or microphone is moved along the line MN, alternate loud (L) and soft (S) sounds are heard according to whether the receiver of sound is on a line of reinforcement (constructive interference) or cancellation (destructive interference).

Food for Thought…

What happens if the connections to one of the speakers are reversed such that the sound waves oscillate in antiphase?

Two-source Interference of Microwaves

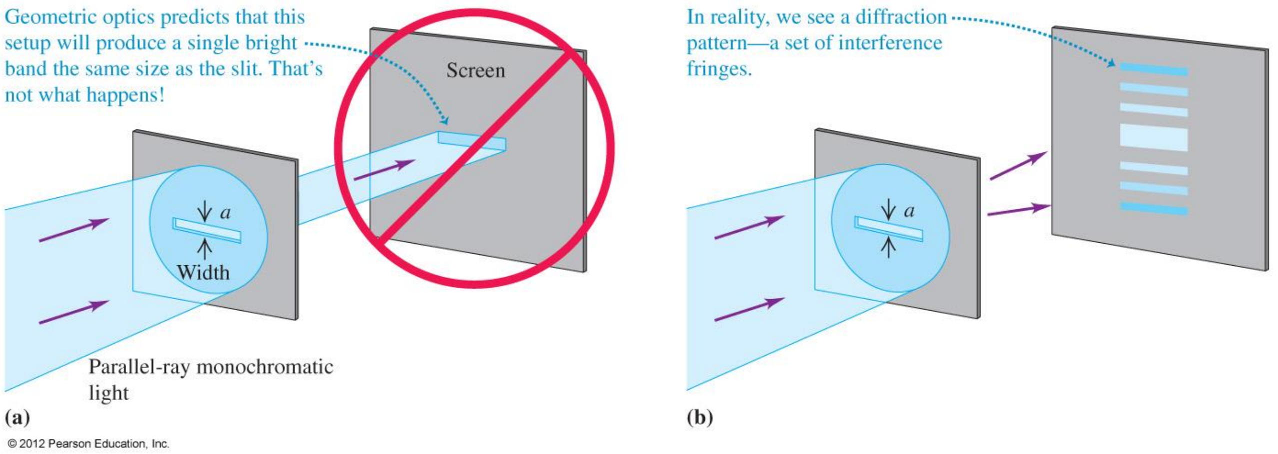


Interference occurs between the two wavetrains emerging from the 3 cm wide slits which act as two coherent sources of diffracted microwaves. These microwaves overlap and interference occurs in the region in front of the slits.

Moving a microwave receiver around, the microammeter registers a current that varies from maximum to minimum alternately showing regions of constructive and destructive interference respectively.

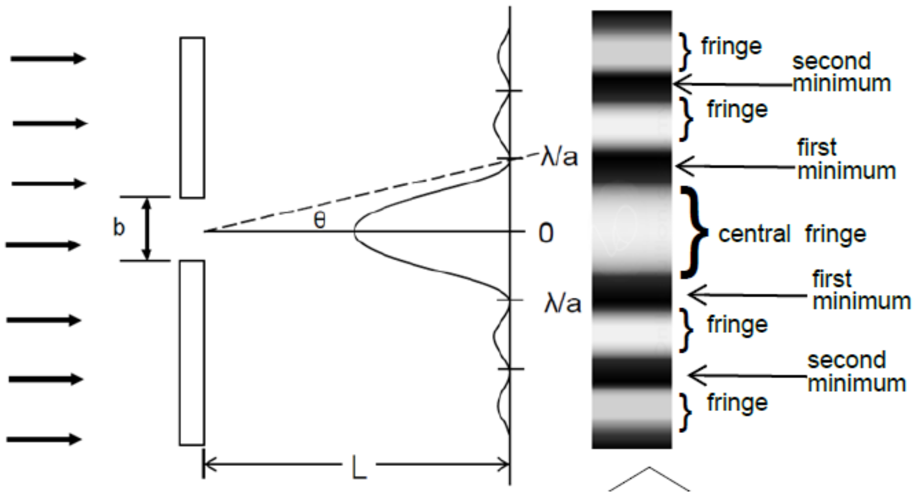
5 SINGLE SLIT DIFFRACTION

When a light is shone through a single slit, geometric optics predicts that a single bright band with the same width of the slit will be produced given that light travels in straight lines. But that's not what happens!



It is observed that:

- an interference pattern of bright and dark fringes is formed.
- the central fringe is much brighter than the others.
- the central fringe has twice the width of each of the other fringes.



