XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME	Mark Scheme [Draft 3]		
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **23 August 2024**

Secondary 4 Express **2 hour 15 minutes**

Setter : Mr Johnson Chua
Vetter : Ms Low Yan Jin
Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		***Marks Awarded***	
Presentation		***Marks Penalised***	

For Examiner's Use
90

Parent's/Guardian's Signature:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\displaystyle \binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan (A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A triangle has a base of length $\left(6+2\sqrt{7}\right)$ cm and an area of $\left(17+7\sqrt{7}\right)$ cm^2.

Find, **without using a calculator**, the perpendicular height to the base of the triangle, in cm, in the form $\left(a+b\sqrt{7}\right)$, where a and b are integers. [3]

$$\frac{1}{2}\left(6+2\sqrt{7}\right)\times h=\left(17+7\sqrt{7}\right) \text{ --- [M1]}$$

$$h=\frac{2\left(17+7\sqrt{7}\right)}{6+2\sqrt{7}}$$

$$h=\frac{\left(17+7\sqrt{7}\right)\left(3-\sqrt{7}\right)}{\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right)} \text{ --- [M1]}$$

$$h=\frac{51-17\sqrt{7}+21\sqrt{7}-49}{3^2-7}$$

$$h=\frac{2+4\sqrt{7}}{2}$$

$$h=1+2\sqrt{7} \text{ ---[A1]}$$

Alternative

$$h=\frac{\left(34+14\sqrt{7}\right)\left(6-2\sqrt{7}\right)}{\left(6+2\sqrt{7}\right)\left(6-2\sqrt{7}\right)} \text{ --- [M1]}$$

$$h=\frac{204-68\sqrt{7}+84\sqrt{7}-196}{36-4(7)}$$

$$h=\frac{8+16\sqrt{7}}{8}$$

$$h=1+2\sqrt{7} \text{ ---[A1]}$$

2 Solve the equation $\sqrt{5-\sqrt{x+1}}=\sqrt{x}$. [4]

$$5-\sqrt{x+1}=x \text{ --- [M1]}$$

$$5-x=\sqrt{x+1}$$

$$\left(5-x\right)^2=x+1$$

$$25-10x+x^2=x+1$$

$$\left.\begin{aligned}&x^2-11x+24=0\\&\left(x-3\right)\left(x-8\right)=0\end{aligned}\right\}$$

Students must show the relevant method in solving quad. equation (either factorisation or quad. formula). Failure to do so would result in the loss of M1; A1 would still be awarded accordingly.

$$\underbrace{x=3}_{\text{[A1]}} \text{ or } \underbrace{x=8}_{\text{[A1: no A1 if students do not reject]}} \text{ (rej)}$$

3 The equation of a curve is $y=\dfrac{4x+2}{\sqrt{x+1}}$, where $x > -1$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$, leaving your answer in the form $\dfrac{ax+b}{\sqrt{(x+1)^n}}$. [2]

Method 1: Quotient Rule	**Method 2: Product Rule**
$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{4\sqrt{x+1}-(4x+2)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}}{x+1}$ --- [M1] $=\dfrac{4\sqrt{x+1}-\frac{2x+1}{\sqrt{x+1}}}{x+1}$ $=\dfrac{4(x+1)-(2x+1)}{(x+1)^{\frac{3}{2}}}$ $=\dfrac{2x+3}{\sqrt{(x+1)^3}}$ ---[A1]	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=(4x+2)\left(-\frac{1}{2}\right)(x+1)^{-\frac{3}{2}}(1)+(x+1)^{-\frac{1}{2}}(4)$ --- [M1] $=-\dfrac{2x+1}{\sqrt{(x+1)^3}}+\dfrac{4}{\sqrt{x+1}}$ $=\dfrac{-2x-1+4(x+1)}{\sqrt{(x+1)^3}}$ $=\dfrac{-2x-1+4x+4}{\sqrt{(x+1)^3}}$ $=\dfrac{2x+3}{\sqrt{(x+1)^3}}$ --- [A1]

(b) Explain why the curve is an increasing function. [2]

