TEMASEK JC INTEGRATED PROGRAMME
YEAR FOUR
2020 END-OF-YEAR EXAMINATION

INTERMEDIATE MATHEMATICS

Date: 1 October 2020

Duration: 2 hours 30 minutes

Name: _____

Class: _____

No additional materials required.

READ THESE INSTRUCTIONS FIRST.

_____
Parent's Signature

Write in dark blue or black pen in the spaces provided in the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The calculator value for π should be used unless the question requires the answer in terms of π.
The use of an approved scientific calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers. Marks will be deducted for poor or unclear presentation.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total score for this paper is 100 marks.

This paper consists of **13** printed pages.

Question	Marks
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Presentation Deduction	– 1
Total	

MATHEMATICAL FORMULAE

Compound Interest

$$A = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

Volume of a cone = $\frac{1}{3}\pi r^2 h$

Volume of a sphere = $\frac{4}{3}\pi r^3$

Area of triangle ABC = $\frac{1}{2}ab\sin C$

Arc length $= r\theta$, where θ is in radians

Sector area $= \frac{1}{2}r^2\theta$, where θ is in radians

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all questions in the spaces provided. Show all working clearly.

1 **(a)** Express $\dfrac{4+2\sqrt{3}}{3\sqrt{3}+5}$ in the form $a\sqrt{3}+b$, where a and b are integers. [4]

(b) Express $\dfrac{5^x+3\left(5^x\right)}{20}\times\left(25^{x-5}\right)$ in the form 5^{cx+d}, where c and d are integers. [4]

2 Figure 1 (not drawn to scale) shows a solid metal paperweight with a uniform cross-section and height 3 cm.
Figure 2 (not drawn to scale) shows the cross-section of the paperweight where *ABC* is the arc of a circle of radius 4 cm and centre *O*. *AC* is a straight line of length 4 cm.

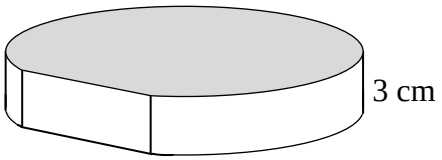


Figure 1

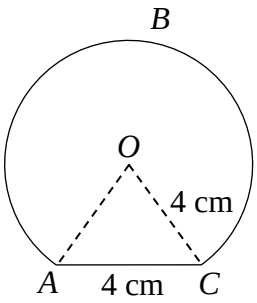


Figure 2

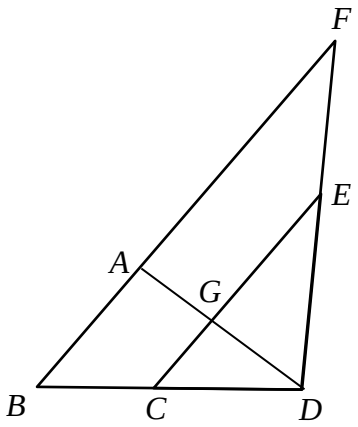
(i) Show that the uniform cross-sectional area of the paperweight is $\left(\dfrac{40}{3}\pi+4\sqrt{3}\right)$ cm^2. [2]

(ii) Find the total surface area of the paperweight, leaving your answer correct to 3 significant figures. [4]

(iii) The paperweight is melted and recast into several identical solid cones. The cones have a base radius of 2 cm and height of 5 cm. Find the maximum number of cones that can be formed. [4]

[Turn over

3 In the following diagram (not drawn to scale), the points *A*, *C* and *E* lie on the lines *BF*, *DB* and *FD* respectively such that ΔCDE is congruent to ΔBAD.