Applying the concept of path difference, we derive the equation indicating the position of the first minima which is given by

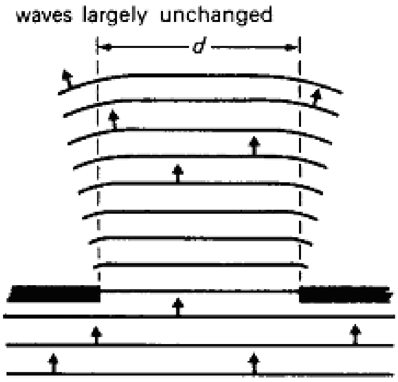
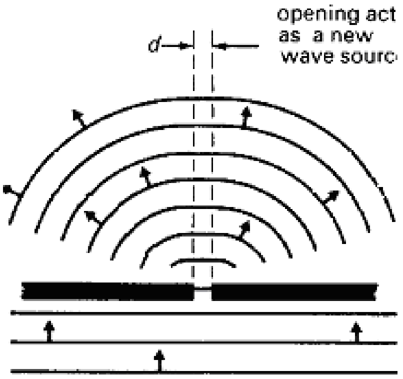
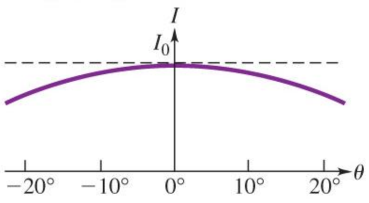
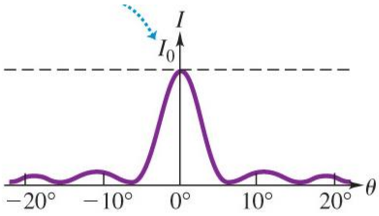
$$\boxed{\sin \theta = \lambda / b}$$

where θ is the angle that the first minima makes with the central maxima

λ is the wavelength of the light

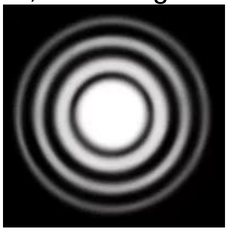
b is the width of the slit

Note: The derivation of this relationship is discussed in Appendix.

b >> λ	b ≈ λ
	
 One broad maxima	 Series of maxima & minima
When b>> λ, sin θ and hence θ is very small. The minima and maxima are so close to each other, such that practically the whole of the light is then in a direction immediately in front of the incident direction. Therefore no spreading occurs.	When b ≈ λ, then sin θ and hence θ will be large. The light waves now spread on either side of the slit, that is, the effect of diffraction is significant.
Very little diffraction	Appreciable diffraction

5.1 DIFFRACTION BY CIRCULAR APERTURE

Cameras, telescopes, binoculars, microscopes-practically all optical instruments, including the human eye, admit light through circular apertures. Thus, the diffraction of light through a circular aperture is of great importance.

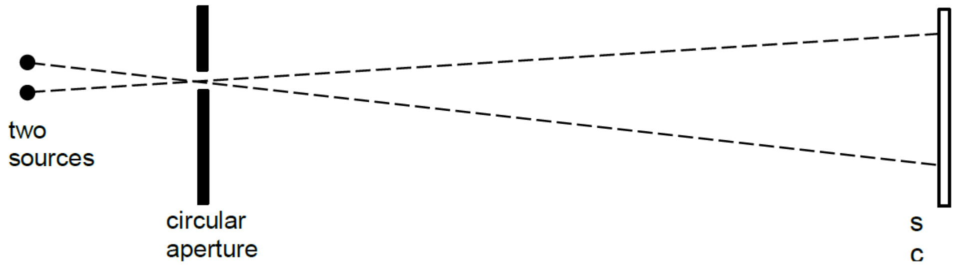


When light passes through a circular aperture of diameter b, the light spreads out in all directions. The diffraction pattern has a wide, bright central maximum, beyond which minima and weaker maxima alternate.

