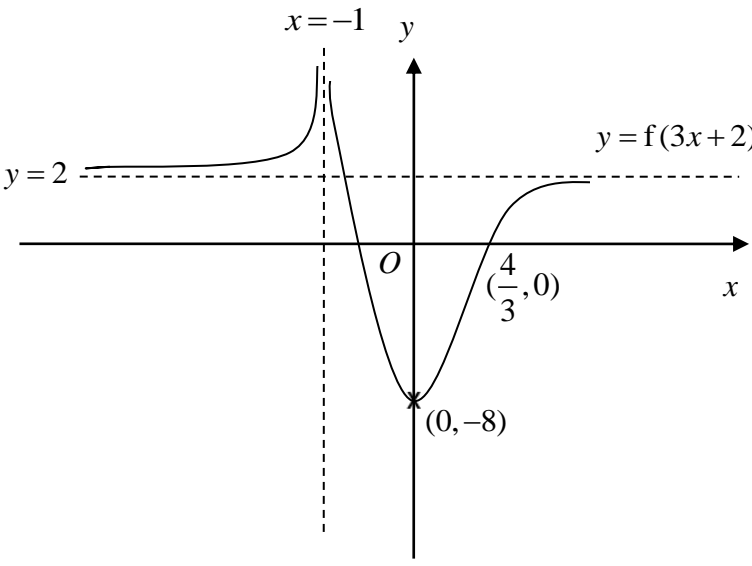
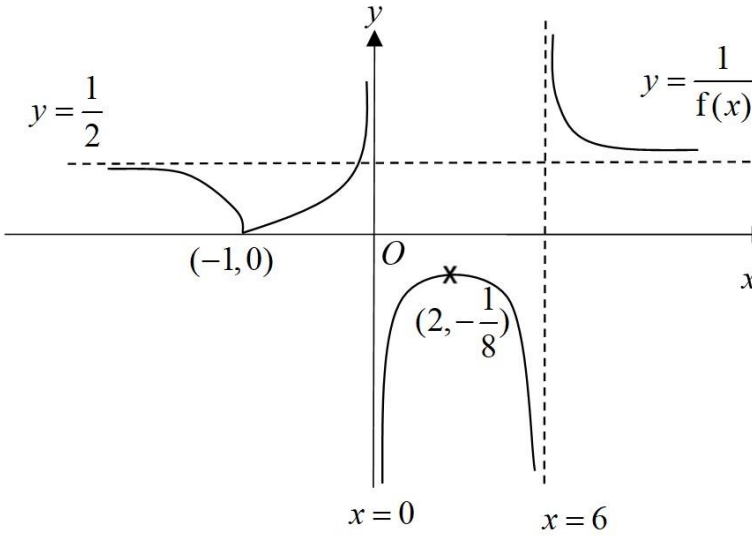