Since $x>-1$,
$\sqrt{(x+1)^3}>0,$ and $2x+3>0$ } M1

$\therefore \dfrac{2x+3}{\sqrt{(x+1)^3}}>0,$ for $x>-1 \rightarrow \dfrac{dy}{dx}>0$

Since $\dfrac{dy}{dx}>0, y$ is an increasing function. } A1

4 The line $2x+3y=12$ intersects the curve $y^2=4x-8$ at points A and B.

Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$. [6]

$2x+3y=12$ --- (1)

$y^2=4x-8$ --- (2)

From (1):

$2x=12-3y$ ---(3)

Subst (3) into (2):

$y^2=2(12-3y)-8$ --- [M1]

$y^2=24-6y-8$

$y^2+6y-16=0$ --- [M1] No M1 if students do not show working for factorisation/ quadratic formula. Award the remaining marks accordingly.

$(y-2)(y+8)=0$

$y=2$ or $y=-8$

when $y=2$,

$2x=12-3(2)$

$x=3$

$\therefore A(3,2)$ --- [A1]

when $y=-8$,

$2x=12-3(-8)$

$x=18$

$\therefore B(18,-8)$ --- [A1]

Length of $AB:\sqrt{(18-3)^2+(-8-2)^2}$ --- [M1, allow ecf]

$=\sqrt{325}$

$=5\sqrt{13}$

$\therefore [\,p=5, q=13\,]$--- [A1]

Alternative

$2x+3y=12$ --- (1)

$y^2=4x-8$ --- (2)

From (1):

$y=\dfrac{12-2x}{3}$ ---(3)

Subst (3) into (2):

$\left(\dfrac{12-2x}{3}\right)^2=4x-8$ --- [M1]

$\dfrac{(12-2x)^2}{9}=4x-8$

$144-48x+4x^2=36x-72$

$4x^2-84x+216=0$ ---[M1]

$x^2-21x+54=0$

$(x-3)(x-18)=0$

$x=3$ or $x=18$

when $x=3$,

$y=\dfrac{12-2(3)}{3}$

$y=2$

$\therefore A(3,2)$ --- [A1]

when $x=18$,

$y=\dfrac{12-2(18)}{3}$

$y=-8$

$\therefore B(18,-8)$ --- [A1]

Length of $AB:\sqrt{(18-3)^2+(-8-2)^2}$ --- [M1]

$=\sqrt{325}$

$=5\sqrt{13}$

$\therefore [\,p=5, q=13\,]$--- [A1]

5 The height, h m, of a baseball above ground t seconds after it has been hit is given by $h = c + 24t - 4t^2$, where c is a constant.

(a) If c = 1.65, express h in the form $h = p + q(t+r)^2$ where p, q and r are constants to be determined. Hence, state the maximum height attained by the baseball and the time at which this occurs. [4]

$h = 1.65 + 24t - 4t^2$

$= 1.65 - 4(t^2 - 6t)$

$= 1.65 - 4(t^2 - 6t \underline{+3^2}) + \underline{4(3^2)}$ --- [M1 for completing the square. Awd if wrong c val subst.]

Note: M1 is not awarded if students wrote $(-3)^2$

$= 37.65 - 4(t-3)^2$ --- [A1]

max. height: 37.65 ---[A1]
time: 3 seconds or $t = 3$ ---[A1]

Awarded even if completed square form is partially correct.

(b) Find the range of values of c if the baseball did not reach a height of 40 m. [2]

$-4t^2 + 24t + c = 40$

$-4t^2 + 24t + c - 40 = 0$

Since ball did not reach 40m,
there is no solution to the equation.