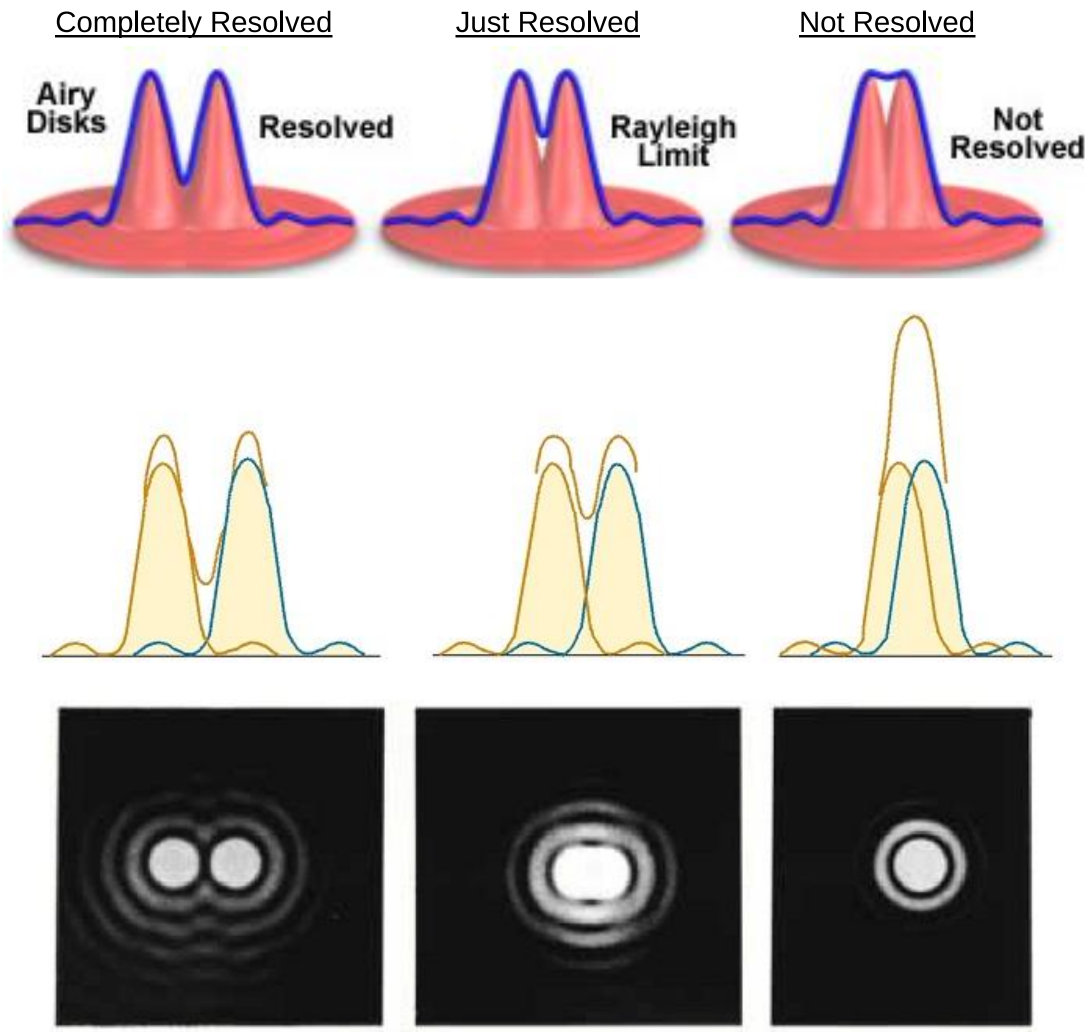
5.2 RESOLUTION POWER

If an instrument is to resolve (distinguish) two objects as being separate entities, it must form separate images of the two. If diffraction spreads out the image of each object enough that they overlap, the instrument cannot resolve them.

Suppose light from two sources travels through vacuum (or air) and enters a circular aperture of diameter b.



Rayleigh's criterion states that two sources are just resolved if the central maximum of one diffraction pattern falls on the first minimum of the diffraction pattern of the other.



When the angular separation θ of the two sources is small, sin θ is approximately θ. Therefore the limit of the angular separation θ_{res} such that the sources are just resolved is given by

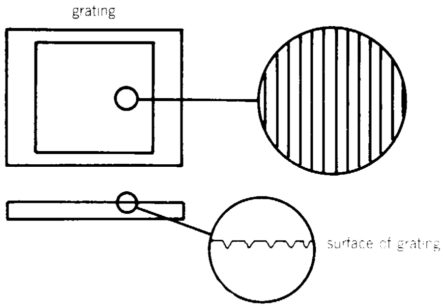
$$\boxed{\theta_{res} = \lambda/b}$$

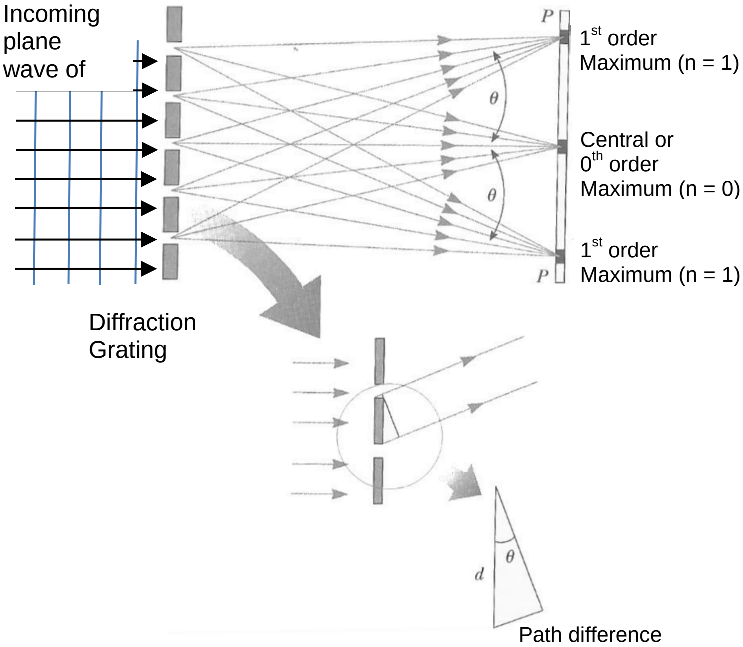
6 DIFFRACTION GRATING

We have examined how the wavelength of light can be determined from Young's Double Slit experiment. Another useful experiment to do so involves the use of a diffraction grating.

