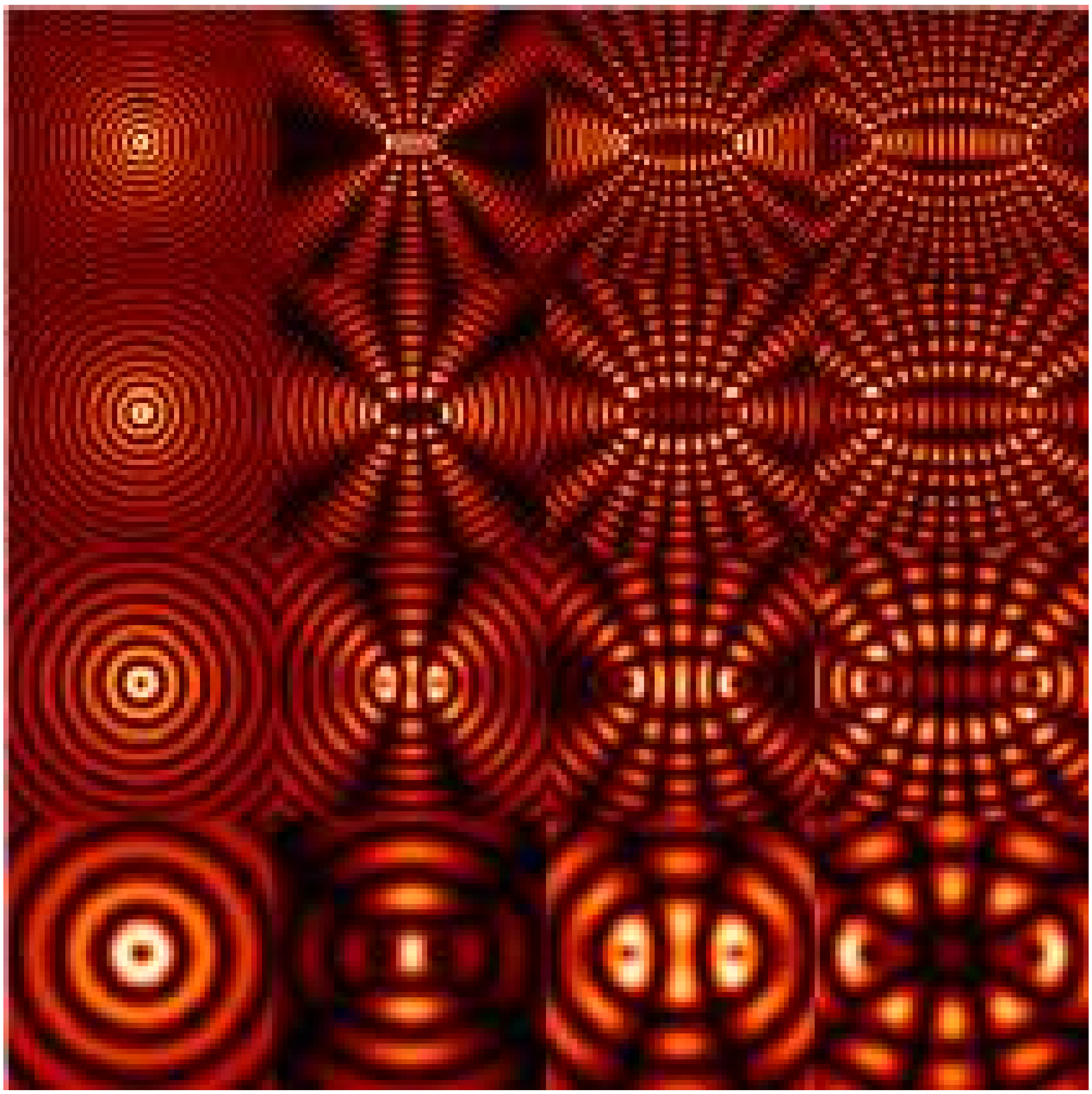
# Chapter 10

# SUPERPOSITION



Interference pattern produced by two sources placed at different distances apart and of different wavelengths. Taken from www.arthistoryclub.com

Playlist of Lecture Example Solutions and Others available at
https://youtube.com/playlist?list=PL_b5cjrUKDlbeRspxU7s_UswuIC_U1VMJ

# Syllabus 9749

## Content

- Principle of Superposition
- Stationary Waves
- Diffraction
- Two-source interference
- Single slit and multiple slit diffraction

## Learning Outcomes

**Candidates should be able to:**

(a) explain and use the principle of superposition in simple applications.

(b) show an understanding of the terms: interference, coherence, phase difference and path difference.

(c) show an understanding of experiments which demonstrate stationary waves using microwaves, stretched strings and air columns.

(d) explain the formation of a stationary wave using a graphical method, and identify nodes and antinodes.

(e) explain the meaning of the term diffraction.

(f) show an understanding of experiments which demonstrate diffraction including the diffraction of water waves in a ripple tank with both a wide gap and a narrow gap.

(g) show an understanding of experiments which demonstrate two-source interference using water waves, sound waves, light waves and microwaves.