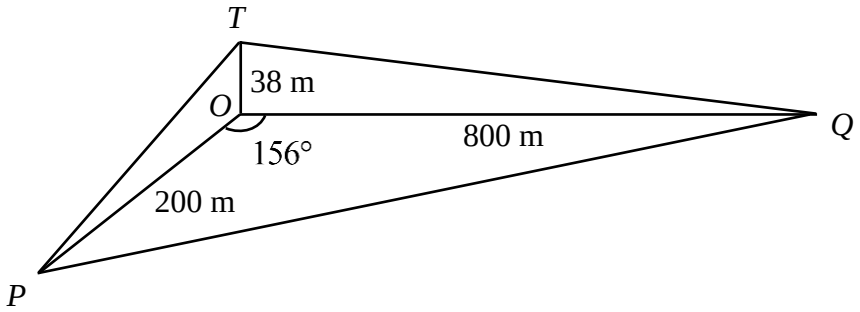
(i) Show that *FB* is parallel to *EC*. [2]

(ii) Prove that ΔCGD is similar to ΔBAD. [2]

The ratio of *AG* : *GD* is 1 : 1.

(iii) If the area of ΔCDG = 3 units^2, find the area of the quadrilateral *AGEF*. [4]

4 In the diagram shown below (not drawn to scale), O, P and Q are three points on level ground. John stands at the top of a wildlife observation deck OT that is 38 metres tall. At the same time, a rescue truck at *P*, which is located 200 metres away from O, travels along PQ to save a trapped animal at *Q*, which is 800 metres due West of *O*.



Given that $\angle POQ = 156°$, find

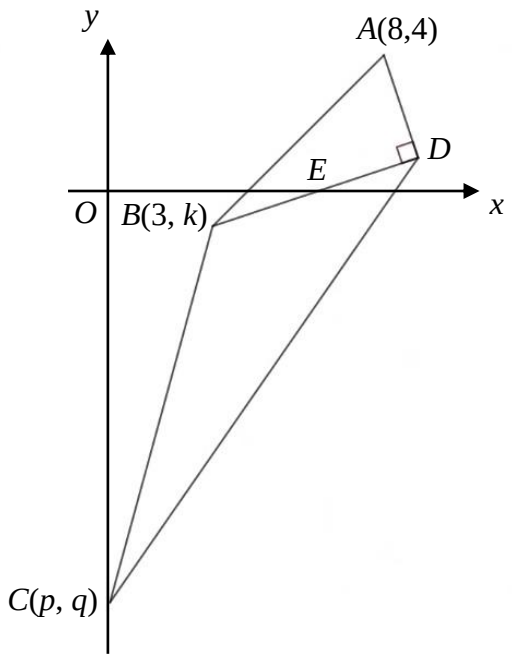
(i) PQ, [2]

(ii) $\angle OPQ$, [3]

(iii) the greatest angle of depression of the rescue truck from John during its travel from P to Q. [3]

[Turn over

5 The diagram shown below (not drawn to scale) shows a quadrilateral *ABCD*, with vertices *A*(8, 4), $B(3,k)$, *C*(*p*, *q*) and *D*. It is given that the line *BD* intersects the *x*-axis at the point *E* and $\angle ADB = 90°$.



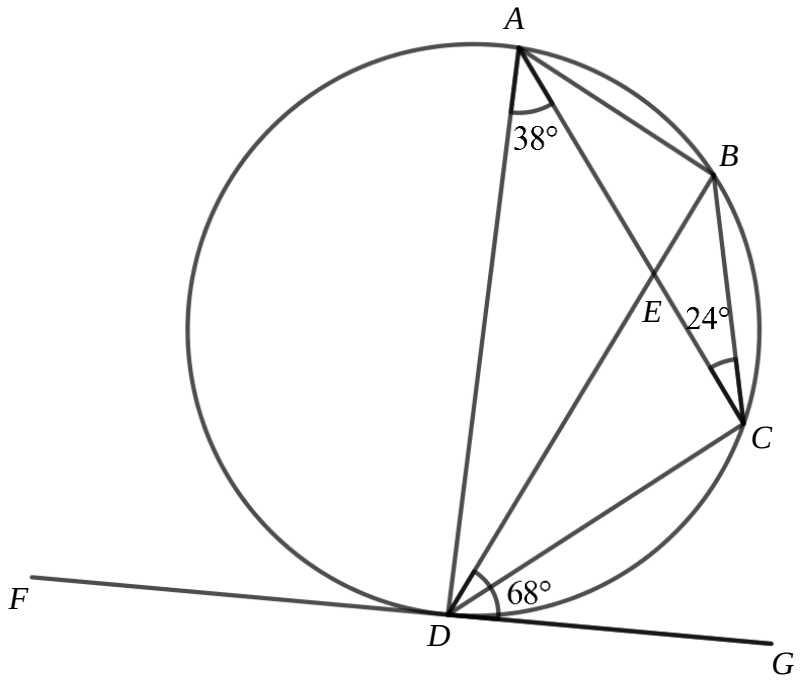
(i) Given that $AB = \sqrt{50}$ units, find the value of *k*. [3]

