

Answer all the questions.

- 1 Estimate of populations in the year 2012 is shown in the table below.

World	6.98×10^9
China	1.35×10^9
Japan	1.27×10^8
Singapore	4.84×10^6

- (a) Find the total population of China and Japan, giving your answer in standard form.

$$\begin{aligned}
 &1.35 \times 10^9 + 1.27 \times 10^8 \text{ --- M1} \\
 &= 1477\ 000\ 000 \\
 &= \underline{1.477 \times 10^9} \text{ --- A1 Answer} \quad (a) \dots\dots\dots 1.477 \times 10^9 \quad [2]
 \end{aligned}$$

- (b) Find the ratio of the population of Singapore to that of the world.

$$\text{Answer} \quad (b) \dots\dots\dots 121 : 174\ 500 \quad \text{B1} \quad [1]$$

- 2 Use your calculator to evaluate $\frac{\sqrt[3]{434.25 + 21.6^3}}{76.15}$, giving your answer correct to 3 significant figures.

$$\begin{aligned}
 &\frac{\sqrt[3]{434.25 + 21.6^3}}{76.15} = 132.6137 \rightarrow \\
 &\approx 133 \quad \text{[M1]}
 \end{aligned}$$

$$\text{Answer} \dots\dots\dots 133 \quad \text{B2} \quad [2]$$

- 3 Make m the subject of the formula $P = 2m + 2b$.

$$\begin{aligned}
 &P = 2m + 2b \\
 &P - 2b = 2m \\
 &2m = P - 2b \text{ --- M1} \\
 &m = \frac{P - 2b}{2} \text{ Answer}
 \end{aligned}$$

$$\text{or } \frac{P}{2} - b \text{ --- A1}$$

$$\text{penalise if } \frac{P - 2b}{2}$$

[2]

[Turn over]

- 4 A car travels a total distance of 480 km in 4 hours 48 minutes.

(a) Express 4 hours 48 minutes in hours.

Answer (a) $4\frac{4}{5}$ h [1] B1

(b) Find the car's average speed in

(i) kilometres per hour,

$$\frac{480}{4\frac{4}{5}} = 100$$

Answer (i) 100 km/h [1] B1

(ii) metres per second.

$$480000 \div 17280 = 27\frac{7}{9}$$

Answer (ii) $27\frac{7}{9}$ m/s [1] B1

- 5 A map is drawn to a scale of 1 : 300 000.

(a) This scale can be written in the form 1 cm : y km.

Find y.

$$1 \text{ cm} : 300000 \text{ cm}$$

$$1 \text{ cm} : 3 \text{ km}$$

Answer (a) $y = 3$ [1] B1

$$\left. \begin{array}{l} 1 \text{ cm} : 3 \text{ km} \\ (1)^2 \text{ cm}^2 : (3)^2 \text{ km}^2 \\ 1 \text{ cm}^2 : 9 \text{ km}^2 \end{array} \right\} \text{ m1}$$

- (b) On the map, the area of a lake is 14 cm^2 .

Calculate the actual area of the lake, giving your answer in square kilometres.

$$1 \text{ cm}^2 : 9 \text{ km}^2 \text{ --- m1}$$

\therefore Actual area of lake

$$= 14 \times 9$$

$$= 126 \text{ km}^2 \text{ --- A1}$$

Answer

(b) 126 km^2 [2]

- 6 (a) Simplify $\frac{(p^2q)^3}{p^3q^4}$ and express your answer in positive index notation.

$$\frac{(p^2q)^3}{p^3q^4} = \frac{p^6q^3}{p^3q^4} \text{ --- m1} = \frac{p^3}{q}$$

$$\times = p^3q^{-1}$$

$$\checkmark = \frac{p^3}{q} \text{ --- A1}$$

Answer

(a)

..... $\frac{p^3}{q}$ [2]

- (b) Given that $4^x = 8^3$, write down the value of x .

change to common base \rightarrow

$$\underline{4}^x = \underline{8}^3$$

$$(\underline{2})^{2x} = (\underline{2}^3)^3 \text{ --- m1}$$

$$2x = \underline{9}$$

$$x = 4\frac{1}{2} \text{ --- A1}$$

Answer

(b)

$x =$ $4\frac{1}{2}$ [2]

- 7 (a) Factorise $4w^2 - 9$ completely.

