Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O-LEVEL PRELIMINARY EXAMINATIONS 2023

LEVEL & STREAM	:	SECONDARY 4 EXPRESS/ 5 NORMAL ACADEMIC
SUBJECT (CODE)	:	ADDITIONAL MATHEMATICS (4049)
PAPER NO	:	02
DATE (DAY)	:	12 SEPTEMBER 2023 (TUESDAY)
DURATION	:	2 HOURS 15 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks in this paper is 90.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Student's Signature		**Parent's Signature**	
Date		**Date**	

/ **90**

This document consists of **18** printed pages including this cover page
Setter : Mr Eric Bay

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 It is given that $f(x) = 2e^x(\sin x - \cos x)$.

(a) Show that $f'(x) = 4e^x \sin x$. [4]

$f(x) = 2e^x(\sin x - \cos x)$

$f'(x) = 2e^x(\sin x - \cos x) + 2e^x(\cos x + \sin x)$ -------------------- M1

$= 2e^x[(\sin x - \cos x) + (\cos x + \sin x)]$ -------------------- M1

$=2e^x(2\sin x)$

$=4e^x \sin x$ (shown)------------------- A1

(b) Hence evaluate $\int_0^{\pi} e^x \sin x \, dx$. [4]

$\int 4e^x \sin x \, dx = 2e^x(\sin x - \cos x) + c$-------------------- M1

$4\int e^x \sin x \, dx = 2e^x(\sin x - \cos x) + c$

$\int e^x \sin x \, dx = \frac{1}{2}e^x(\sin x - \cos x) + c$------------------- M1

$\int_0^{\pi} e^x \sin x \, dx = \left[\frac{1}{2}e^x(\sin x - \cos x)\right]_0^{\pi}$ ------------------- M1

$= \left[\frac{1}{2}e^{\pi}(\sin \pi - \cos \pi) - \frac{1}{2}e^0(\sin 0 - \cos 0)\right]$

$= \frac{1}{2}e^{\pi} + \frac{1}{2}$ ------------------- A1

2 **(a)** Prove that $\sin 3x = 3\sin x - 4\sin^3 x$. [3]

$\text{LHS} = \sin 3x$

$= \sin(2x + x)$ ------------------- M1

$= \sin 2x \cos x + \cos 2x \sin x$

$= 2\sin x \cos x \cos x + (1 - 2\sin^2 x)\text{sinx}$ ------------------- M1

$= 2\sin x \cos^2 x + \sin x - 2\sin^3 x$

$= 2\sin x(1 - \sin^2 x) + \sin x - 2\sin^3 x$------------------- M1

$= 2\sin x - 2\sin^3 x + \sin x - 2\sin^3 x$

$= 3\sin x - 4\sin^3 x$

$= \text{RHS}$ ------------------- A1

(b) Hence solve the equation $6\sin x - 8\sin^3 x = 1$ for $0° < x < 120°$. [4]

$6\sin x - 8\sin^3 x = 1$

$2(3\sin x - 4\sin^3 x) = 1$

$2\sin 3x = 1$ ------------------- M1

$\sin 3x = \frac{1}{2}$

$\alpha = 30°$ ------------------- M1

$3x = 30, 180 - 30$

$3x = 30, 150$ ------------------- M1

$x = 10°, 50°$ ------------------- A1

3 **(a)** Show that $x^4+3x^2-4=(x+1)(x-1)(x^2+4)$. [2]

LHS = x^4+3x^2-4

$=\left(x^2-1\right)\left(x^2+4\right)$ -------------------- M1

$=\left(x+1\right)\left(x-1\right)\left(x^2+4\right)$ -------------------- A1

(b) Hence express $\dfrac{3x^2+7}{x^4+3x^2-4}$ in partial fractions. [6]

$$\frac{3x^2+7}{(x+1)(x-1)(x^2+4)}=\frac{A}{(x+1)}+\frac{B}{(x-1)}+\frac{Cx+D}{(x^2+4)}$$ -------------------- M1

$3x^2+7=A(x-1)(x^2+4)+B(x+1)(x^2+4)+(Cx+D)(x+1)(x-1)$

sub $x=1$

$10=B(10)$

$B=1$-------------------- M1

sub $x=-1$

$10=-10A$

$A=-1$-------------------- M1

sub $x=0$

$7=-4A+4B-D$

$7=4+4-D$

$D=1$-------------------- M1

compare coeff of x^3, — or when $x=2$

$0=A+B+C$ — $19=-8+24+6c+3$

$0=-1+1+C$ — $c=0$

$C=0$-------------------- M1

$$\frac{3x^2+7}{(x+1)(x-1)(x^2+4)}=\frac{-1}{(x+1)}+\frac{1}{(x-1)}+\frac{1}{(x^2+4)}$$

$$=\frac{1}{(x-1)}-\frac{1}{(x+1)}+\frac{1}{(x^2+4)}$$ -------------------- A1

4 A curve y, is such that $\frac{d^2y}{dx^2} = 2x$ and the point $P(0,-3)$ lies on the curve. The gradient of the curve at P is 5.