It is given that the equation of line *AD* is $y = -3x + 28$.

(ii) Find the coordinates of *E*. [4]

(iii) The point *C* lies on the line *AE* produced such that $AE : EC = 1:3$. Find the value of p and of q, showing your working clearly. [2]

6 In the diagram shown below (not drawn to scale), A, B, C and D are points that lie on a circle. AC and BD intersect at E. FG is tangent to the circle at D. It is given that $\angle BCA = 24°$, $\angle BDG = 68°$ and $\angle CAD = 38°$.



(i) Find $\angle ADB$. [1]

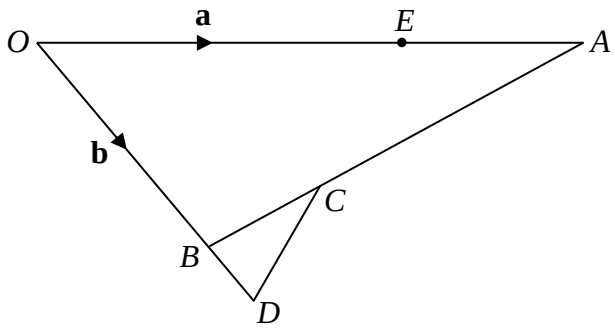
(ii) Prove that AD is not a diameter of the circle. [2]

(iii) Find $\angle AEB$. [1]

(iv) Find $\angle BDC$. [3]

[Turn over

7 In the diagram (not drawn to scale), $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. C is the point on AB such that $AB = 3CB$. The point D lies on OB produced such that $OD = 4BD$.



(i) Express, as simply as possible, $\overrightarrow{OC}$ in terms of $\mathbf{a}$ and/or $\mathbf{b}$. [2]

(ii) Show that $\overrightarrow{CD} = -\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$. [2]