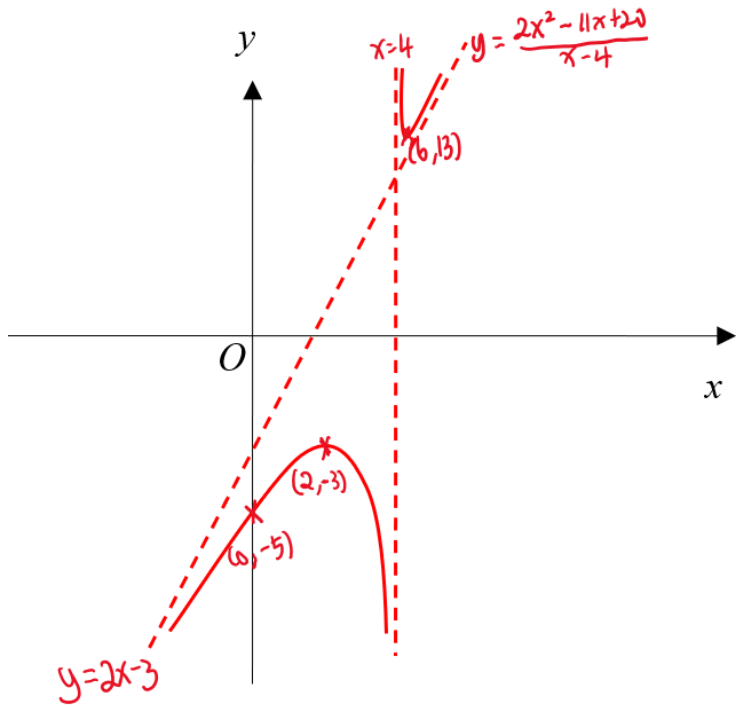
2024 JC1 H2 MATH

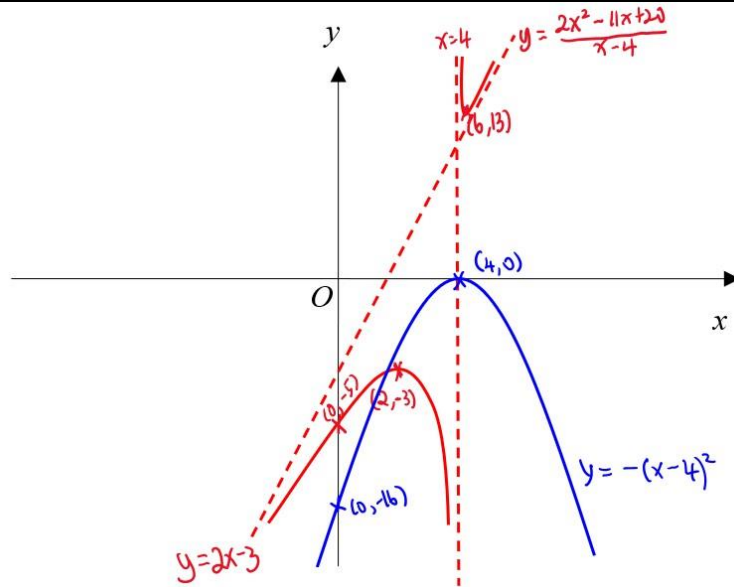
1	Solution [5] Solving system of linear equations	
	$y = \frac{a}{x^2} + \frac{b}{x} + c$ <p>Substitute $\left(-2, -\frac{11}{2}\right)$</p> $\Rightarrow \frac{a}{4} - \frac{b}{2} + c = -\frac{11}{2}$ $\Rightarrow a - 2b + 4c = -22 \text{ ---(1)}$ $\frac{dy}{dx} = -\frac{2a}{x^3} - \frac{b}{x^2}$ <p>When $x = 1, \frac{dy}{dx} = 0$</p> $\Rightarrow -2a - b = 0$ $\Rightarrow 2a + b = 0 \text{ ---(2)}$ <p>When $x = 2, \frac{dy}{dx} = -\frac{3}{2}$</p> $\Rightarrow -\frac{a}{4} - \frac{b}{4} = -\frac{3}{2}$ $\Rightarrow a + b = 6 \text{ ---(3)}$ <p>Solving (1), (2) and (3), from GC, $a = -6, b = 12, c = 2$</p> <p>Equation of C is $y = -\frac{6}{x^2} + \frac{12}{x} + 2$</p>	

2	Solution [5] Transformation of graphs	
(i)	 <p>Graph of $y = f(3x+2)$ showing a vertical asymptote at $x = -1$ and a horizontal asymptote at $y = 2$. The curve passes through the points $(0, -8)$ and $(\frac{4}{3}, 0)$. The origin is labeled O.</p>	
(b)	 <p>Graph of $y = \frac{1}{f(x)}$ showing a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = \frac{1}{2}$. The curve passes through the points $(-1, 0)$ and $(2, -\frac{1}{8})$. The origin is labeled O.</p>	

3	Solution [5] Solving inequalities	
(i)	$\frac{9}{x^2 - 5} > -1$ $\frac{9}{x^2 - 5} + 1 > 0$ $\frac{x^2 + 4}{x^2 - 5} > 0$ <p>Since $x^2 + 4 > 0$ as $x^2 \geq 0$, hence $x^2 - 5 > 0$ $(x + \sqrt{5})(x - \sqrt{5}) > 0$</p> $\begin{array}{ccc} + & - & + \\ \hline -\sqrt{5} & & \sqrt{5} \end{array}$ $x < -\sqrt{5} \text{ or } x > \sqrt{5}$	
(b)	<p>Replacing x with e^x,</p> $\frac{9}{e^{2x} - 5} > -1$ <p>Using result in (i), $e^x < -\sqrt{5}$ or $e^x > \sqrt{5}$ (Reject as $e^x > 0$ for all real x) $x > \ln(\sqrt{5})$ $x > \frac{1}{2} \ln 5$</p> <p>Therefore $x > \frac{1}{2} \ln 5$.</p>	