- A diffraction grating is a large number of close parallel equidistant slits, ruled on glass or metal by a diamond point.

- There are typically 400 to 1200 slits per mm, causing its slit separation *a* to be very much smaller than a pair of double slit (like in Young's Experiment).





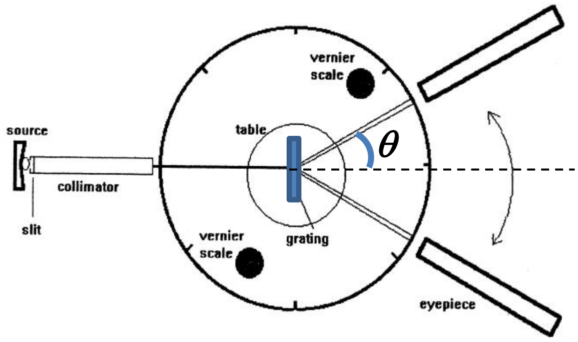


Fig. 5.1 Experimental Setup

- Referring to the experimental setup in Fig. 5.1, a diffraction grating is mounted on a spectrometer. The bright fringes are located by revolving the eyepiece to measure angle θ.

- The slit separation, *a,* which was introduced in Young's Double Slit Experiment is denoted by <u>*d*, grating spacing in Diffraction Grating</u> to represent distance between adjacent slits.

- Diffraction gratings are usually specified by the number of lines per unit length. N. The relationship between d and N is $d = \frac{1}{N}$. Eg. for a grating with 500 lines per mm, the spacing (*d*) between adjacent lines (or slits) is 1/500 mm = 2 x 10^{-6} m.

- For the nth order maxima,

$$\boxed{d \sin \theta = n \lambda}$$

Where *d* is the separation between slits
λ is the wavelength of light used
n is the order of the maxima
θ is the angle that the nth order maxima makes with the central maxima

Note: The derivation of this relationship is discussed in Appendix.

- In the double slit experiment, we are more concerned with fringe separation. But in diffraction grating, the fringe separation is no longer constant, in fact they increase as the order increases. Hence we are now more concerned about the angular position of each maxima from the principle axis.

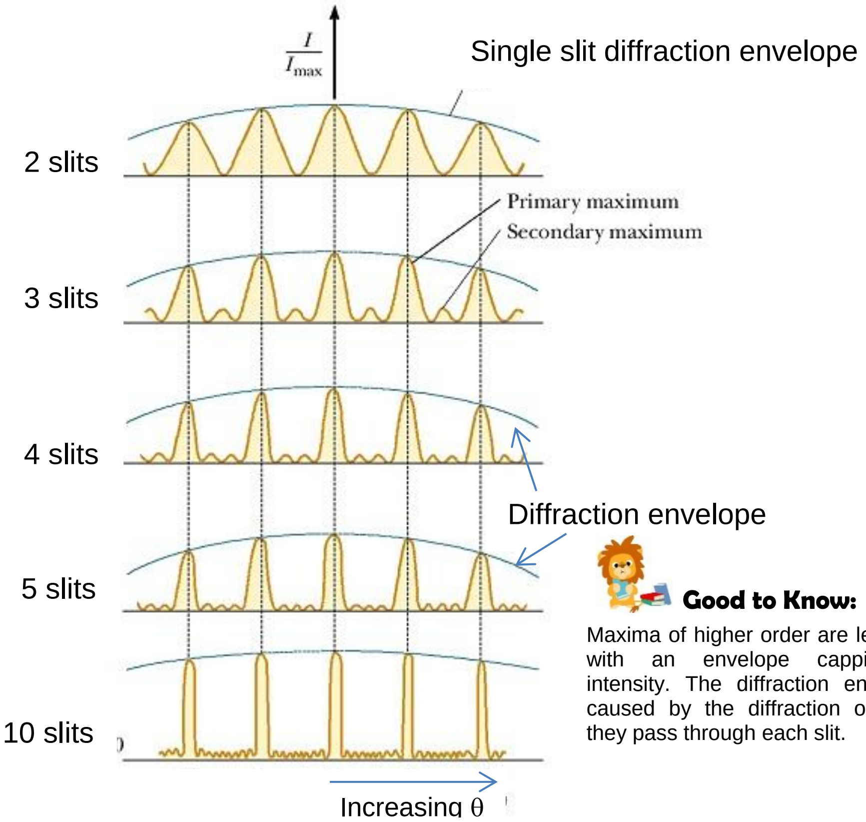
- The pattern is symmetrical around the central maxima. If there are n maxima above the central maxima, then there will also be n maxima below it.