Answer

(a)

$$(2w+3)(2w-3)$$

B2
[2]

Completing square.

- (b) (i) Given that $x^2 + 8x - 9 = (x+a)^2 + b$, find a and b .

$$x^2 + 8x - 9 = (x-4)^2 - 25$$

Answer (b) (i)

$$a = 4$$

$$b = -25$$

B2
[2]

- (ii) Solve $x^2 + 8x - 9 = 0$.

x	$+$	9	$+9x$
x	$-$	1	$-x$
x^2	$-$	9	$+8x$

]

m1

$$(x+9)(x-1) = 0$$

$$x = -9 \text{ or } x = 1 \text{ — A1}$$

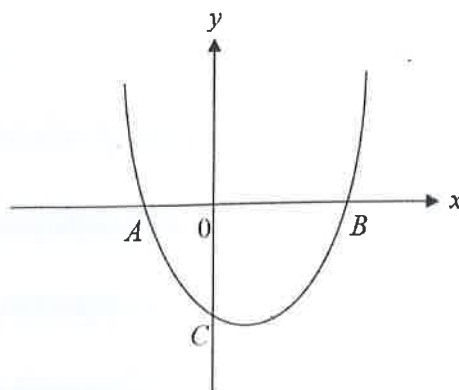
Answer (b) (ii)

$$x = -9 \text{ or } 1$$

[2]

[Turn over]

- 8 The diagram shows the graph of $y = x^2 - 2x - 15$.



- (a) The graph cuts the y -axis at C .
Write down the coordinates of C .

Answer

(a) $C = (0, -15)$ [1] B1

- (b) The graph cuts the x -axis at $A(a, 0)$ and $B(b, 0)$.
Find the value of

- (i) a ,

Answer

(b)(i) $a = -3$ [1] B1

- (ii) b .

Answer

(b)(ii) $b = 5$ [1] B1

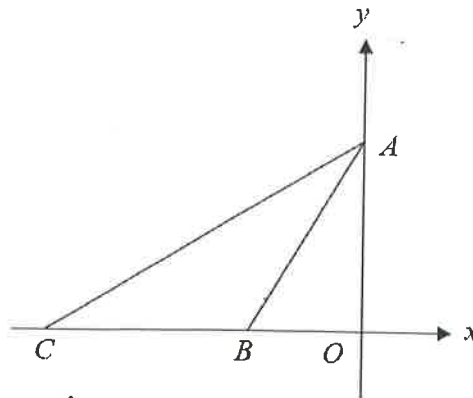
- (c) Find the equation of the line of symmetry of the graph.

$$x = \frac{-3+5}{2} = 1$$

Answer

(c) $x = 1$ [1] B1

- 9 In the diagram, the equation of the line AC is $3y - 2x - 30 = 0$. The length of the line BC is 10 units.



- (a) Find the coordinates of C.

At C, $y = 0$

$\therefore 3(0) - 2(x) - 30 = 0$ — M1

$2x = -30$

Answer

(a)

$C(-15, 0)$

[2]

$x = -15$ — A1

- (b) Find the coordinates of B.

Since the length is 10 units (BC),

$\therefore x$ coordinate of B = $-15 + 10$

Answer = -5

(b)

$(-5, 0)$

B1

[1]

- (c) Find the area of triangle ABC.