(h) show an understanding of the conditions required for two-source interference fringes to be observed.

(i) recall and solve problems using the equations $\lambda = a\,x / D$ for double slit interference.

(j) recall and use the equation $\sin\theta = \lambda/b$ to locate the position of the first minima for single slit diffraction.

(k) recall and use the Rayleigh criterion $\theta \approx \lambda/b$ for the resolving power of a single aperture.

(l) recall and use the equation $d \sin\theta = n\lambda$ to locate the positions of the principal maxima produced by a diffraction grating.

(m) describe the use of a diffraction grating to determine the wavelength of light. (the structure and use of a spectrometer are not required.)

# Contents

## 10.1 Introduction

In your secondary schools, you have seen that all waves exhibit *reflection* and *refraction.*

For the A-levels, we extend our understanding of waves by looking at two other phenomena that are exhibited by waves: *interference* and *diffraction*.

We will learn about *Young's double slit experiment*, *the measurement of wavelengths* of light and sound, and the formation of musical notes in strings and woodwind instruments through the formation of *stationary waves*.

## 10.2 Principle of Superposition

Imagine throwing two pebbles into a still pond. Each pebble makes its own ripples and as the ripples spread out, they kind of overlap. What determines the resultant “ripple” that we see?

The **displacement** of the resultant wave is determined by the **Principle of Superposition**:

> ***<u>Principle of Superposition:</u>***
>
> The principle of superposition states that **when two or more waves of the same kind overlap**, the **resultant displacement** at any point at any instant is given by the **vector sum of the individual displacements** that each individual wave would cause at that point at that instant.
>
> $$y_{resultant} = y_1 + y_2 + y_3 + .... + y_N$$
>
> where $y_{resultant}$ is the displacement of the resultant wave and $y_1$, $y_2$, ..., $y_N$ are the individual displacements of the waves at the point of consideration.

The statement of the Principle of Superposition is a popular exam question. Be careful to use the word “**displacement**” and not “amplitude” in your statement.

Fig. 10.2.1 and 10.2.2 are pictorial representations of the superposition of two wave pulses, $y_1$ and $y_2$, travelling in opposite directions. When the waves begin to overlap, the resultant displacement of the waveform on the string at every point at any time is given by $y_1 + y_2$.

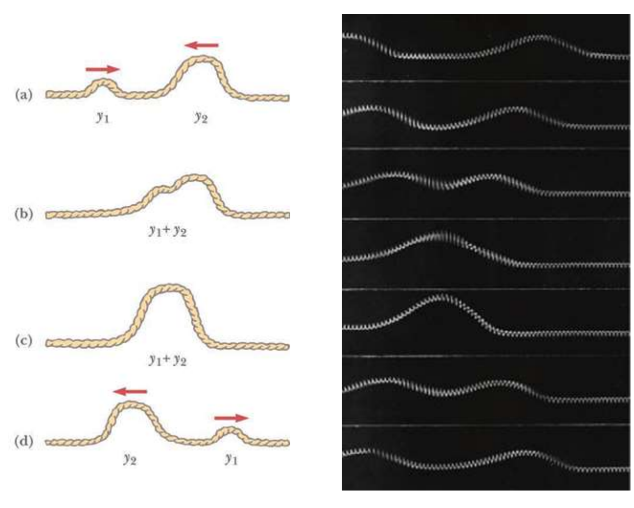



**Fig. 10.2.1.** Constructive interference of two wave pulses.

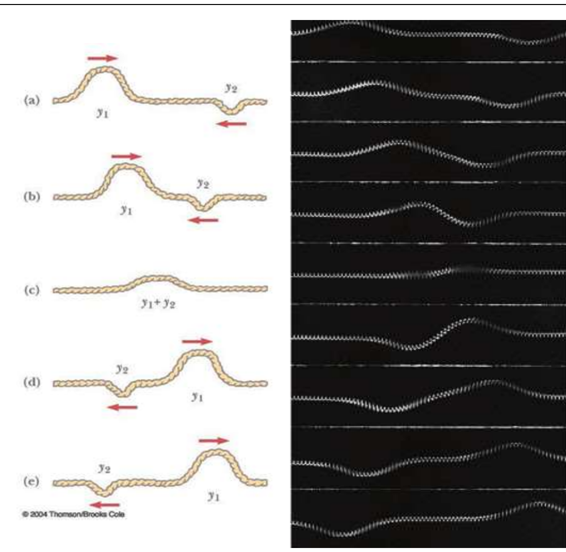



**Fig. 10.2.2.** Destructive interference of two wave pulses.

Two pulses travelling on a stretched string in opposite directions pass through each other. When the pulses overlap the net displacement of the string is equal to the sum of the displacements produced by each pulse.

Note that when the two pulses separate, they will continue to move in their original directions as if nothing has happened. From this illustration, we also see that:

> After two travelling waves have passed through each other, **the pulse shapes of each wave are unchanged, as if the two pulses had never met.**

### 10.3 Interference

This combination of separate waves in the same region of space at the same time to produce a resultant wave is called **interference**:

> **<u>Interference</u>**
>
> Interference is the **superposing** or overlapping of two or more waves to give a resultant wave whose **displacement** is given by the **Principle of Superposition,** which states that **displacement** of the resultant wave at any point is the **vector sum** of the **displacements** of the individual waves at that point.