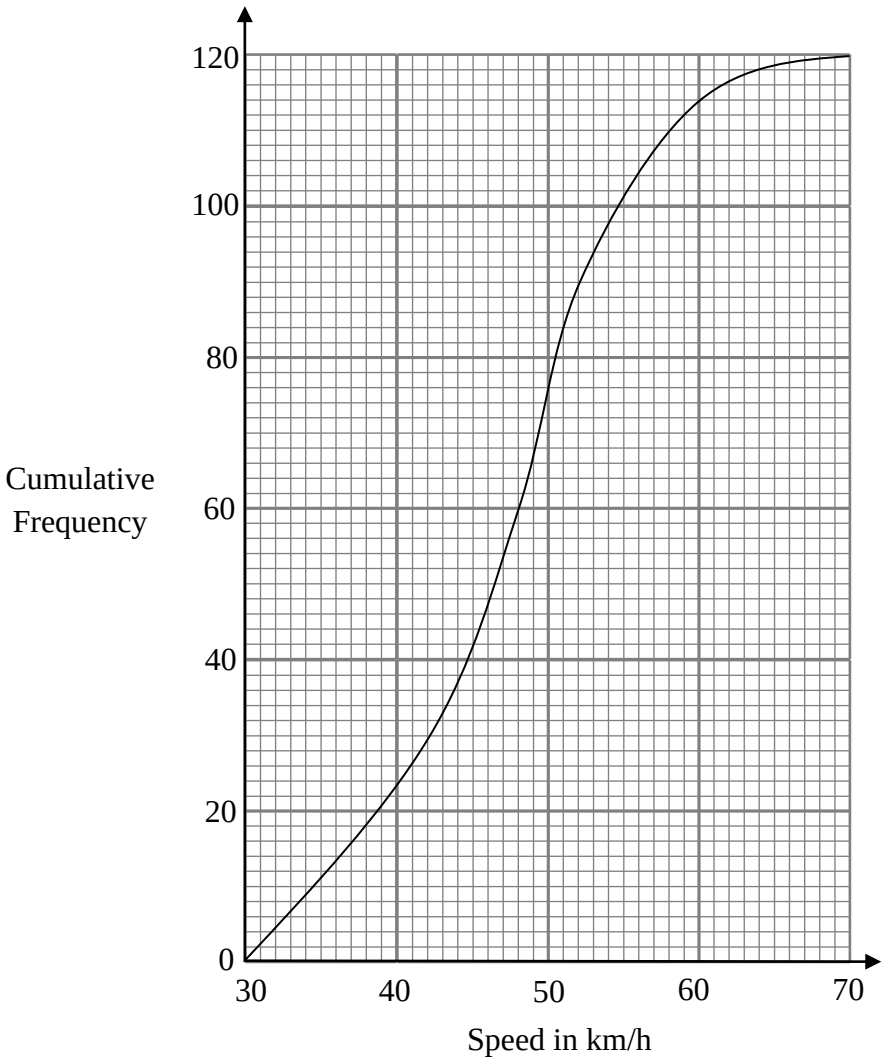
E is a point on OA such that $\overrightarrow{OE} = \frac{2}{3}\mathbf{a}$.

(iii) Show that $\overrightarrow{ED} = k\overrightarrow{CD}$, where k is a constant. [3]

(iv) Write down two facts about ED and CD. [2]

[Turn over

8 The speeds of 120 cars on road *A* were measured. The cumulative frequency curve for the speeds of the cars on road *A* is shown below.



(a) Complete the grouped frequency table for the speeds of the 120 cars on road *A*. [2]

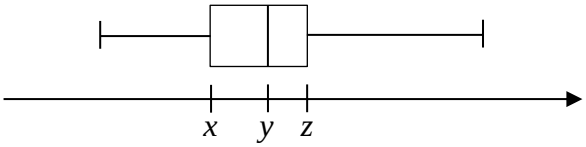
Speed (x km/h)	$30 < x \leq 40$	$40 < x \leq 50$	$50 < x \leq 60$	$60 < x \leq 70$
Frequency	24			6

(b) Using the grouped frequency table in **(a)**, calculate

(i) the mean speed, [1]

(ii) the standard deviation. [1]

(c) The box-and-whisker plot below was obtained **using the data from the cumulative frequency graph above**.



(i) State the values of x, y and z. [3]

(ii) Hence find the interquartile range. [1]

(d) The speeds of cars on road B were measured. The data for road B has the same median as the data from road A but with a larger interquartile range. Describe how the box-and-whisker plot for road B will differ from the one given in **(c)**. [1]

9 The quadratic curve $y = \mathrm{f}(x)$ has x-intercepts -2 and 3.

(i) Given that the coefficient of x^2 is 1, show that $\mathrm{f}(x) = \left(x - \frac{1}{2}\right)^2 - \frac{25}{4}$. [2]

(ii) State the coordinates of the y-intercept of the curve of $\mathrm{f}(x)$. [1]

(iii) State the solution to the inequality

(a) $\mathrm{f}(x) > -\frac{25}{4}$, [1]

(b) $\mathrm{f}(x) < -\frac{25}{4}$. [1]

Another quadratic curve $y = \mathrm{g}(x)$ has the same x-intercepts as the curve of $y = \mathrm{f}(x)$. However, the y-intercept of the curve of $y = \mathrm{g}(x)$ is different from that of the curve of $y = \mathrm{f}(x)$.