Area of $\triangle ABC = \frac{1}{2} \times BC \times OA$

$= \frac{1}{2} \times 10 \times 10$

$= 50 \text{ units}^2$

B1

Answer

(c)

50

units² [1]

- 10 Mr Lee wants to deposit \$10 000 in a bank for 5 years. He compares the interest rate offered by 2 different banks.

<p>ABC Bank 1.65% per year Simple Interest</p>

<p>XYZ Bank 1.5% per year Compound Interest</p>
--

Which bank should he choose? Justify your answer with the relevant working.

Circle your answer:

ABC Bank

/

XYZ Bank

[1]

AI

Working [2]:

Simple interest for ABC Bank

$$= \frac{(10000)(1.65)(5)}{100}$$

$$= \$825 + 10000 = \$10825$$

Amount for XYZ Bank

$$= 10000 \left(1 + \frac{1.5}{100} \right)^5$$

$$= \$10722.84004$$

① ABC Bank

② Correct formula used. (at least one)

③ Even the wrong answer, it is ok.

M1 + M1

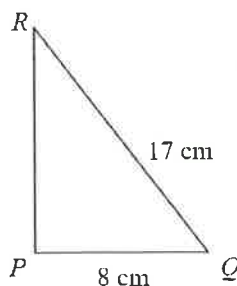
Amount of ABC

Amount of XYZ

Since \$10825 > \$10722.84004,

∴ ABC Bank is a better choice.

- 11 In the diagram, $PQ = 8$ cm, $PR = 15$ cm and $QR = 17$ cm. PQR is a right-angled triangle.



Prove that $PR = 15$.

- (a) ~~Find the length of PR .~~

By Pythagoras' Theorem,

$$PR^2 = RQ^2 - PQ^2$$

$$= (17)^2 - (8)^2 \text{ --- M1}$$

$$= 15 \text{ cm} \text{ --- A1}$$

Answer

(a)

15

cm

[2]

- (b) (i) Find the area of triangle PQR .

Answer

(b) (i)

60

cm²

[1]

- (b) (ii) Hence or otherwise, find the shortest distance from P to QR .

let d to be the shortest distance,

$$\text{Area of } \triangle PQR = \frac{1}{2} \times 17 \times d$$

$$60 = \frac{1}{2} \times 17 \times d \text{ --- M1}$$

$$d = 7\frac{1}{17} \text{ --- A1}$$

Answer

(b) (ii)

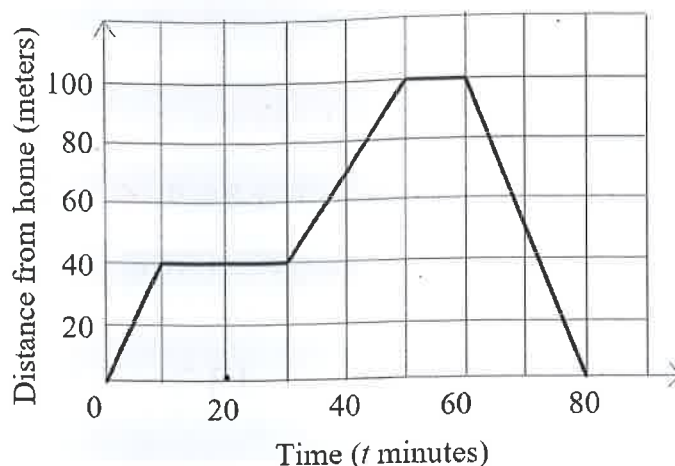
$7\frac{1}{17}$

7.06

cm

[2]

- 12 Mary walked from her home to a fast food restaurant and stopped to buy a burger. She then walked to the library and stopped to return a book, before heading home. The diagram below shows her distance-time graph.



- (a) Find Mary's speed if she took 10 minutes to walk to the restaurant.