$\therefore\ b^2 - 4ac < 0$

$a = -4,\ b = 24,\ c = c - 40$

$24^2 - 4(-4)(c-40) < 0$ --- [M1]

$576 - 640 + 16c < 0$

$16c < 64$

$c < 4$ --- [A1]

Alternatively

$h = c + 24t - 4t^2$

$= -4\left(t^2 - 6t - \frac{c}{4}\right)$

$= -4[(t-3)^2 - (3)^2 - \frac{c}{4}]$

$= -4[(t-3)^2 - 9 - \frac{c}{4}]$

$= -4(t-3)^2 + 36 + c$ --- [M1]

$36 + c < 40$

$c < 4$ --- [A1]

6 A curve is such that $\frac{d^2y}{dx^2}=12e^{6x}+15e^{-3x}$.

The point $P(0,-2)$ lies on the curve and the normal to the curve at P is parallel to the y-axis. Find the equation of the curve. [6]

$$\frac{dy}{dx}=\int 12e^{6x}+15e^{-3x}\ dx$$

$$=\frac{12e^{6x}}{6}+\frac{15e^{-3x}}{-3}+c \text{ --- [M1]}$$

$$=2e^{6x}-5e^{-3x}+c$$

at $x=0$, $\frac{dy}{dx}=2-5+c$ --- [M1 for subst. $x=0$ into $\frac{dy}{dx}$]

$$=c-3$$

$m_{\text{tangent at P}}:\ 0$

$\therefore c-3=0$ --- [M1]

$c=3$

$$\therefore\ \frac{dy}{dx}=2e^{6x}-5e^{-3x}+3$$

$$y=\int 2e^{6x}-5e^{-3x}+3\ dx$$

$$=\frac{2e^{6x}}{6}-\frac{5e^{-3x}}{-3}+3x+d \text{ --- [M1, allow ecf]}$$

$$=\frac{e^{6x}}{3}+\frac{5e^{-3x}}{3}+3x+d$$

at $x=0$, $y=-2$,

$\frac{1}{3}+\frac{5}{3}+d=-2$ ---[M1, allow ecf]

$2+d=-2$

$d=-4$

$$\therefore y=\frac{e^{6x}}{3}+\frac{5e^{-3x}}{3}+3x-4 \text{ ---[A1]}$$

7 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x-axis. [3]

$a = k, b = k, c = p$

$k^2 - 4(k)(p) < 0$ ---[M1]

$k(k - 4p) < 0$

Since $k > 0, k - 4p < 0$ ---[M1, need to see both inequalities formed]

$\therefore \quad 4p > k$

$p > \frac{k}{4}$ ---[A1]

(b) In the case where k = 2 and p = 4, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

at $k = 2, p = 4$: $y = 2x^2 + 2x + 4$

$2x^2 + 2x + 4 = mx - 4$ --- [M1]

$2x^2 + (2 - m)x + 8 = 0$

Since $y = mx - 4$ is a tangent to the curve,

$(2 - m)^2 - 4(2)(8) = 0$ --- [M1]

$4 - 4m + m^2 - 64 = 0$

$m^2 - 4m - 60 = 0$

$(m + 6)(m - 10) = 0$

Students must show the relevant method in solving quad. equation (either factorisation or quad. formula). Failure to do so would result in the loss of M1; A1 would still be awarded accordingly.

$m = -6$ or $m = 10$ ---[A1]

8 **(a)** Divide $2x^3 + 5x^2 + 5x + 9$ by $x^3 + 3x$. [1]

$$2 + \frac{5x^2 - x + 9}{x^3 + 3x} \text{ --- [B1]}$$

(b) Express $\dfrac{2x^3 + 5x^2 + 5x + 9}{x^3 + 3x}$ in partial fractions. [5]

$$\frac{5x^2 - x + 9}{x^3 + 3x} = \frac{5x^2 - x + 9}{x(x^2 + 3)}$$

$$\frac{5x^2 - x + 9}{x(x^2 + 3)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 3} \text{ --- [M1]}$$

$$5x^2 - x + 9 = A(x^2 + 3) + (Bx + C)x$$

when $x = 0$, $9 = 3A \Rightarrow A = 3$ ---[M1]

By comparing coefficients: $-1 = C$ ---[M1]

$5 = A + B \Rightarrow B = 2$ ---[M1]

> If students made error in partial fraction decomposition or long division, award M1 (ecf) for correctly applying substitution/ comparing coefficients to find numerators. Max. 1mark.