(a) Determine if the curve passes through point $Q(3,21)$. [5]

$\frac{d^2y}{dx^2} = 2x$

$\frac{dy}{dx} = x^2 + C$ ------------------ M1

$x = 0, \frac{dy}{dx} = 5$ ------------------ M1

$C = 5$

$\frac{dy}{dx} = x^2 + 5$

$y = \frac{1}{3}x^3 + 5x + D$------------------ M1

$x = 0, y = -3$

$D = -3$

$y = \frac{1}{3}x^3 + 5x - 3$

when $x = 3$ ------------------ M1

$y = 9 + 15 - 3$

$y = 21$

the curve passes through the point (3,21) ------------------ A1

(b) Explain why the curve has no turning point. [2]

$\frac{dy}{dx} = x^2 + 5$

for turning point, $\frac{dy}{dx} = 0$,

$x^2 + 5 = 0$

$x^2 = -5$

$x = \sqrt{-5}$---------------- M1

no Solution, therefore no turning point ---------------- A1

or as $x^2 \geq 0$, $\frac{dy}{dx} \geq 5$---------------- M1

since $\frac{dy}{dx}$ can never be zero,

$\therefore$ the curve has no turning point. ---------------- A1

(c) Determine whether the curve is an increasing or decreasing function. [2]

$\frac{dy}{dx} = x^2 + 5$

$x^2 \geq 0$

$x^2 + 5 > 0$--------------- A1

Therefore the curve is an increasing function for all value of x--------------- A1

5 The points $A(-2,1)$, $B(3,-4)$ and $C(3,1)$ lies on a circle.

(a) Show that the centre of the circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$. [6]

$$A(-2,1), B(3,-4)$$

$$m_{AB}=\frac{-4-1}{3-(-2)}$$

$$=-1 \text{-------------------- M1}$$

$$m_{\perp AB}=1$$

$$midpoint=\left(\frac{-2+3}{2},\frac{1+(-4)}{2}\right)$$

$$=\left(\frac{1}{2},-\frac{3}{2}\right) \text{-------------------- M1}$$

$$y=mx+c$$

$$-\frac{3}{2}=\frac{1}{2}+c$$

$$c=-2$$

$$y=x-2 \text{-------------------- M1}$$

$$B(3,-4), C(3,1)$$

$$m_{BC}=\frac{-4-1}{3-3}$$

$$=\text{undefine -------------------- M1}$$

$$m_{\perp BC}=0$$

$$\text{midpoint}=\left(3,-\frac{3}{2}\right)$$

$$y=mx+c$$

$$-\frac{3}{2}=0+c$$

$$c=-\frac{3}{2}$$

$$y=-\frac{3}{2} \text{-------------------- M1}$$

$$y=x-2$$

$$y=-\frac{3}{2}$$

$$x=\frac{1}{2}$$

Therefore centre of circle is $\left(\frac{1}{2},-\frac{3}{2}\right)$ -------------------- A1

(b) Explain why *AB* is the diameter of the circle. [1]

Midpoint of *AB* is the center of the circle.
Or student show that

$m_{BC} \times m_{AC} = -1$-----------B1

(c) Find the equation of the circle. [3]

$$r^2 = \left(3-\frac{1}{2}\right)^2 + \left(-4+\frac{3}{2}\right)^2$$ -------------------- M1

$$= \frac{25}{4} + \frac{25}{4}$$

$$= \frac{25}{2}$$ -------------------- M1

$$\left(x-\frac{1}{2}\right)^2 + \left(y+\frac{3}{2}\right)^2 = \frac{25}{2}$$ -------------------- A1

(d) Show that point $D(2,2)$ lies outside the circle. [2]

Distance of D from centre $= \sqrt{\left(2-\frac{1}{2}\right)^2 + \left(2+\frac{3}{2}\right)^2}$ ------------------- M1

$$= \sqrt{\frac{9}{4} + \frac{49}{4}}$$

$= \sqrt{\frac{29}{2}} > \sqrt{\frac{25}{2}}$ -therefore point D lies out side the circle------------------- A1