Answer

(a)

..... ⁴ m/min [1] ^{B1}

- (b) Find Mary's average speed for the entire journey in m/min.

$$\begin{aligned} \text{Average speed} &= \frac{200}{80} \text{ m/min} \text{ (when students identify that total distance is 200m)} \\ &= 2.5 \text{ m/min} \text{ — A1} \end{aligned}$$

Answer

(b)

..... ^{2.5} m/min [2] ^{~~B1~~}

- 13 In a Mathematics lesson on Quadratic Equations, Jacer wrote the following working. However, his good friend Lance claimed that Jacer's working is incorrect.

Jacer's working:

$$4x^2 - 3x = 8$$

$$4x^2 - 3x + 8 = 0 \quad \text{M1: incorrect}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(4)(8)}}{2}$$

$$x = \frac{-3 \pm \sqrt{-119}}{2}$$

Hence, the equation has no real solution.

Do you agree with Lance? Justify your answer with the relevant working.

Circle your answer:

Agree

Disagree

[1]

A1

Working [2]:

$$4x^2 - 3x = 8$$

$$4x^2 - 3x - 8 = 0 \quad \text{M1}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-8)}}{2(4)} \quad \text{M1}$$

$$= \frac{3 \pm \sqrt{137}}{8}$$

$$= 1.84 \text{ or } -1.09 \text{ (3 sf) } \text{### M1}$$

$\therefore x$ has solutions which are 1.84 and -1.09.

[Turn over]

14 Machine A can assemble a computer in x minutes.

- (a) Write down an expression, in terms of x , for the number of computers Machine A can assemble in one hour.

$$\begin{array}{l} \div \quad \begin{array}{l} x \text{ min} \rightarrow 1 \\ 60 \text{ min} \rightarrow \end{array} \quad \frac{60}{x} \times 1 = \frac{60}{x} \quad \frac{60}{x} \quad B1 \\ \text{Answer} \quad (a) \quad \dots\dots\dots [1] \end{array}$$

Machine B takes 5 minutes less than Machine A to assemble a computer.

- (b) Write down an expression, in terms of x , for the number of computers Machine B can assemble in one hour.

$$\begin{array}{l} (x-5) \text{ min} \rightarrow 1 \\ 60 \text{ min} \rightarrow \frac{60}{x-5} \times 1 = \frac{60}{x-5} \quad \frac{60}{x-5} \quad B1 \\ \text{Answer} \quad (b) \quad \dots\dots\dots [1] \end{array}$$

- (c) In an hour, Machine B can assemble 4 computers more than Machine A. Form an equation in x and show that it reduces to $x^2 - 5x - 75 = 0$.

$$\frac{60}{x-5} - \frac{60}{x} = 4 \quad \text{--- M1}$$

$$60x - 60(x-5) = 4x(x-5)$$

$$60x - 60x + 300 = 4x^2 - 20x$$

$$4x^2 - 20x - 300 = 0$$

$$x^2 - 5x - 75 = 0 \quad \text{--- A1}$$

(shown)

Answer (c) Show in your working [2]

[Turn over]

- (d) Solve the equation $x^2 - 5x - 75 = 0$ shown in part (c) to find the **maximum** number of computers Machine A can assemble in 5 hours.

$$x^2 - 5x - 75 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-75)}}{2(1)} \quad \text{--- m1}$$

$$= \frac{5 \pm \sqrt{325}}{2}$$

$$x = 11.5138 \quad \text{or} \quad x = -6.5138 \text{ (rej)} \quad \text{--- m1}$$

Note: Award m1 if students get both values of x .
Students must reject negative x values.