- Since the angular position cannot exceed 90°, there is a maximum number of orders visible. To find the maximum order, let $sin\theta \leq 1$

$$\frac{n\lambda}{d} \leq 1$$

$$n \leq \frac{d}{\lambda}$$

- Let us compare the diffraction pattern from a Young's Double Slit fringe pattern:
    - Positions of maxima remain fixed
    - As the number of slits increases, each maxima becomes narrower (ie. fringes become sharper)
    - As the number of slits increases, intensity of maxima increases. This is because each maximum is the result of constructive interference of light from each slit.



Good to Know:
Maxima of higher order are less bright, with an envelope capping their intensity. The diffraction envelope is caused by the diffraction of light as they pass through each slit.

- Therefore, for a diffraction grating with thousands of slits per mm, the fringes are much sharper and brighter.

Food for Thought…
What are the advantages of using diffracting grating over double slits to determine the wavelength of light?

Example 8

Monochromatic light of wavelength 500 nm is incident normally on a grating. If the third order maximum of the diffraction pattern is observed at 32.0°, how many diffraction maxima can be observed using the eyepiece?

$d \sin\theta = n\lambda$

$d \sin 32.0 = 3(500 \times 10^{-9})$ --- (1)

$d \sin 90.0 = n_{max}(500 \times 10^{-9})$ --- (2)

$n_{max}/3 = \sin 90.0/\sin 32.0$

$n_{max} = 5$ (calculated to be 5.7)

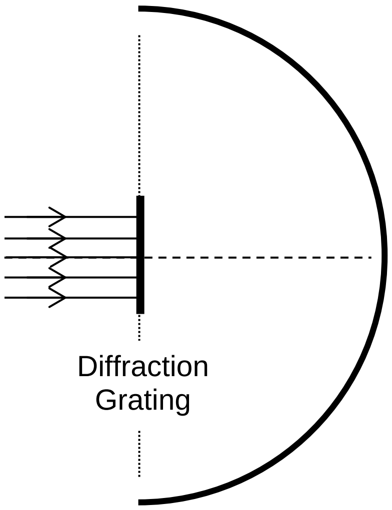
Total no. of maxima = 5 x 2 +1 = 11

Example 9

White light of wavelength 400 nm to 700 nm passes through a diffraction grating of 500 lines per mm.

(a) On the semi-circular screen below, mark the limits of the 1st, 2nd and 3rd order spectrum.

(b) Determine the number of complete spectra observed on the screen.



$d = 1/N = 2 \times 10^{-6}$ m

1st order spectrum:

$d \sin\theta = n\lambda$

$2 \times 10^{-6} \sin \theta_{1v} = 1(4\times10^{-7})$

$\theta_{1v} = 11.5°$

$2 \times 10^{-6} \sin \theta_{1R} = 1(7\times10^{-7})$

$\theta_{1R} = 20.5°$

1st order spectrum spans from 11.5° to 20.5° on both sides of the zeroth order maximum.

$d = 1/N = 2 \times 10^{-6}$ m

2nd order spectrum:

$d \sin\theta = n\lambda$

$2 \times 10^{-6} \sin \theta_{2V} = 2(4\times10^{-7})$

θ_{2V} = 23.6°

2 x 10^{-6} sin θ_{2R} = 2 (7x10^{-7})

θ_{2R} = 44.4°

2nd order spectrum spans from 23.6° to 44.4° on both sides of the zeroth order maximum.

d = 1/N = 2 x 10^{-6} m

3rd order spectrum:

d sinθ = nλ

2 x 10^{-6} sin θ_{3V} = 3(4x10^{-7})

θ_{3V} = 36.9°

2 x 10^{-6} sin θ_{3R} = 3 (7x10^{-7})

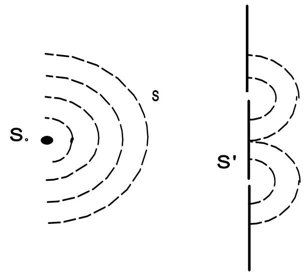
θ_{3R} = impossible!

An incomplete 3rd order spectrum spans from 36.9° onwards on both sides of the zeroth order maximum.