6 Solve the following equations.

(a) $3+\log_2(x+4)=2\log_2(3x-4)$. [4]

$$3+\log_2(x+4)=2\log_2(3x-4)$$

$$\log_2(3x-4)^2-\log_2(x+4)=3$$

$$\log_2\frac{(3x-4)^2}{(x+4)}=3 \text{ -------------------- M1}$$

$$\frac{(3x-4)^2}{(x+4)}=2^3 \text{ -------------------- M1}$$

$$(3x-4)^2=8(x+4)$$

$$9x^2-24x+16=8x+32$$

$$9x^2-32x-16=0 \text{ -------------------- M1}$$

$$(x-4)(9x+4)=0$$

$$x=4 \quad \text{or } x=-\frac{4}{9} \text{ (Rej) ------------------ A1}$$

(b) $2\log_3 y-\log_y 3=1$. [5]

$$2\log_3 y-\log_y 3=1$$

$$2\log_3 y-\frac{\log_3 3}{\log_3 y}=1 \text{ -------------------- M1}$$

$$\text{let } x=\log_3 y$$

$$2x-\frac{1}{x}=1 \text{ -------------------- M1}$$

$$2x^2-1=x$$

$$2x^2-x-1=0 \text{ -------------------- M1}$$

$$(2x+1)(x-1)=0$$

$$x=-\frac{1}{2} \quad \text{or} \quad x=1$$

$$\log_3 y=-\frac{1}{2} \quad \text{or} \quad \log_3 y=1 \text{ -------------------- M1}$$

$$y=\frac{1}{\sqrt{3}} \quad \text{or} \quad y=3 \text{ -------------------- A1}$$

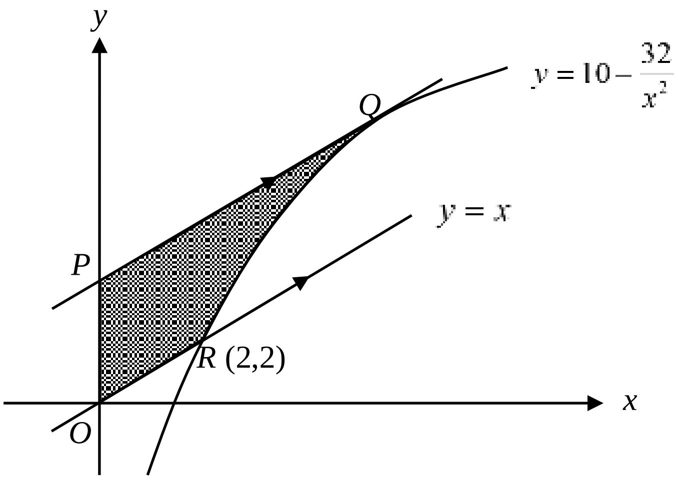
7 **(a)** Factorise $(x-1)^3-8$ completely. [3]

$$(x-1)^3-8=\left[(x-1)-2\right]\left[(x-1)^2+2(x-1)+4\right]\text{-------------------- M1}$$
$$=(x-1-2)(x^2-2x+1+2x-2+4)\text{-------------------- M1}$$
$$=(x-3)(x^2+3)\quad\text{-------------------- A1}$$

(b) Hence show that $(x-1)^3-8=0$ has only 1 solution. [3]

$$(x-1)^3-8=0$$
$$(x-3)(x^2+3)=0\quad\text{------------------ M1}$$
$$x^2+3=0 \qquad \text{or} \qquad x-3=0$$
$$b^2-4ac=0^2-4(1)(3) \qquad \text{or} \qquad x=3$$
$$=-12<0 \ \text{(no solution)} \qquad \text{------------------- M1}$$
$$\therefore\ (x-1)^3-8=0 \text{ has only 1 solution } x=3\text{-----------------------A1}$$

8



The diagram shows part of the curve $y = 10 - \frac{32}{x^2}$ and two parallel lines *OR* and *PQ*. The equation of *OR* is $y = x$ and the line intersects the curve at point $R(2, 2)$. *PQ* is the tangent to the curve at point *Q*.