4	Solution [6] Sigma Notation	
(a)	$\sum_{r=n}^{2n} (r+1)^3 = \sum_{k=n+1}^{2n+1} k^3$ $= \sum_{k=1}^{2n+1} k^3 - \sum_{k=1}^n k^3$ $= \frac{1}{4}(2n+1)^2(2n+2)^2 - \frac{1}{4}n^2(n+1)^2$ $= (n+1)^2(2n+1)^2 - \frac{1}{4}n^2(n+1)^2$ $= \frac{1}{4}(n+1)^2[4(2n+1)^2 - n^2]$ <p>Therefore $a=4$</p>	
(b)	$1^3 + 3^3 + \dots + (2k-1)^3 = \sum_{r=1}^{2k} r^3 - \sum_{r=1}^k (2r)^3$ $= \sum_{r=1}^{2k} r^3 - 8 \sum_{r=1}^k r^3$ $= \frac{1}{4}(2k)^2(2k+1)^2 - \frac{1}{4}(8)(k)^2(k+1)^2$ $= (k)^2(2k+1)^2 - 2(k)^2(k+1)^2$ $= (k)^2(2k^2-1)$ $= 2k^4 - k^2$	

5	Solution [8] Graphing techniques	
(i)	<p>Since C has asymptote $x=4 \Rightarrow b=-4$</p> <p><u>Method 1</u></p> $y = 2x - 3 + \frac{k}{x-4} = \frac{(2x-3)(x-4)+k}{x-4}, \text{ where } k \text{ is a constant}$ <p>By comparing $y = \frac{2x^2 + ax + 20}{x-4} = \frac{2x^2 - 11x + (k+12)}{x-4}$, we obtain $a = -11$.</p> <p><u>Method 2</u></p> <p>By long division,</p> $y = \frac{2x^2 + ax + 20}{x-4} = 2x + a + 8 + \frac{4a+52}{x-4}$ <p>By comparing $y = 2x - 3 = 2x + a + 8$, we obtain $a = -11$.</p>	
(b) (i)	<p>When $a = -11$, $b = -4$, $y = \frac{2x^2 - 11x + 20}{x-4}$</p>  <p>The graph shows a rational function with a vertical asymptote at $x=4$ and a slant asymptote $y=2x-3$. The curve passes through the points $(0, -5)$, $(2, -3)$, and $(6, 13)$. The region between the curve and the asymptotes is shaded red.</p>	
(iii)	$2x^2 + ax + 20 = -(x+b)^3$ $\frac{2x^2 + ax + 20}{x+b} = -(x+b)^2 \quad \text{--- (1)}$	

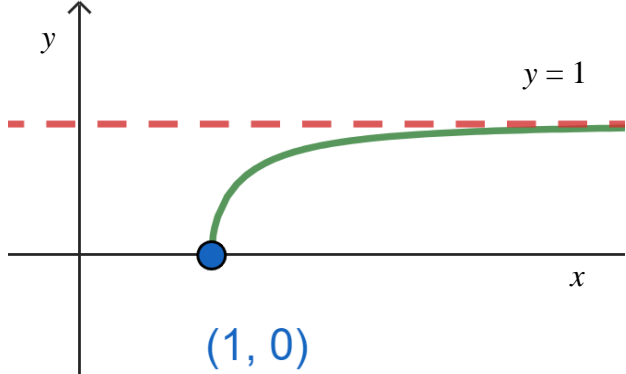


There is 1 intersect between the graph of C and $y = -(x-4)^2$, implying 1 real root for equation (1).

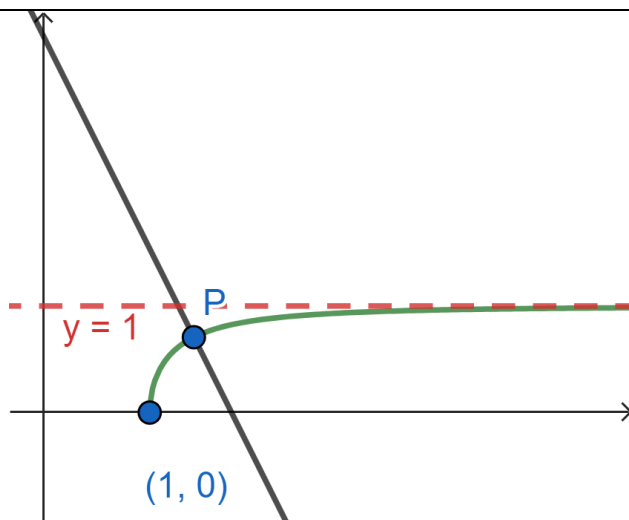
6	Solution [8] Integration Techniques	
(a) (i)	$\int \frac{x^3}{1+x^4} dx = \frac{1}{4} \int \frac{4x^3}{1+x^4} dx$ $= \frac{1}{4} \ln(1+x^4) + c$	
(a) (ii)	$\int \frac{x}{1+x^4} dx = \frac{1}{2} \int \frac{2x}{1+(x^2)^2} dx$ $= \frac{1}{2} \tan^{-1} x^2 + c$	
(b)	$\int x \tan^{-1} x^2 dx$ $= (\tan^{-1} x^2) \left(\frac{1}{2} x^2 \right) - \int \left(\frac{1}{2} x^2 \right) \left(\frac{(2x)}{1+x^4} \right) dx$ $= \left(\frac{1}{2} x^2 \right) \tan^{-1} x^2 - \int \frac{x^3}{1+x^4} dx$ $= \left(\frac{1}{2} x^2 \right) \tan^{-1} x^2 - \frac{1}{4} \int \frac{4x^3}{1+x^4} dx$ $= \left(\frac{1}{2} x^2 \right) \tan^{-1} x^2 - \frac{1}{4} \ln(1+x^4) + c$	