$$\therefore \frac{2x^3 + 5x^2 + 5x}{x^3 + 3x} = 2 + \frac{3}{x} + \frac{2x - 1}{x^2 + 3} \text{ ---[A1]}$$

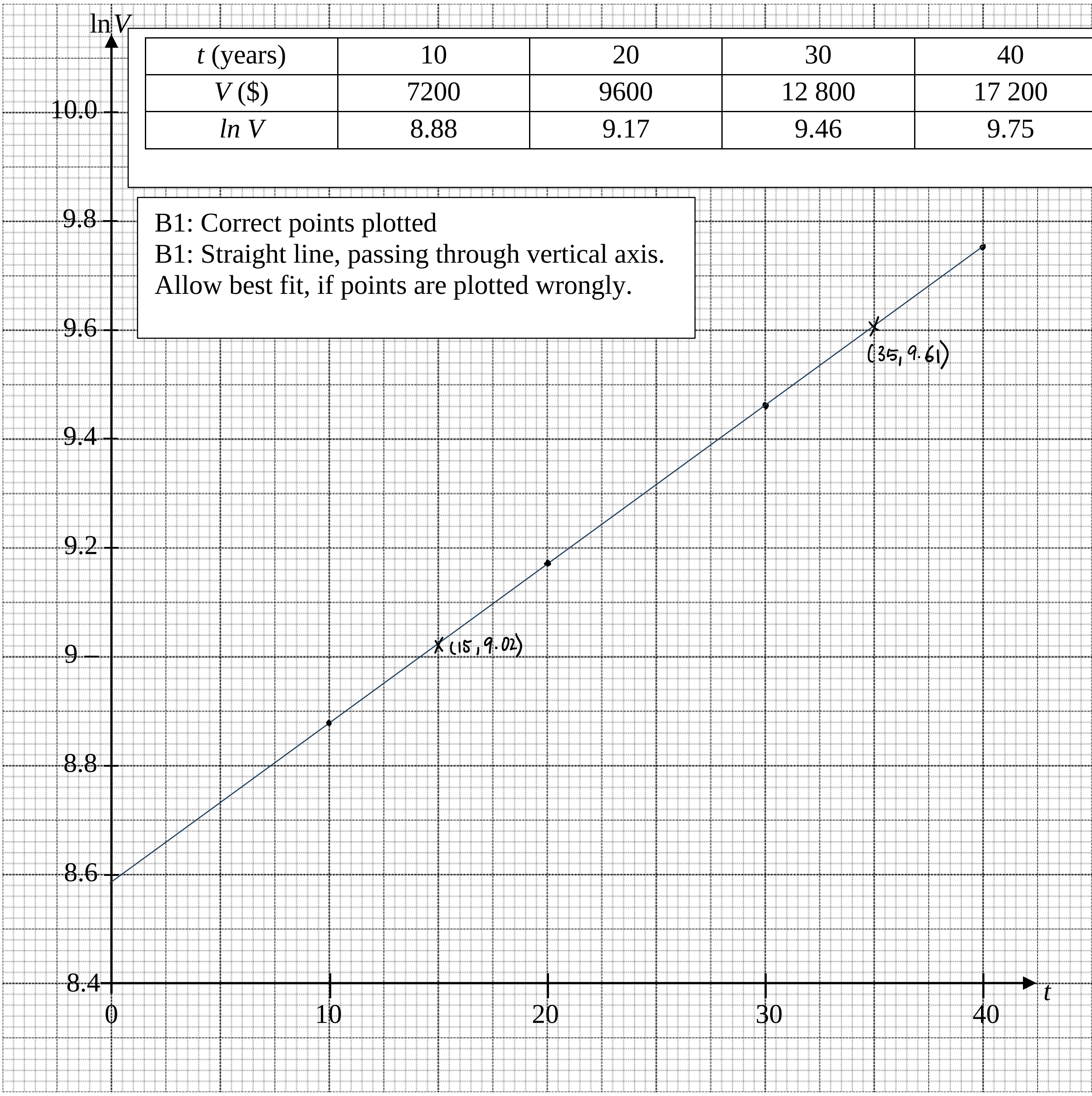
9 The value, $\$V$, of a watch is related to t, the number of years since 1980.
The table below gives the value of the watch in 1990, 2000, 2010, 2020.