$$\therefore \text{No of computers} = \frac{60}{11.5138} \times 5$$

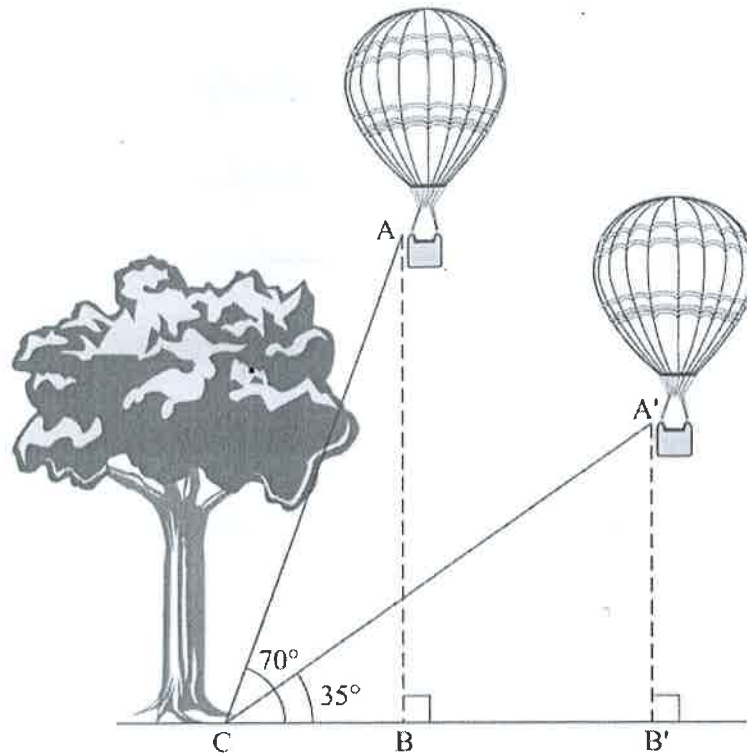
$$\text{Part (a)} \rightarrow = 26.055$$

$$\approx 26 \quad \# \quad \text{--- A1}$$

Answer (d) 26 computers [3]

[Turn over]

- 15 A hot air balloon was tied with a 300 m rope to the bottom of a tree. At first, the angle between the rope and the ground was 70° and finally decreased to 35° .



- (a) Find the distance AB .

$$\sin 70 = \frac{AB}{300} \text{ --- M1}$$

$$\therefore AB = \sin 70 \times 300$$

$$= 281.908 \approx 282 \text{ --- A1}$$

Answer (a) 282 m [2]

- (b) Find the vertical distance that the hot air balloon descended.

$$\sin 35 = \frac{AB'}{300} \text{ --- M1}$$

$$AB' = 172.073$$

$$\therefore \text{Height descended} = 281.908 - 172.073$$

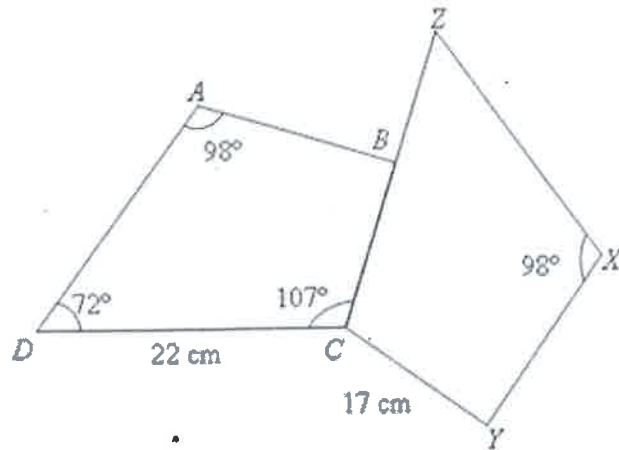
$$= 109.835$$

$$\approx 110 \text{ (3sf) --- A1}$$

Answer (b) 110 m [2]

[Turn over]

- 16 In the diagram, quadrilaterals $ABCD$ and $XYZZ$ are congruent.



Find

- (a) $\angle ABC$,

Answer

(a)

83

B1

[1]

- (b) the length of BZ .