7 APPENDIX

7.1 COHERENCE OF LIGHT

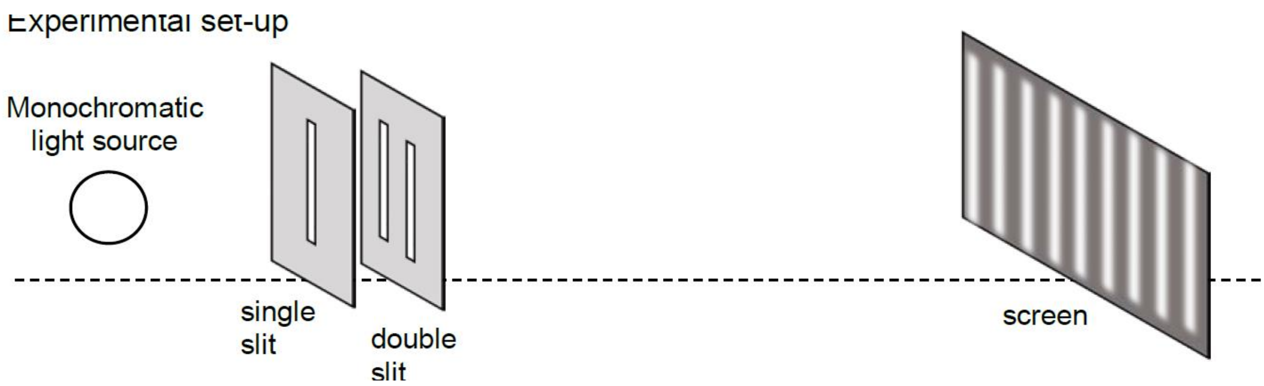
In the Young's Double Slit Experiment, why do we need to split a single light source into two sources instead of directly using two separate light sources?



- Light sources do not generate continuous streams of light waves. Instead, electrons within atoms of the source lose energy and radiate light in an unsynchronized and random manner, generating light in pulses.
- This results in light waves from any source having minor imperfections in them as shown below.

- Hence separate beams of light emitted from two different sources have no fixed phase relation with each other.
- If, on the other hand, light from a single source is split into two, the two secondary sources will have all of their imperfections occurring simultaneously. The light in these two secondary sources is said to be **coherent** i.e. there is a constant phase relationship between the two waves.

7.2 PROOF OF FRINGE SEPARATION IN YOUNG'S DOUBLE SLIT EXPERIMENT $x = \lambda D/a$



Top View

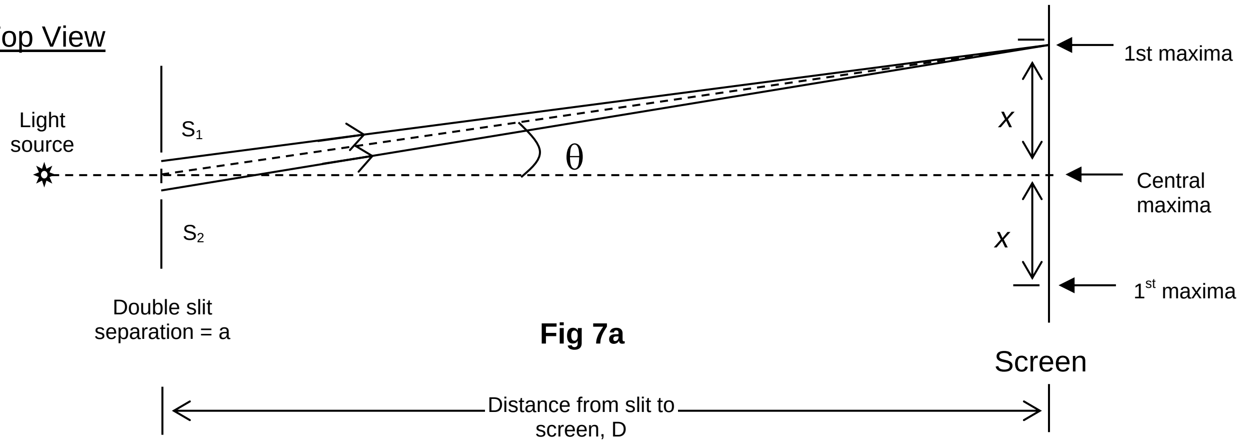
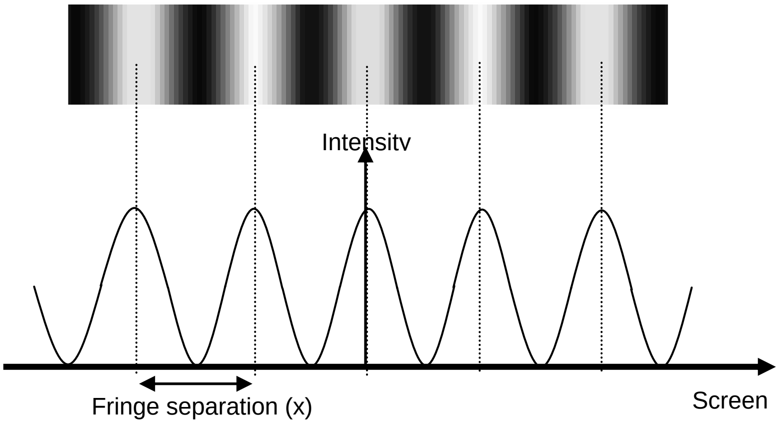


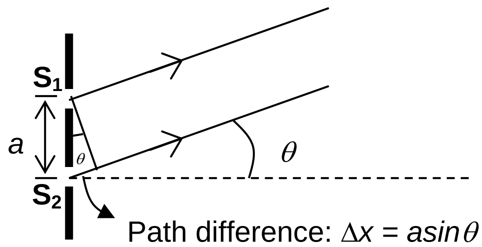
Fig 7a



- Assume that S_1 and S_2 are equidistant from the light source $\Rightarrow \Delta\phi_{source} = 0$
- For regions of constructive interference, path difference = nλ where n = 0, integer.