Year	1990	2000	2010	2020
t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200

(a) Plot $\ln V$ against t and draw a straight line graph to illustrate the information. [2]

t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200
ln V	8.88	9.17	9.46	9.75

B1: Correct points plotted
B1: Straight line, passing through vertical axis.
Allow best fit, if points are plotted wrongly.



(b) Find the gradient and the intercept of the vertical axis of your straight line graph. Hence, express V in the form Ae^{kt}, where A and k are constants. [4]

Using 2 points on straight line graph plotted:
$(15, 9.02)$ and $(35, 9.61)$

$$m = \frac{9.61 - 9.02}{35 - 15}$$
$= 0.0295$ --- [M1]
(Acceptable range: 0.0275 to 0.0315)

$C = 8.59$ (Accept: 8.57, 8.58, 8.6, 8.61) ---[A1]

$\ln V = 0.0295t + 8.59$ ---[M1]
$V = e^{0.0295t+8.59}$
$V = e^{0.0295t} \cdot e^{8.59}$
$V = 5380e^{0.0295t}$ (3 s.f.) ---[A1]

(c) Explain what the constant A represents. [1]

A represents the **value** of the watch in **1980**. --- [B1]

10 (a) (i) Write down the first four terms in the expansion of $(3-2x)^7$. [2]

$$(3-2x)^7 = 3^7 + \binom{7}{1}(3)^6(-2x)^1 + \binom{7}{2}(3)^5(-2x)^2 + \binom{7}{3}(3)^4(-2x)^3 + \ldots \text{ ---[M1]}$$

$$= 2187 + 5103(-2x) + 5103(4x^2) + 2835(-8x^3) + \ldots$$

$$= 2187 - 10206x + 20412x^2 - 22680x^3 + \ldots$$

[A1: do not award A1 if students did not give four terms]

(ii) Find the coefficient of x^3 in the expansion of $(1-7x^2)(3-2x)^7$. [2]

$(1-7x^2)(2187-10206x+20412x^2-22680x^3+\ldots)$

Coefficient of x^3 : $1(-22680)-7(-10206)$ ---[M1: award if seen/implied; allow ecf]

= 48762 ---[A1]

10 (b) In the binomial expansion of $\left(x+\frac{k}{x}\right)^{11}$, where k is a positive constant, the coefficient of $\frac{1}{x^5}$ is 8 times the coefficient of x^3. Find the value of k. [5]

$$T_{r+1}=\binom{11}{r}x^{11-r}\left(\frac{k}{x}\right)^r \text{ --- [M1]}$$
$$=\binom{11}{r}x^{11-2r}k^r$$

For x^3 term:
$11-2r=3$
$2r=8$
$r=4$

Coefficient of x^3: $\binom{11}{4}k^4=330k^4$

M1 for either correct

For x^{-5} term:
$11-2r=-5$
$2r=16$
$r=8$

M1 for either correct

Coefficient of x^{-5}: $\binom{11}{8}k^8=165k^8$

$\therefore \frac{165k^8}{330k^4}=8$ --- [M1]
$k^4=16$
$k=2$ --- [A1]

11 The triangle ABC is such that A is $(9,9)$, B is $(1,-3)$ and C is (p,q) where $q > p$.

C lies on the perpendicular bisector of AB and area of triangle ABC is 26 units^2.

(a) Find the equation of the perpendicular bisector of AB. [4]

$$m_{AB} = \frac{9-(-3)}{9-1} = \frac{3}{2}$$

$$m_{\text{perpendicular bisector}} = -\frac{2}{3} \text{ ---[M1]}$$

$$\text{midpoint of AB}: \left(\frac{9+1}{2}, \frac{9-3}{2}\right) = (5,3) \text{ ---[M1]}$$