Answer

(b)

5

B1

cm [1]

[Turn over]

17

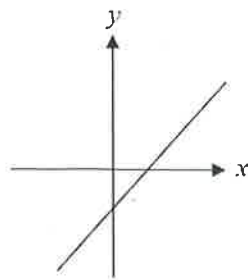


Figure 1

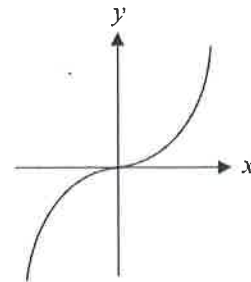


Figure 2

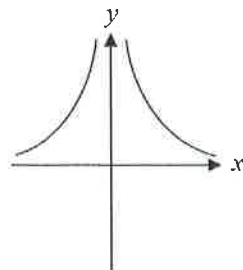


Figure 3

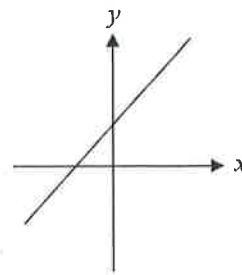


Figure 4

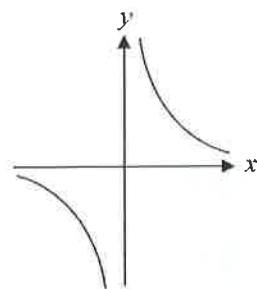


Figure 5

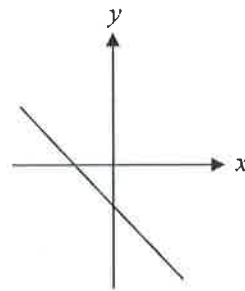


Figure 6

Which of the graphs shown above could be the graph of

(a) $y = \frac{1}{x^2}$,

Answer

(a) Figure ³ ^{BI} [1]

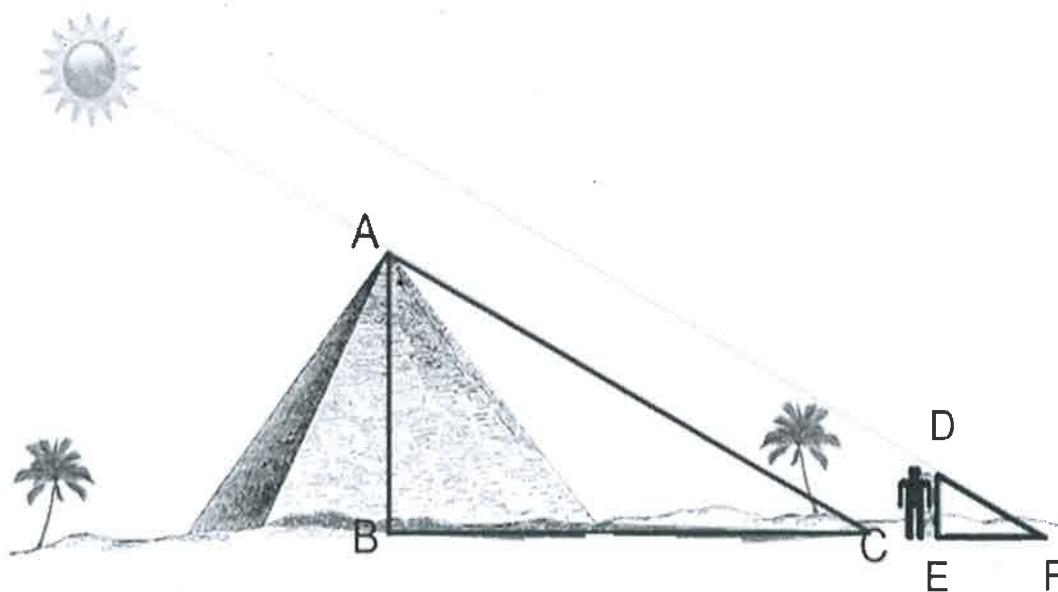
(b) $y = 3x - 1$?