(iv) Write down a possible expression for $\mathrm{g}(x)$ in the form $ax^2 + bx + c$, where the curve of $y = \mathrm{g}(x)$ concaves downwards. [1]

10 The curves C_1 and C_2 have equations $y = 2^x - 4$ and $y = -\frac{4}{x^2} + 3$ respectively.

(i) State the equations of the asymptotes of the curves C_1 and C_2 respectively. [3]

(ii) Sketch the graphs of C_1 and C_2 on the same diagram, indicating the intercepts with the axes and equations of any asymptotes. [4]

(iii) Hence, deduce the number of real roots of the equation $2^x + \frac{4}{x^2} - 7 = 0$, showing your method clearly. [2]

[Turn over

11 The owners of TJ Bubble Tea created recipes for three new flavours of brown sugar bubble tea. The mass of brown sugar and volume of fructose corn syrup required to make a cup of each of these new flavours of bubble tea are shown in the table below.

	Tangerine Brown Sugar Bubble Tea	**Jackfruit Brown Sugar Bubble Tea**	**Cranberry Brown Sugar Bubble Tea**
Mass of brown sugar (g)	5	6	4
Volume of fructose corn syrup (ml)	36	38	40

To promote the three new flavours of bubble tea, the owners of TJ Bubble Tea intend to sell them at the promotional prices as shown in the table below.

Tangerine Brown Sugar Bubble Tea	**Jackfruit Brown Sugar Bubble Tea**	**Cranberry Brown Sugar Bubble Tea**
\$3 per cup	\$4 per cup	\$5 per cup

On a particular day, x cups of Tangerine Brown Sugar Bubble Tea, y cups of Jackfruit Brown Sugar Bubble Tea and z cups of Cranberry Brown Sugar Bubble Tea were sold. It was recorded that 6507 g of brown sugar and 53316 ml of fructose corn syrup were used. The owners collected \$6138 in sales.

(i) Formulate an equation in terms of x, y and z to represent the total amount of money collected from the sale of the three new flavours of bubble tea. [1]

(ii) Formulate an equation in terms of x, y and z to represent the total mass of brown sugar used to make the three new flavours of bubble tea. [1]

(iii) Formulate an equation in terms of x, y and z to represent the total volume of fructose corn syrup used to make the three new flavours of bubble tea. [1]

(iv) Using your answers to parts **(i), (ii)** and **(iii)**, formulate a matrix equation in the form $\mathbf{A} \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{B}$, that will allow you to solve the equations simultaneously. [1]

(v) Using a matrix method, solve the matrix equation in part **(iv)** and determine the number of cups of each new flavour of bubble tea sold. [3]

(vi) Using your understanding of matrices, explain why the number of cups of each new flavour of bubble tea sold cannot be determined if 4.5 g instead of 6 g of brown sugar was used to make each cup of Jackfruit Brown Sugar Bubble Tea. (You may assume that all other values given remain unchanged.) [1]

12 Mr Lau has a bag containing 4 blue, 3 purple, 2 red and 1 green markers. At the start of his lesson, he draws 3 markers out of the bag at random, one after another without replacement.

(i) Show that the probability of drawing a blue, a purple and a red marker in this order is $\frac{1}{30}$. [1]

Find the probability that

(ii) a purple marker was drawn first, [1]

(iii) a red marker was drawn second, [2]

(iv) either a purple marker was drawn first, or a red marker was drawn second, or a purple marker was drawn first and a red marker was drawn second, [3]

(v) at least one blue marker was drawn. [2]

End of Paper

[ANSWERS]

1 (a) $\sqrt{3}-1$ (b) 5^{3x-11}

2 (ii) 172 cm^2 (3 s.f.) (iii) 6 (rounded down to nearest integer)

3 (iii) 27 units^2

4 (i) 986 m (3 s.f.) (ii) 19.3° (1 d.p.) (iii) 29.9° (1 d.p.)