7	Solution [9] Functions	
(a)	<p>Let $y = 6x - x^2 - 2, x \leq -1$</p> $y = -x^2 + 6x - 2$ $y = -(x^2 - 6x + 2)$ $y = -[(x-3)^2 - 9 + 2]$ $y = 7 - (x-3)^2$ $(x-3)^2 = 7 - y$ $x-3 = \pm\sqrt{7-y}$ $x = 3 \pm \sqrt{7-y}$ $x = 3 - \sqrt{7-y} \quad (\text{reject } x = 3 + \sqrt{7-y} \text{ as } x \leq -1)$ $D_{f^{-1}} = R_f = (-\infty, -9]$ $f^{-1}: x \mapsto 3 - \sqrt{7-x}, x \in \square, x \leq -9$	
(b) (i)	$g^2(x) = \frac{a\left(\frac{ax}{x-a}\right)}{\left(\frac{ax}{x-a}\right) - a}$ $= \frac{\frac{a^2x}{x-a}}{\frac{ax-a(x-a)}{x-a}}$ $= \frac{a^2x}{ax - ax + a^2}$ $= \frac{a^2x}{a^2}$ $= x$	
(ii)	$g^2(x) = x$ $g^{-1}[g^2(x)] = g^{-1}(x)$ $g^{-1}[g(g(x))] = g^{-1}(x)$ $g(x) = g^{-1}(x) \text{ (Shown)}$	
(iii)	<p>For fg^{-1} to exist, $R_{g^{-1}} \subseteq D_f$.</p> <p>Since $R_{g^{-1}} = D_g = (-\infty, a)$ and $D_f = (-\infty, -1]$,</p>	

	$\therefore a \leq -1$ Alternatively, from (ii), since $g(x) = g^{-1}(x)$, Since $R_{g^{-1}} = R_g = (-\infty, a)$ and $D_f = (-\infty, -1]$ $\therefore a \leq -1$	
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8	Solution [10] Integration	
(a)	 <p>A graph on a Cartesian coordinate system. The horizontal axis is labeled x and the vertical axis is labeled y. A blue dot marks the point $(1, 0)$ on the x-axis. A green curve starts at this point and increases, approaching a horizontal dashed red line at $y = 1$ as x increases. The label $y = 1$ is placed near the dashed line.</p>	

(ii)



$$y = \frac{1}{x} \sqrt{x^2 - 1} = \frac{(x^2 - 1)^{\frac{1}{2}}}{x}$$

$$\frac{dy}{dx} = \frac{(x) \left(\frac{1}{2} \right) (x^2 - 1)^{-\frac{1}{2}} (2x) - (x^2 - 1)^{\frac{1}{2}}}{x^2} = \frac{1}{x^2 \sqrt{x^2 - 1}}$$

When $x = \sqrt{2}$,

$$\left. \frac{dy}{dx} \right|_{x=\sqrt{2}} = \frac{1}{2} \quad (\text{This value can be obtained via GC})$$

Equation of normal:

$$y - \frac{1}{\sqrt{2}} = (-2) \left(x - \sqrt{2} \right)$$

$$y = -2x + \frac{5}{\sqrt{2}}$$

Alternatively

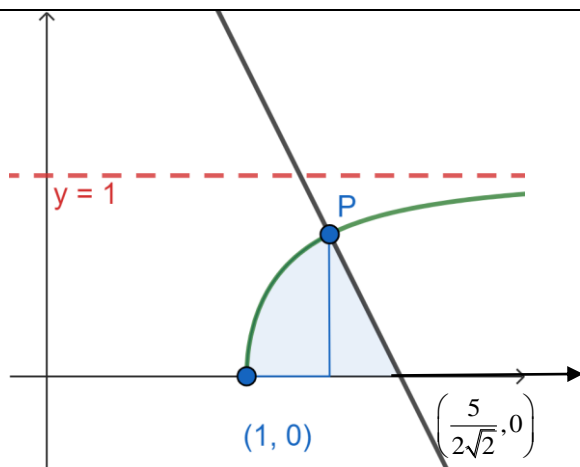
$$y = \frac{1}{x} \sqrt{x^2 - 1} = \sqrt{\frac{x^2 - 1}{x^2}} = \sqrt{1 - \frac{1}{x^2}}$$