Answer

(b) Figure ¹ ^{BI} [1]

[Turn over]

- 18 A wise man looks up at a pyramid and wants to find the height of the pyramid. After some observation, he sketches the diagram shown below. It is given that $\triangle ABC$ is similar to $\triangle DEF$, $\angle ABC = 90^\circ$, $\angle DEF = 90^\circ$, $BC = 20$ m and $EF = 3$ m. The man's height is 1.8 m.



- (a) If $\angle ACB = 38^\circ$ find $\angle EDF$.

$$\angle ACB = \angle OFE = 38^\circ$$

$$\therefore \angle EDF = 180^\circ - 90^\circ - 38^\circ \text{ --- M1}$$

$$= 52^\circ \text{ --- A1} \quad 52$$

Answer (a) $^\circ$ [2]

- (b) If DE represents the man's height, find the height of the pyramid, AB .

$$\frac{BC}{EF} = \frac{AB}{DE}$$

$$\therefore AB = 12 \text{ m --- A1}$$

$$\frac{20}{3} = \frac{AB}{1.8} \text{ --- M1}$$

Answer (b) 12 m [2]

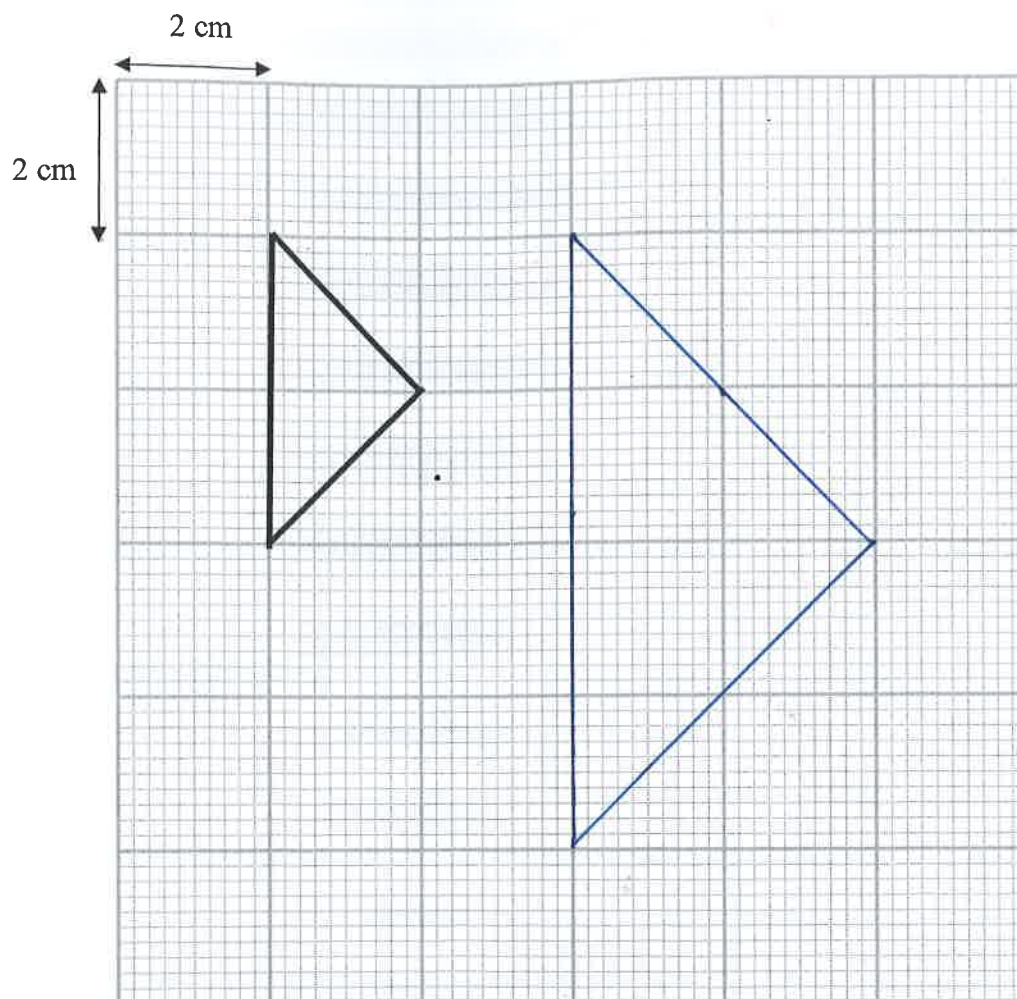
$$b) \frac{20}{\sin 52} = \frac{AB}{\sin 38}$$