5 (i) -1 (ii) (6, 0) (iii) $p=0,\ q=-12$

6 (i) 24° (iii) 62° (iv) 30°

7 (i) $\frac{1}{3}\mathbf{a}+\frac{2}{3}\mathbf{b}$ (iii) $\overrightarrow{ED}=2\overrightarrow{CD}$

(iv) ED // CD or C, D and E are collinear

$ED=2CD$

8 (a) 52, 38 (b)(i) 47.2 (ii) 8.18 (3 s.f.)

(c)(i) 42, 48, 52 (ii) 10

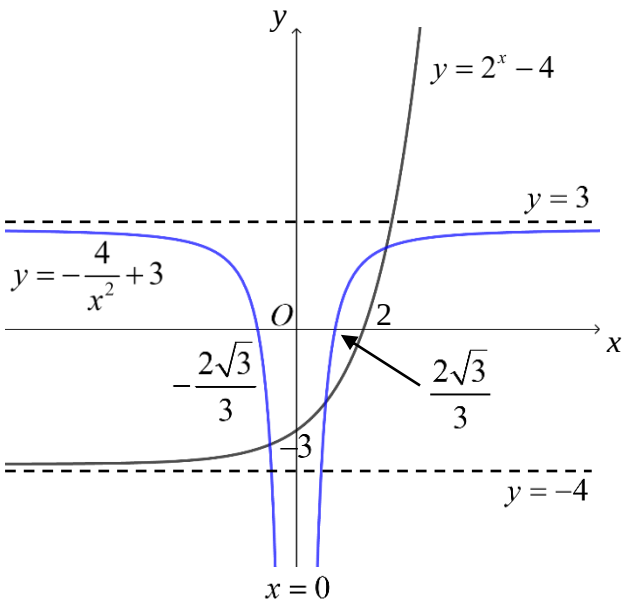
(d) The "box" in the box-and-whisker plot for road B will be wider/more spread out than the "box" in the box-and-whisker plot for road A.

9 (ii) $(0,-6)$ (iii)(a) $x\in\mathbb{R},\ x\neq\frac{1}{2}$ (b) No real solution

(iv) any function $\text{g}(x)$ in the form $ax^2-ax-6a,\ a<0$

10 (i) Asymptotes for C_1: $y=-4$, Asymptotes for C_2: $x=0,\ y=3$

(ii)



(iii) 3

11 (i) $3x+4y+5z=6138$ (ii) $5x+6y+4z=6507$ (iii) $36x+38y+40z=53316$

(iv) $\begin{pmatrix} 3 & 4 & 5 \\ 5 & 6 & 4 \\ 36 & 38 & 40 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 6138 \\ 6507 \\ 53316 \end{pmatrix}$

(v) $\begin{pmatrix} x \\ y \\ z \end{pmatrix}=\begin{pmatrix} 123 \\ 456 \\ 789 \end{pmatrix}$

123 cups of Tangerine Brown Sugar Bubble Tea, 456 cups of Jackfruit Brown Sugar Bubble Tea and 789 cups of Cranberry Brown Sugar Bubble Tea were sold on the product launch day.

(iv) This will result in **A** becoming $\begin{pmatrix} 3 & 4 & 5 \\ 5 & 4.5 & 4 \\ 36 & 38 & 40 \end{pmatrix}$ and $|\mathbf{A}|=0$. Hence **A** is a singular matrix and $\mathbf{A}^{-1}$ does not exist. The equations thus cannot be solved to determine the number of cups of each flavour of bubble tea sold on the product launch day.

12 (ii) $\frac{3}{10}$ (iii) $\frac{1}{5}$ (iv) $\frac{13}{30}$ (v) $\frac{5}{6}$