$$\frac{dy}{dx} = \left(\frac{1}{2} \right) \left(1 - \frac{1}{x^2} \right)^{-\frac{1}{2}} \left(\frac{2}{x^3} \right)$$

$$\frac{dy}{dx} = \left(\frac{1}{2} \right) \left(\frac{x^2}{x^2 - 1} \right)^{\frac{1}{2}} \left(\frac{2}{x^3} \right)$$

$$\frac{dy}{dx} = \frac{1}{x^2 \sqrt{x^2 - 1}}$$

(iii)



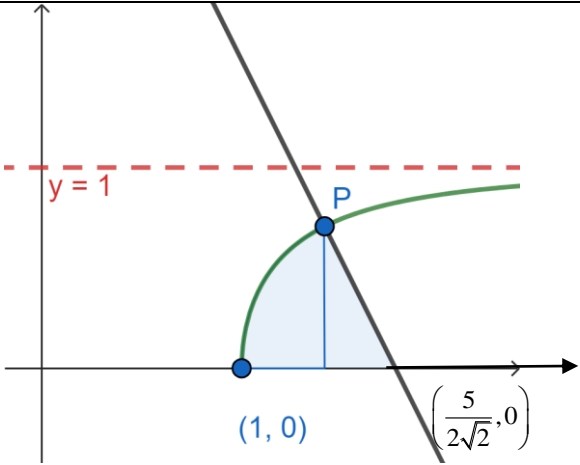
$$\begin{aligned}
 \text{Area} &= \int_1^{\sqrt{2}} \frac{1}{x} \sqrt{x^2 - 1} \, dx \\
 &= \int_0^{\frac{\pi}{4}} \frac{1}{\sec \theta} \sqrt{\sec^2 \theta - 1} \sec \theta \tan \theta \, d\theta \\
 &= \int_0^{\frac{\pi}{4}} \tan^2 \theta \, d\theta \\
 &= \int_0^{\frac{\pi}{4}} (\sec^2 \theta - 1) \, d\theta \\
 &= [\tan \theta - \theta]_0^{\frac{\pi}{4}} \\
 &= 1 - \frac{\pi}{4} \text{ units}^2
 \end{aligned}$$

$$x = \sec \theta$$

$$\frac{dx}{d\theta} = \sec \theta \tan \theta$$

$$\text{When } x = 1, \theta = 0$$

$$\text{When } x = \sqrt{2}, \theta = \frac{\pi}{4}$$

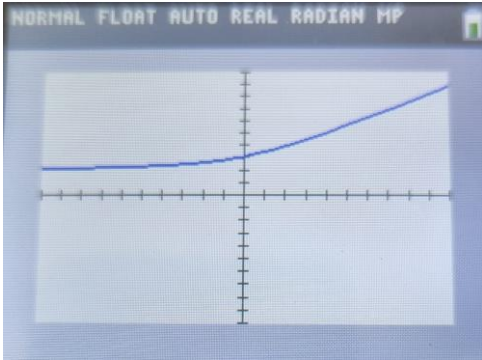
(iv)	 <p>Area = $\int_1^{\sqrt{2}} \frac{1}{x} \sqrt{x^2-1} \, dx + \frac{1}{2} \left(\frac{5}{2\sqrt{2}} - \sqrt{2} \right) \left(\frac{1}{\sqrt{2}} \right)$</p> $= \left[1 - \frac{\pi}{4} \right] + \frac{1}{2} \left(\frac{1}{2\sqrt{2}} \right) \left(\frac{1}{\sqrt{2}} \right)$ $= \left[1 - \frac{\pi}{4} \right] + \frac{1}{8}$ $= \frac{9}{8} - \frac{\pi}{4}$ $= \frac{9-2\pi}{8} \text{ units}^2$	
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9	Solution [10] AP, GP contextual question							
(a)	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{77}{2}[2(1250) + (77-1)(12)]$ $= 131\,362$ <p>The theoretical maximum amount of waste that the landfill can contain is 131 362 tonnes.</p>							
(b) (i)	$S_n = \frac{a(1-r^n)}{1-r}$ $= \frac{1500(1-0.98^n)}{1-0.98}$ <p><u>Method 1</u></p> $\frac{1500(1-0.98^n)}{1-0.98} > 50\,000$ $1500(1-0.98^n) > 1000$ $1-0.98^n > \frac{2}{3}$ $0.98^n < \frac{1}{3}$ $n > \frac{\ln\left(\frac{1}{3}\right)}{\ln(0.98)}$ $n > 54.4 \text{ (3 s.f.)}$ <p><u>Method 2</u></p> $\frac{1500(1-0.98^n)}{1-0.98} > 50\,000$ <p>By GC,</p> <table><tr><td>n</td><td>$\frac{1500(1-0.98^n)}{1-0.98}$</td></tr><tr><td>54</td><td>$49\,808 < 50\,000$</td></tr><tr><td>55</td><td>$50\,311 > 50\,000$</td></tr></table> <p>Thus least $n=55$ and the least number of years is 55.</p>	n	$\frac{1500(1-0.98^n)}{1-0.98}$	54	$49\,808 < 50\,000$	55	$50\,311 > 50\,000$	
n	$\frac{1500(1-0.98^n)}{1-0.98}$							
54	$49\,808 < 50\,000$							
55	$50\,311 > 50\,000$							