(a) Find the coordinates of *Q* and of *P*. [5]

$y = 10 - \frac{32}{x^2}$

$y = 10 - 32x^{-2}$

$\frac{dy}{dx} = 64x^{-3}$ -------------------- M1

$64x^{-3} = 1$ -------------------- M1

$x = 4$

$y = 8$

$Q(4, 8)$ -------------------- A1

$y = mx + c$

$8 = 4 + c$ -------------------- M1

$c = 4$

$P(0, 4)$ -------------------- A1

(b) Find the area of the shaded region *OPQR*. [5]

Area of trapezium $= \frac{1}{2}(4+8)\times 4$

$= 24$ -------------------------------------M1

Area of triangle$= \frac{1}{2}\times 2\times 2$

$= 2$ -------------------------------------M1

Area under the curve$= \int_{2}^{4} 10 - 32x^{-2}\ \mathrm{d}x$

$= \left[10x + 32x^{-1}\right]_{2}^{4}$ -------------------------------------M1

$= \left[10(4) + \frac{32}{4}\right] - \left[10(2) + \frac{32}{2}\right]$

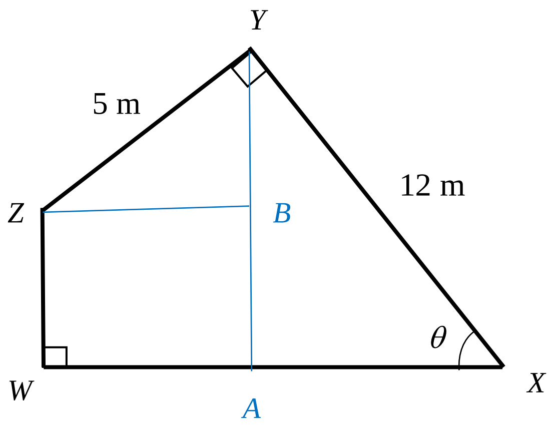
$= 48 - 36$

$= 12$ -------------------------------------M1

Area of shaded region *OPQR* $= 24 - 2 - 12$

$= 10$ units^2 -------------------------------------A1

9



In the diagram $WXYZ$ is a quadrilateral with $XY = 12$ m, $YZ = 5$ m and $\angle WXY = \theta$.

(a) Show that the perimeter, P cm, of $WXYZ$ is $17\sin\theta + 7\cos\theta + 17$. [4]

$\sin\theta = \frac{AY}{12}$ $\cos\theta = \frac{AX}{12}$ $\sin\theta = \frac{BZ}{5}$ $\cos\theta = \frac{BY}{5}$

$AY = 12\sin\theta$ $AX = 12\cos\theta$ $BZ = 5\sin\theta$ $BY = 5\cos\theta$ ----------------------M2

M1 for any pair

$P = 12\cos\theta + 5\sin\theta + 12\sin\theta - 5\cos\theta + 12 + 5$----------------------------------M1

$= 17\sin\theta + 7\cos\theta + 17$ (Shown) -----------------------------A1

(b) Express P in the form $R\sin(\theta + \alpha) + k$, where $R > 0$, $k > 0$ and $0° < \alpha < 90°$. [3]

$17\sin\theta + 7\cos\theta = R\sin(\theta + \alpha)$

$R = \sqrt{17^2 + 7^2}$

$= \sqrt{338}$ -----------------------------M1

$\alpha = \tan^{-1}\frac{7}{17}$

$\alpha = 22.3801°$ -----------------------------M1

$P = 17\sin\theta + 7\cos\theta + 17$

$= 13\sqrt{2}\sin(\theta + 22.4°) + 17$

OR

$= 18.4\sin(\theta + 22.4°) + 17$------------------------A1

(c) Find the maximum value of P and the corresponding value of θ. [2]