  For regions of destructive interference, path difference = (n + ½)λ, where n = integer.
  ****Note that these formulae are only true for sources which are in phase.**
- Referring to the diagram on the right, path difference between the waves from S_1 and S_2, $\Delta x = S_2P - S_1P = a\sin\theta$
- For the nth maximum, $\Delta x = a\sin\theta = n\lambda$

  For the nth minimum, $\Delta x = a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$



- For interference of light, θ is small, hence by small angle approximation,

  sinθ ≈ tanθ $= \frac{x}{D}$ [see Fig 7a]

- For nth maximum, using $a\sin\theta = n\lambda$:

  $a\left(\frac{x_n}{D}\right) = n\lambda \quad \Rightarrow \quad x_n = \frac{n\lambda D}{a}$ ---------- {7.1}

  Similar, for (n+1)th maximum:

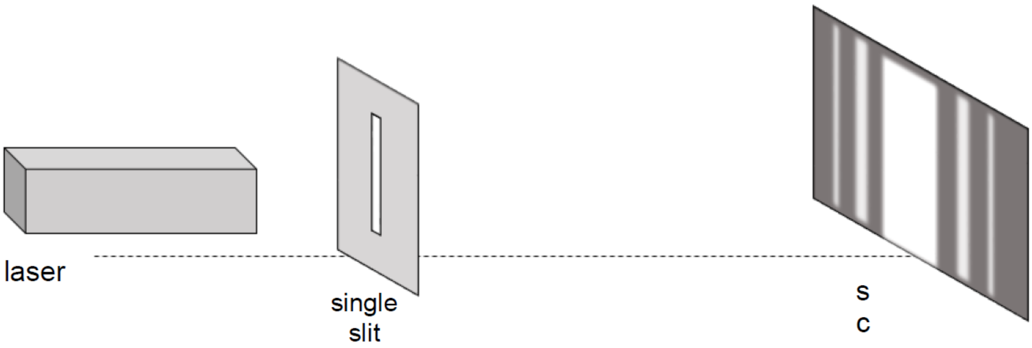
  $a\left(\frac{x_{n+1}}{D}\right) = (n+1)\lambda \quad \Rightarrow \quad x_{n+1} = \frac{(n+1)\lambda D}{a}$ ---------- {7.2}
- The distance between two maxima or bright fringes is called Fringe separation, $x = x_{n+1} - x_n$

$$\boxed{x = \frac{\lambda D}{a}}$$

Where λ is the wavelength
D is the distance between the screen and the double slit
a is the distance separating the two slits (slit separation)

Note: The above formula is derived based on the condition that the slit separation *a* is much smaller than the distance between the slits and the detector/screen. Hence, use it **only** when such conditions are met.

7.3 PROOF OF ANGULAR POSITION OF 1ST ORDER MINIMA OF A SINGLE SLIT DIFFRACTION PATTERN

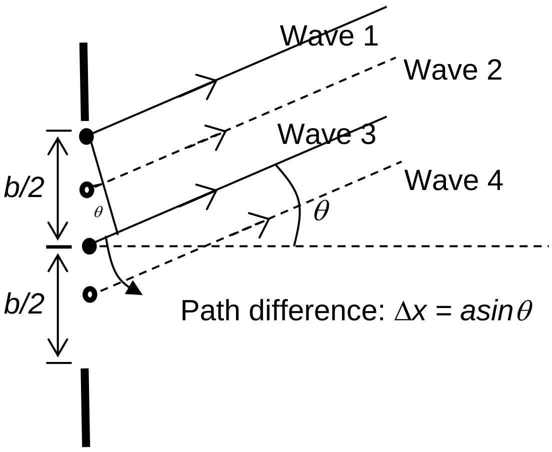


The result of having an interference pattern should be surprising as it appears that there is only a single source of light. We shall now examine the position of minima can be determined.

- According to Huygens's principle, each portion of the slit acts as a source of waves. Hence light from one portion of the slit can interfere with light from another portion.

- First, we divide the slit into two halves. All waves that originate from the slit are in phase. As the screen is distant, the rays are approximately parallel.