<div>(b)</div> <div>(ii)</div>	<div><u>Method 1</u></div> <div>$S_{\infty} = \frac{a}{1-r}$$= \frac{1500}{1-0.98}$$= 75\,000$</div> <div><u>Method 2</u></div> <div>As $n \rightarrow \infty$, $\frac{1500(1-0.98^n)}{1-0.98} \rightarrow \frac{1500}{1-0.98} = 75\,000$</div> <div>Yes, the plant can cope with the total amount of waste of 75 000 tonnes to be incinerated.</div>							
<div>(c)</div>	<div>In the nth year from 2014, amount of waste deposited in landfill is $1250 + (n-1)(12)$.</div> <div>In the nth year from 2014, amount of waste incinerated is $1500(0.98)^{n-1}$.</div> <div>$1250 + (n-1)(12) > 1500(0.98)^{n-1}$$1250 + (n-1)(12) - 1500(0.98)^{n-1} > 0$<p>By GC, $n > 7.17$</p><p>By GC,</p><table><tr><td>n</td><td>$1250 + (n-1)(12) - 1500(0.98)^{n-1}$</td></tr><tr><td>7</td><td>$-6.764 < 0$</td></tr><tr><td>8</td><td>$31.812 > 0$</td></tr></table></div> <div>Thus $n = 8$ and it is the 8th year after the year 2014 that the amount of waste deposited in the landfill first exceeds the amount of waste incenerated in the incineration plant.</div>	n	$1250 + (n-1)(12) - 1500(0.98)^{n-1}$	7	$-6.764 < 0$	8	$31.812 > 0$	
n	$1250 + (n-1)(12) - 1500(0.98)^{n-1}$							
7	$-6.764 < 0$							
8	$31.812 > 0$							

10	Solution [10] Maclaurin series	
(a) (i)	$y = \ln(2 + \sin 2x)$ $e^y = 2 + \sin 2x$ Differentiate w.r.t. x : $e^y \frac{dy}{dx} = 2 \cos 2x \text{ --- (1)}$ Differentiate w.r.t. x : $e^y \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx} \right)^2 = -4 \sin 2x$ $e^y \left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right) + 4 \sin 2x = 0 \text{ --- (2)}$	
	<p><u>Alternative method</u></p> $y = \ln(2 + \sin 2x)$ Differentiate w.r.t. x : $\frac{dy}{dx} = \frac{2 \cos 2x}{2 + \sin 2x}$ Differentiate w.r.t. x : $\frac{d^2y}{dx^2} = \frac{-4 \sin 2x(2 + \sin 2x) - 2 \cos 2x(2 \cos 2x)}{(2 + \sin 2x)^2}$ $= \frac{-4 \sin 2x}{2 + \sin 2x} - \left(\frac{2 \cos 2x}{2 + \sin 2x} \right)^2$ $= \frac{-4 \sin 2x}{e^y} - \left(\frac{dy}{dx} \right)^2$ $e^y \frac{d^2y}{dx^2} = -4 \sin 2x - e^y \left(\frac{dy}{dx} \right)^2$ $e^y \frac{d^2y}{dx^2} + e^y \left(\frac{dy}{dx} \right)^2 + 4 \sin 2x = 0$ $e^y \left(\frac{d^2y}{dx^2} + \left(\frac{dy}{dx} \right)^2 \right) + 4 \sin 2x = 0$	