$$\max P = \sqrt{338} + 17$$
$$= 35.38477$$
$$= 35.4 \text{--------------B1}$$

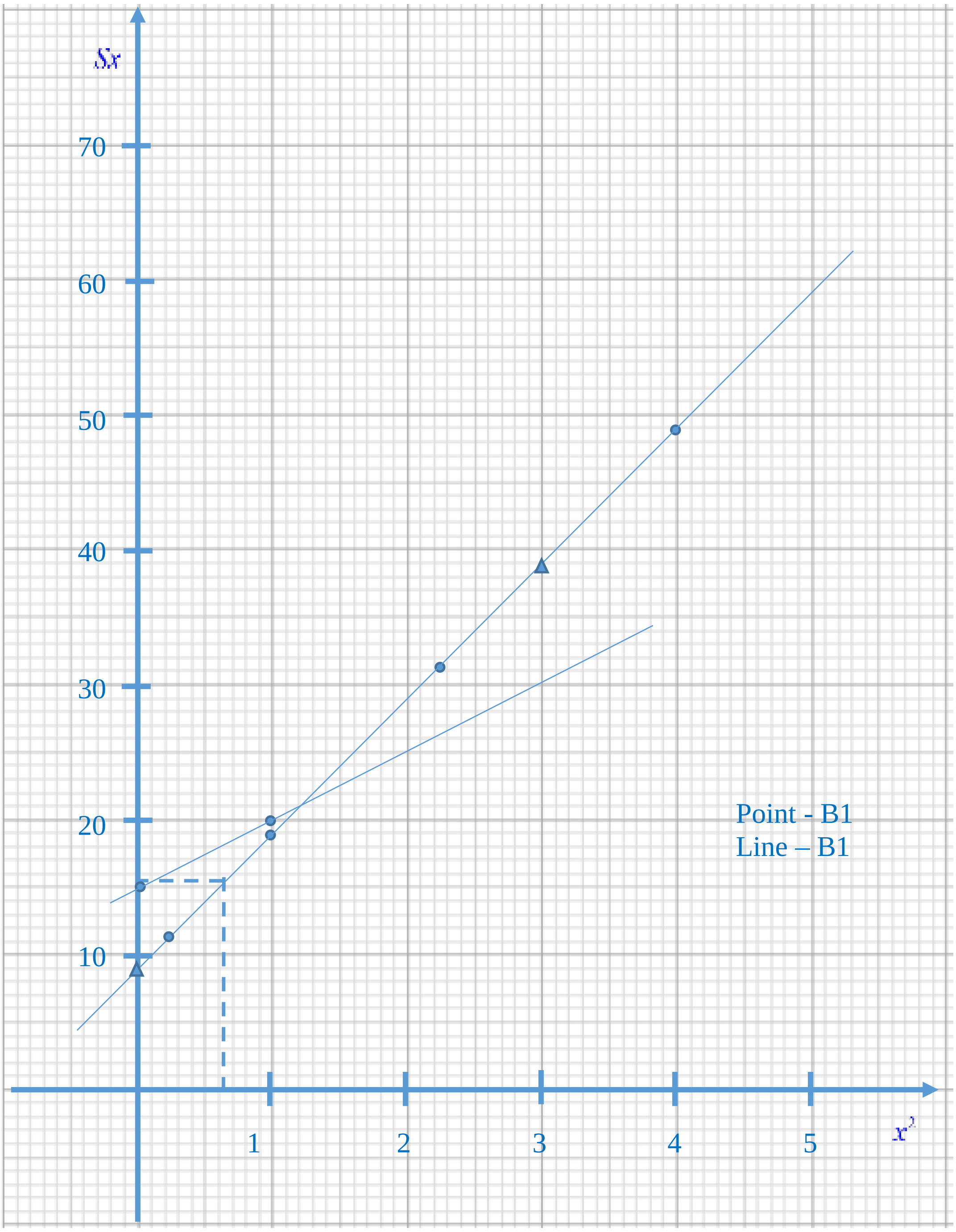
$$\sin(\theta + 22.3801°) = 1$$
$$\theta = 90 - 22.3801$$
$$= 67.6199$$
$$= 67.6° (1\text{dp}) \text{-------------B1}$$

10 A cylindrical pipe of surface area S m^2 has a circumference of $\left(a+\dfrac{b}{x^2}\right)$ m and length of x m. Corresponding values of x and S are shown in the table below.

x	0.5	1.0	1.5	2.0
S	23	19	21	24.5

(a) Draw a straight line graph of Sx against x^2. [2]

x^2	0.25	1.0	2.25	4.0
Sx	11.5	19.0	31.5	49.0



(b) Use the graph to estimate

(i) the value of each of the constants *a* and *b*, [4]

$c = 9$-----------------------------M1

$m = \frac{39-9}{3-0}$

$= 10$-----------------------------M1

$S = \left(a + \frac{b}{x^2}\right)x$

$S = ax + \frac{b}{x}$

$Sx = ax^2 + b$-----------------------------M1

a=10

b=9-----------------------A1

(ii) the surface area of the pipe with a length of 0.8 m. [3]

$x^2 = 0.64$-----------------------------M1

from the graph,

$Sx = 15.5$-----------------------------M1

$S = \frac{15.5}{0.8}$

$= 19.375$

$= 19.4 \text{ m}^2$ (3sf)-------------------A1

(c) By drawing a suitable straight line, find the length of the pipe when its surface area is $5\left(x + \frac{3}{x}\right)$ cm^2. [3]

$S = 5\left(x + \frac{3}{x}\right)$

$Sx = 5x^2 + 15$-----------------------------M1

From the graph,

$x^2 = 1.2$-----------------------------M1

$x = \sqrt{1.2}$

$= 1.095$

$= 1.10$

the length is 1.10 cm-------------------A1

END OF PAPER