$$AB = 15.625$$

$$\approx 15.6 \text{ m}$$

[Turn over]

- 19 (a) Draw on the grid an enlargement of the given shape A , with a scale factor of 2. [1]



- (b) Shape B is a reduction of Shape A using scale factor x .
The longest side of shape B has length 3 cm.

Find x .

scale factor > 1 : enlargement

< 1 : reduction.

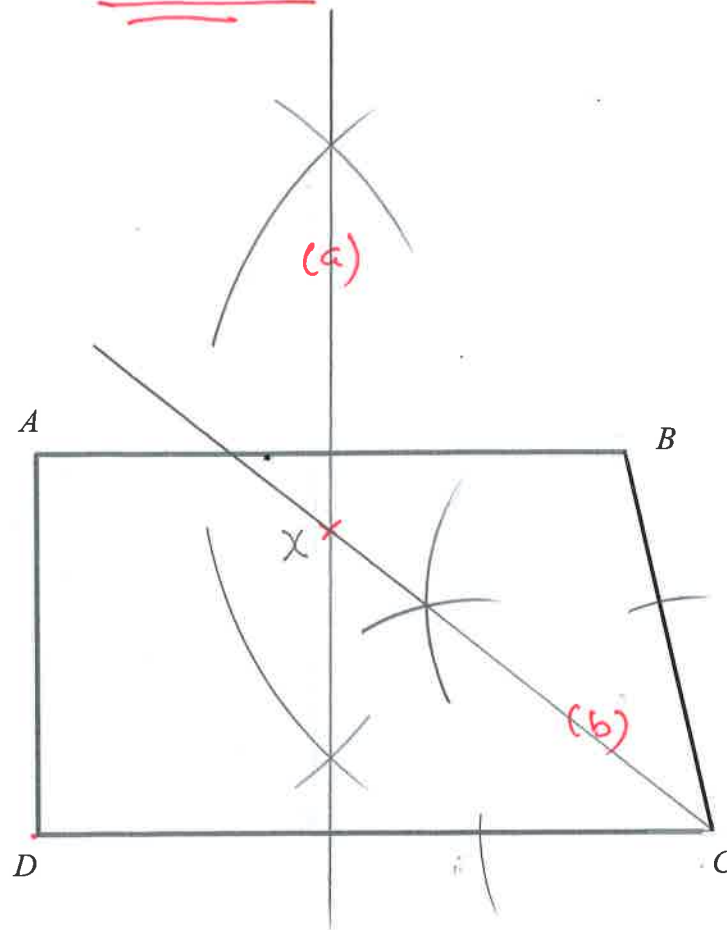
Answer

(b)

0.75 or $\frac{3}{4}$

81
[1]

- 20 The diagram shows a scale drawing of a field $ABCD$.
It is drawn to a scale of 1 cm to 10 km.



(a) Construct the perpendicular bisector of AB .

[1] B1

(b) Construct the angle bisector of angle BCD .

[1] B1

(c) (i) A treasure is hidden at point X which is equidistant from lines CB and CD as well as points A and B . Mark the point X .

[1] B1

(ii) Measure the length DX , giving your answer in kilometres

54, 55, 56 km. $DX = 5.5 \text{ cm } (\pm 0.1)$ ←

1 cm : 10 km

Answer (cii)

55

54, 55, 56

km [1]

B1

END OF PAPER