<p>(a) (ii)</p>	<p>Differentiate implicitly w.r.t. x:</p> $e^y \frac{d^3 y}{dx^3} + e^y \left(\frac{dy}{dx} \right) \left(\frac{d^2 y}{dx^2} \right) + 2e^y \left(\frac{dy}{dx} \right) \left(\frac{d^2 y}{dx^2} \right) + e^y \left(\frac{dy}{dx} \right)^3 + 8 \cos 2x = 0$ <p>When $x = 0$, $y = \ln 2$, $\frac{dy}{dx} = 1$, $\frac{d^2 y}{dx^2} = -1$, $\frac{d^3 y}{dx^3} = -2$,</p> $y = \ln 2 + x + \frac{-1}{2!} x^2 + \frac{-2}{3!} x^3 + \dots$ $y \approx \ln 2 + x - \frac{1}{2} x^2 - \frac{1}{3} x^3 \quad (\text{up to } x^3 \text{ term})$	
<p>(a) (iii)</p>	<p>$\ln(2 + \sin 2x) = \ln 2 + x - \frac{1}{2} x^2 - \frac{1}{3} x^3 + \dots$</p> <p>Differentiate w.r.t. x:</p> $\frac{2 \cos 2x}{2 + \sin 2x} = 1 - x - x^2 \quad (\text{up to } x^2 \text{ term})$	
<p>(b)</p>	$\frac{2 \cos 2x}{2 + \sin 2x} \approx 2 \left(1 - \frac{(2x)^2}{2} \right) (2 + 2x)^{-1}$ $\approx 2(1 - 2x^2)(2)^{-1}(1 + x)^{-1}$ $\approx (1 - 2x^2) \left(1 - x + \frac{(-1)(-2)}{2!} (-x)^2 \right)$ $= (1 - 2x^2)(1 - x + x^2)$ $= 1 - x + x^2 - 2x^2 + \dots$ $= 1 - x - x^2 \quad (\text{up to } x^2 \text{ term})$	

11	Solution [12] Tangent & Rate of change	
(a)	$C: x = \frac{3}{\theta} - \theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ 	
(b)	$x = \frac{3}{\theta} - \theta$ $\frac{dx}{d\theta} = \frac{-3}{\theta^2} - 1 = \frac{-3 - \theta^2}{\theta^2}$ $\frac{d\theta}{dt} = \frac{d\theta}{dx} \cdot \frac{dx}{dt} = -\frac{\theta^2}{3 + \theta^2} \cdot 10$ <p>At (2,4), $\frac{3}{\theta} - \theta = 2$ and $\frac{2}{\theta} + 2 = 4$</p> $\theta^2 + 2\theta - 3 = 0$ $\theta = -3 \text{ or } 1 \quad \text{and} \quad \theta = 1$ <p>$\therefore \theta = 1$</p> $\frac{d\theta}{dt} = -\frac{(1)^2}{3 + (1)^2} \cdot 10 = -\frac{5}{2} \text{ units s}^{-1}$	
(c) (i)	$y = \frac{2}{\theta} + 2$ $\frac{dy}{d\theta} = -\frac{2}{\theta^2}$ $\frac{dy}{dx} = \frac{-\frac{2}{\theta^2}}{\frac{-3 - \theta^2}{\theta^2}}$ $= \frac{-2}{\theta^2} \times \frac{\theta^2}{-3 - \theta^2}$ $= \frac{2}{3 + \theta^2}$	

	<p>When $\theta = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{2}$, $x = 2$ and $y = 4$.</p> $y - 4 = \frac{1}{2}(x - 2)$ $y = \frac{1}{2}x + 3$ <p>The tangent at point P is $y = \frac{1}{2}x + 3$.</p>	
(c) (ii)	$y - 4 = \frac{1}{2}(x - 2) \text{ ---(1)}$ <p>Substitute $x = \frac{3}{\theta} - \theta$, $y = \frac{2}{\theta} + 2$ into (1):</p> $\frac{2}{\theta} + 2 - 4 = \frac{1}{2}\left(\frac{3}{\theta} - \theta - 2\right)$ $\frac{2 - 2\theta}{\theta} = \frac{3 - \theta^2 - 2\theta}{2\theta}$ $4 - 4\theta = 3 - \theta^2 - 2\theta$ $\theta^2 - 2\theta + 1 = 0$ $(\theta - 1)^2 = 0$ $\theta = 1 \text{ which corresponds to } P.$ <p>Alternatively,</p> $\theta^2 - 2\theta + 1 = 0$ $b^2 - 4ac = (-2)^2 - 4(1)(1) = 0$ <p>This implies that the equation $\theta^2 - 2\theta + 1 = 0$ has two equal and real roots, which is $\theta = 1$.</p> <p>Hence the tangent will not meet the curve again.</p>	
(c) (iii)	<p>Tangent to C at point P, gradient is $\frac{1}{2}$</p> <p>Tangent to C at point M, gradient is</p> $\frac{2}{3 + \theta^2} = \frac{2}{3 + (2)^2}$ $= \frac{2}{7}$ <p>Therefore, Normal to C at point M has gradient $-\frac{7}{2}$.</p>	