Equation of perpendicular bisector:

$$y-3 = -\frac{2}{3}(x-5) \text{ ---[M1]}$$

$$y = -\frac{2}{3}x + 6\frac{1}{3} \text{ ---[A1]}$$

(b) Find the coordinates of C. [3]

coordinates of $C:(p,q)$

$$\frac{1}{2}\begin{vmatrix} 9 & p & 1 & 9 \\ 9 & q & -3 & 9 \end{vmatrix} = 26 \text{ --- [M1]}$$

$$\frac{1}{2}\left[(9q-3p+9)-(9p+q-27)\right] = 26$$

$$9q-3p+9-9p-q+27 = 52$$

$$8q-12p = 16$$

$$2q-3p = 4 \quad ---(1)$$

$$q = -\frac{2}{3}p + 6\frac{1}{3} \quad ---(2)$$

Sub (2) into (1)

$$2(-\frac{2}{3}p + 6\frac{1}{3}) - 3p = 4$$

$$2(-2p+19) - 9p = 12$$

$$-4p + 38 - 9p = 12$$

$$-13p = -26$$

$$p = 2 \text{ --- [M1]}$$

$$\text{when } p = 2, q = -\frac{2}{3}(2) + 6\frac{1}{3}$$

$$q = 5$$

$$\therefore C(2,5) \text{ --- [A1]}$$

Alternative

$$AB = \sqrt{(9-1)^2 + (9+3)^2}$$

$$= \sqrt{208}$$

$$\frac{1}{2} \times \sqrt{208} \times h = 26 \text{ --- [M1]}$$

$$h = \frac{52}{\sqrt{208}}$$

$$= \frac{13}{\sqrt{13}}$$

$$\sqrt{(x-5)^2 + \left(-\frac{2}{3}x + 6\frac{1}{3} - 3\right)^2} = \frac{13}{\sqrt{13}} \text{ ---[M1]}$$

$$x^2 - 10x + 25 + \left(-\frac{2}{3}x + 3\frac{1}{3}\right)^2 = 13$$

$$x^2 - 10x + 25 + \frac{4}{9}x^2 - \frac{40}{9}x + \frac{100}{9} = 13$$

$$13x^2 - 130x + 208 = 0$$

$$x^2 - 10x + 16 = 0$$

$$(x-8)(x-2) = 0$$

$$x = 8 \text{ or } x = 2$$

when $x = 8, y = 1$ --- (rej)

when $x = 2, y = 5$

$$\therefore C(2,5)\text{---[A1]}$$

12 **(a)** The graph of $y = a\sin bx + 2$ has one maximum point at $\left(\frac{5\pi}{4}, 7\right)$ and the next maximum point after this has coordinates $\left(\frac{9\pi}{4}, 7\right)$. Find the values of a and b. [2]

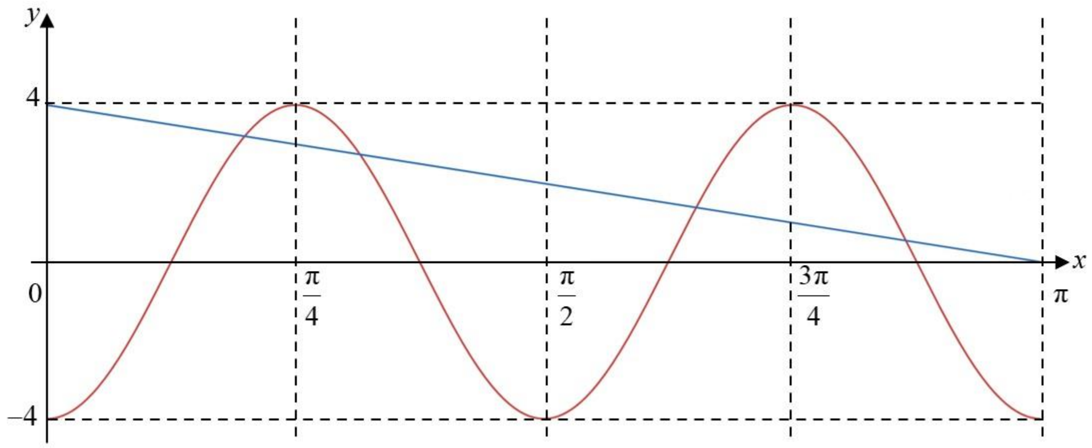
amplitude of graph: $7 - 2 = 5$

$\therefore a = 5$ ---[B1]

period of graph: $\frac{9\pi}{4} - \frac{5\pi}{4} = \pi$

$\frac{2\pi}{b} = \pi \to b = 2$ ---[B1]