- Consider wave 1 and 3 which are b/2 apart. Path difference = b/2 sin θ



- At a point P where the two wave meet, if the paths difference is $\lambda/2$, the two waves meet in anti-phase and destructive intereference occurs. The first minimum is observed.
- This can be applied to any other two waves that are b/2 apart at the slit, because the phase difference of any two of such waves at P will always be an odd multiple of pi. Therefore, waves from the upper half of the slit interfere destructively with waves from the lower half of the slit when

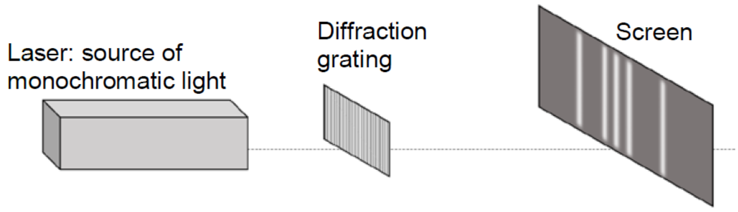
b/2 sin $\theta = \lambda/2$

sin $\theta = \lambda/b$ where θ is the angle of the first minimum

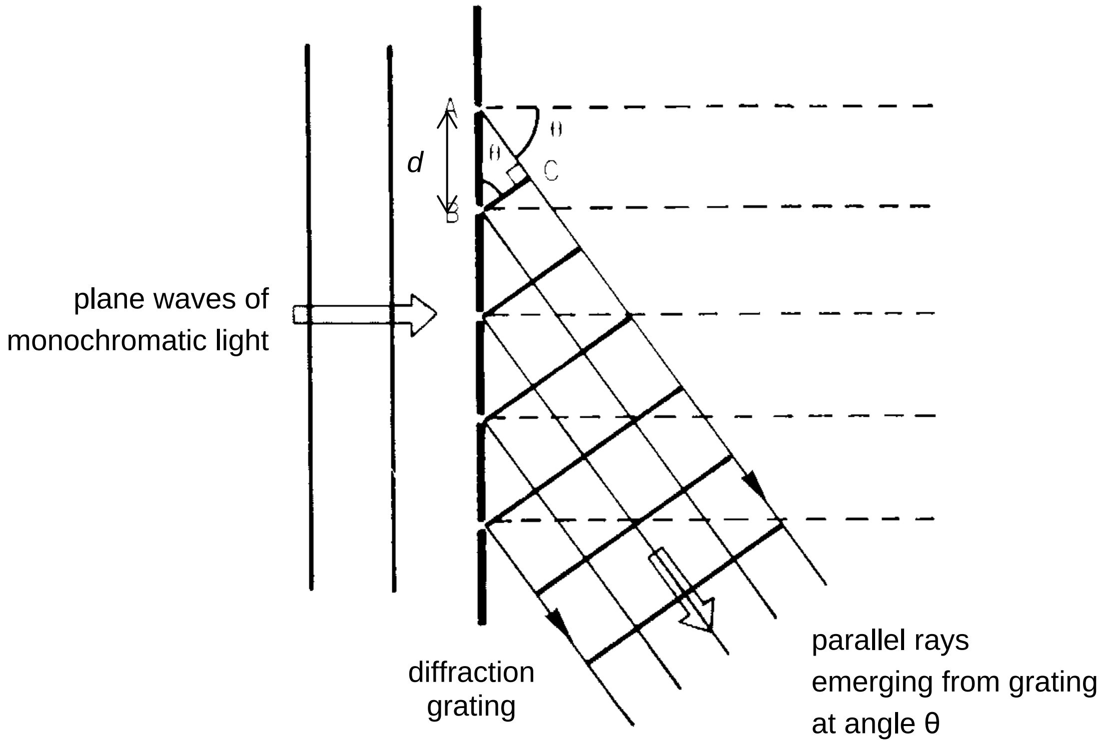
7.4 PROOF OF ANGULAR SEPARATION IN DIFFRACTION GRATING

$d \sin\theta = n\lambda$

Experimental Setup



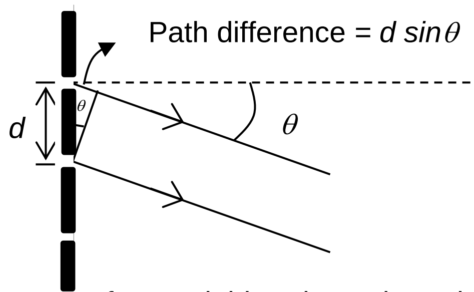
Top View



Suppose plane waves of monochromatic light with wavelength λ fall on a diffraction grating whose slit separation is d.

The waves will diffract at each slit of the diffraction grating.

Consider wavelets coming from points A, B, and other similar points travelling at an arbitrary angle θ to the direction of the incident beam.



Path difference, AC, between waves from neighbouring point = $d \sin\theta$

A condition for constructive interference (bright spot) to occur is that the path difference between all wavelets = $n\lambda$

Thus the equation for grating experiment is $\boxed{d \sin\theta = n\lambda}$