	<p>Gradients of tangent at $P = \frac{1}{2}$</p> <p>$\theta_1 = \tan^{-1} \frac{1}{2}$</p> <p>$\theta_2 = -\tan^{-1} \left(-\frac{7}{2}\right)$</p> <p>Gradients of normal at $M = -\frac{7}{2}$</p> <p>Note: $-\frac{\pi}{2} < \tan^{-1} \theta < \frac{\pi}{2}$, $\tan^{-1} \left(-\frac{7}{2}\right)$ gives a negative value.</p> <p><u>Method 1</u></p> $\tan^{-1} \left(\frac{1}{2}\right) - \tan^{-1} \left(-\frac{7}{2}\right) = 100.6^\circ$ <p>Required acute angle</p> $= 180^\circ - 100.6^\circ$ $= 79.4^\circ$	
	<p><u>Method 2</u></p> <p>Angle between the 2 lines</p> $= \tan^{-1} \left(\frac{1}{2}\right) + \tan^{-1} \left(\frac{7}{2}\right) = 100.6^\circ$ <p>Required acute angle</p> $= 180^\circ - 100.6^\circ$ $= 79.4^\circ$	
(d)	<p>$C: x = \frac{3}{\theta} - \theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$</p> <p>After by a stretch with scale factor 2 in the x-direction, the equation of C becomes</p> $\frac{x}{2} = \frac{3}{\theta} - \theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ $x = \frac{6}{\theta} - 2\theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ <p>After by a reflection in the y-axis, the equation becomes</p> $-x = \frac{6}{\theta} - 2\theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ $D: x = 2\theta - \frac{6}{\theta}, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ <p>Alternatively,</p> <p>After by a reflection in the y-axis, the equation becomes</p> $-x = \frac{3}{\theta} - \theta, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$	

	$x = \theta - \frac{3}{\theta}, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ <p>After by a stretch with scale factor 2 in the x-direction, the equation of becomes</p> $\frac{x}{2} = \theta - \frac{3}{\theta}, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$ <p>D: $x = 2\theta - \frac{6}{\theta}, \quad y = \frac{2}{\theta} + 2, \quad \text{where } \theta > 0$</p>	
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12	Solution [12] Differential Equations	
(a) (i)	$\frac{dv}{dt} = c$	
(a) (ii)	$\frac{dv}{dt} = c$ $\int dv = \int c dt$ $v = ct + v_0$ Given that $c = 10$, $v = 10t + v_0$ When $t = 0$, $v = 0 \Rightarrow v_0 = 0$ Therefore $v = 10t$. When $v = 30 \Rightarrow 30 = 10t \Rightarrow t = 3$ It will take 3 seconds for the velocity to reach 30 ms^{-1}	
(b) (i)	$\frac{dv}{dt} = 10 - k\sqrt{v}$ $\frac{dv}{dt} = 0$ when $v = \frac{100}{k^2}$ $v = \frac{100}{k^2}$ is the constant velocity that the object moves eventually.	

<p>(b) (ii)</p>	<p>Let $u = \sqrt{v} \Rightarrow v = u^2 \Rightarrow \frac{dv}{du} = 2u$</p> $\frac{dv}{dt} = 10 - k\sqrt{v}$ $\frac{dv}{du} \cdot \frac{du}{dt} = 10 - ku$ $2u \frac{du}{dt} = 10 - ku \text{ ---- } (*)$ $2 \int \frac{u}{10 - ku} du = \int dt$ $2 \int \frac{1}{k} \left(\frac{10}{10 - ku} - 1 \right) du = \int dt \text{ using given result}$ <p>Note: Given $\frac{ku}{10 - ku} = \frac{10}{10 - ku} - 1$ where k is a constant</p> $\int \frac{10}{10 - ku} - 1 du = \frac{kt}{2} + C$ $-\frac{10}{k} \ln 10 - ku - u = \frac{kt}{2} + C$ $u + \frac{10}{k} \ln 10 - ku = -\frac{kt}{2} - C$ $\sqrt{v} + \frac{10}{k} \ln 10 - k\sqrt{v} = -\frac{kt}{2} - C$ $2k\sqrt{v} + 20 \ln 10 - k\sqrt{v} = A - k^2t,$ <p style="text-align: center;">where $A = -2Ck$ (Shown)</p>	
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