(b) **(i)** Sketch, on the same diagram, the graph of $y = -4\cos 4x$ and $y = -\frac{4}{\pi}x + 4$ for $0 \le x \le \pi$ radians. [4]



B1: Correct shape and max/min. y value of cosine graph; B1: number of cycles and labelling on *x*-axis for cosine graph B1: Correct slope and *y*-int of linear graph; B1: correct *x*-int of linear graph.

(ii) Hence, state the number of solutions to the equation $\frac{x}{\pi} - \cos 4x - 1 = 0$ for $0 \le x \le \frac{\pi}{2}$. [2]

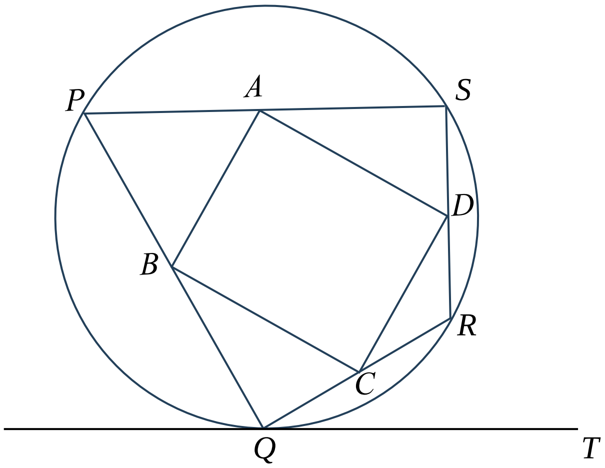
$$\left.\begin{aligned} &\frac{x}{\pi} - \cos 4x - 1 = 0 \\ &-\cos 4x = -\frac{x}{\pi} + 1 \\ &-4\cos 4x = -\frac{4}{\pi}x + 4 \end{aligned}\right\} \text{M1}$$

Students must show the manipulation of equation to contain the expression of the graphs on each side. Do not award any mark if students do not show any manipulation of the equation but wrote down the correct final answer.

From graph, there are 2 intersections for $0 \le x \le \frac{\pi}{2}$.

$\therefore$ no. of solutions: 2 --- [A1]

13



In the diagram, *PQRS* is a kite, where all four points lie on the circumference of the circle. *RS* = *RQ* and *PQ* = *PS*. *QT* is a tangent to the circle at *Q*.

(a) Show that $\angle QPS = 2 \times \angle RQT$. [3]

Let $\angle RQT = \alpha$

$\angle QPR = \angle RQT$ (alt. segment theorem / tan. chord theorem) --- [M1]

$= \alpha$

$\angle QPR = \angle SPR$ --- [M1]

$= \alpha$

$$\left.\begin{aligned} \angle QPS &= \angle QPR + \angle SPR \\ &= \alpha + \alpha \\ &= 2\alpha \\ &= 2 \times \angle RQT \text{ (shown)} \end{aligned}\right\} \text{[A1]}$$

Alternative

Let $\angle RQT = \alpha$

$\angle RQT = \angle QSR$ (alt. segment theorem / tan. chord theorem) --- [M1]

$\angle QSR = \angle SQR$ (base $\angle$, isos. triangles)

$\angle SRQ = 180° - \angle QSR - \angle SQR$ --- [M1]

$= 180° - 2\alpha$

$$\left.\begin{aligned} \angle QPS &= 180° - (180° - 2\alpha) \text{ } (\angle\text{s in opp. segment}) \\ &= 2\alpha \\ &= 2 \times \angle RQT \text{ (shown)} \end{aligned}\right\} \text{[A1]}$$

-1m from the whole of question 13 for missing/wrong geometrical reason(s).

(b) A circle can be drawn passing through *A*, *B*, *C* and *D*, with *BD* as the diameter. Given that *A*, *B*, *C* and *D* are midpoints of *PS*, *PQ*, *QR* and *SR* respectively, what can you deduce about quadrilateral *ABCD*? [4]

Since A and B are midpoints of PS and PQ,

by midpoint theorem, $AB = \frac{1}{2}QS$ and $AB // QS$.

Since C and D are midpoints of RQ and RS,

by midpoint theorem, $CD = \frac{1}{2}QS$ and $CD // QS$.

[M1] for either.

Since $AB = \frac{1}{2}QS$ and $CD = \frac{1}{2}QS \rightarrow AB = CD$

Since $AB // QS$ and $CD // QS \rightarrow AB // CD$

Since $AB = CD$ and $AB // CD$, *ABCD* is a parallelogram. --- [M1]

$\angle BAD = 90°$ (rt. angle in semicircle) --- [M1]

Since *ABCD* is a parallelogram with $\angle BAD = 90°$, *ABCD* is a rectangle. --- [A1]

<u>Alternative</u>

Since *B* and *C* are midpoints of *PQ* and *QR* respectively, by midpoint theorem,

$BC = \frac{1}{2}PR$ and $BC // PR$.

Since *A* and *D* are midpoints of *PS* and *SR* respectively, by midpoint theorem,

$AD = \frac{1}{2}PR$ and $AD // PR$.

[M1] for either.

$\therefore BC = AD$ and $BC // AD$ --- [M1]

Hence, *ABCD* is a parallelogram.

$\angle BAD$ (or $\angle BCD$) $= 90°$ (rt. angle in semi circle). --- [M1]

Since *ABCD* is a parallelogram with $\angle BAD = 90°$, *ABCD* is a rectangle. --- [A1]

-1m from the whole of question 13 for missing/wrong geometrical reason(s).

14 A particle travelling in a straight line, has a velocity, v m/s, at time t seconds, $t \geq 0$, given by $v = 3\sin 2t - 4\cos 2t$.

(a) Find the initial acceleration of the particle. [2]

$a = 3\left(\cos 2t\right)(2) - 4\left(-\sin 2t\right)(2)$

$= 6\cos 2t + 8\sin 2t$ --- [M1]

at $t = 0,\ a = 6\cos 0 + 8\sin 0$

$= 6$ --- [A1]

(b) Find the total distance travelled by the particle in the first 1.5 seconds. [8]

When particle comes to rest, $v = 0$,

$3\sin 2t - 4\cos 2t = 0$ --- [M1]

$3\sin 2t = 4\cos 2t$

$\dfrac{\sin 2t}{\cos 2t} = \dfrac{4}{3}$

$\tan 2t = \dfrac{4}{3}$ --- [M1]

Basic Angle: $\tan^{-1}\left(\dfrac{4}{3}\right) = 0.92729$ (5 s.f.)

$2t = 0.92729,\ \pi + 0.9272,\ ...$

$t = 0.46364$ (5 s.f.), ... --- [A1]

Continuation of working space for question **13(b).**

$s = \int 3\sin 2t - 4\cos 2t \ \mathrm{d}t$

$= -\frac{3}{2}\cos 2t - 2\sin 2t + c$ --- [M1 for correct integration of trigo functions]

at t = 0, s = 0:

$0 = -\frac{3}{2}\cos 0 - 2\sin 0 + c$ --- [M1]

$c = \frac{3}{2}$

$\therefore s = -\frac{3}{2}\cos 2t - 2\sin 2t + \frac{3}{2}$

at t = 0.46364, $s = -\frac{3}{2}\cos(2\times 0.46364) - 2\sin(2\times 0.46364) + \frac{3}{2}$ --- [M1, allow ecf]

$= -0.99999$ (5 s.f.)

at t = 1.5, $s = -\frac{3}{2}\cos(2\times 1.5) - 2\sin(2\times 1.5) + \frac{3}{2}$ --- [M1, allow ecf]

= 2.7027 (5 s.f.)

Total distance: $2\times 0.99999 + 2.7027 = 4.70$ m (3 s.f.) --- [A1]

END OF PAPER