> **<u>Constructive interference</u>**
>
> Constructive interference **occurs at a point** when **two waves** meet **in phase** at that point. The resultant **amplitude** of the oscillation at that point is therefore a **maximum**.
>
> - Phase difference between the two waves at that point = 0, $2\pi$, $4\pi$, …
> - Resultant amplitude $A_R = A_1 + A_2$

*<u>Constructive Interference of Transverse Waves</u>*

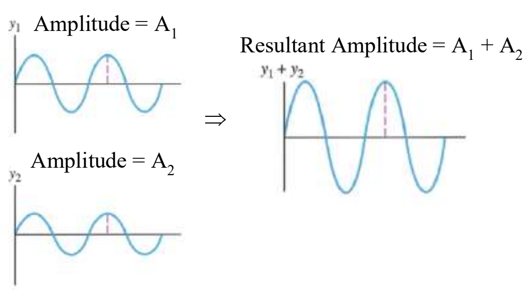



**Fig. 10.3.1.** Two transverse waves in phase undergo constructive interference at every point to produce a wave of maximum amplitude.

*<u>Constructive Interference of Longitudinal Waves</u>*

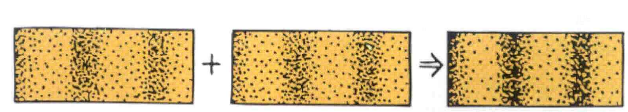



**Fig. 10.3.2.** Two longitudinal waves in phase undergo constructive interference at every point to produce a wave of increased intensity. In this case, since the amplitudes of the 2 longitudinal waves are the same, the resultant amplitude will be double that of each individual wave.

> **<u>Destructive Interference:</u>**
>
> Destructive interference **occurs at a point** when **two waves** meet in **antiphase** (phase difference of $180^o$). The resultant **amplitude** of the oscillation at that point is therefore a **minimum**.
>
> - Phase difference between the two waves at that point = $\pi$, $3\pi$, $5\pi$,…
> - Resultant amplitude $A_R = A_1 - A_2$
> - If the waves have the **same amplitude**, then the **resultant amplitude** would be **zero**.

***Destructive Interference of Transverse Waves***

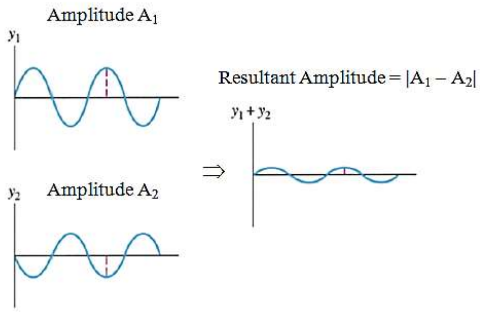



**Fig. 10.3.3** Two identical transverse waves that are out of phase undergo destructive interference at every point when they superimposed

***Destructive Interference of Longitudinal Waves***

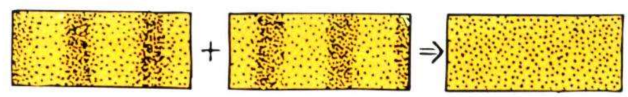
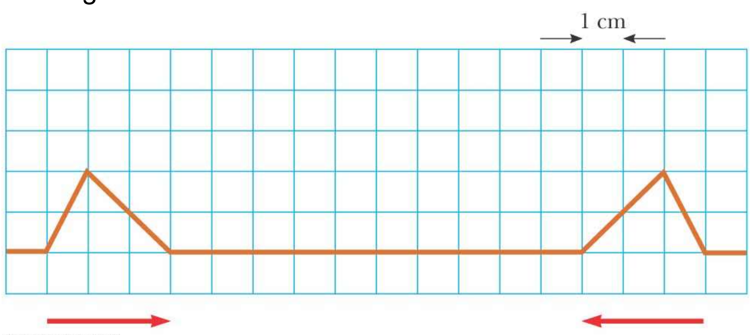


**Fig. 10.3.4.** Two identical longitudinal waves that are out of phase undergo destructive interference at every point when they are superimposed

Note that when interference is neither fully constructive nor fully destructive, it is called *intermediate interference* or *partially constructive or destructive interference*. The amplitude of the resultant waveform will be between the maximum and minimum amplitudes possible (not inclusive).

**Example 10.3.1**

Two pulses are travelling toward each other, at 10 cm $s^{-1}$ on a long string, as shown in the figure below. Sketch the shape of the string after 0.6 s.




**Solution:**

# 10.4 Interference of Two Wave Sources

## 10.4.1 Interference of Waves from Two Sources that are in Phase

Consider two identical sources of monochromatic waves $S_1$ and $S_2$ as shown in Fig. 10.4.1. The two sources produce waves of *same amplitude, same frequency* and are permanently oscillating *in phase*. Both waves have the same speed.

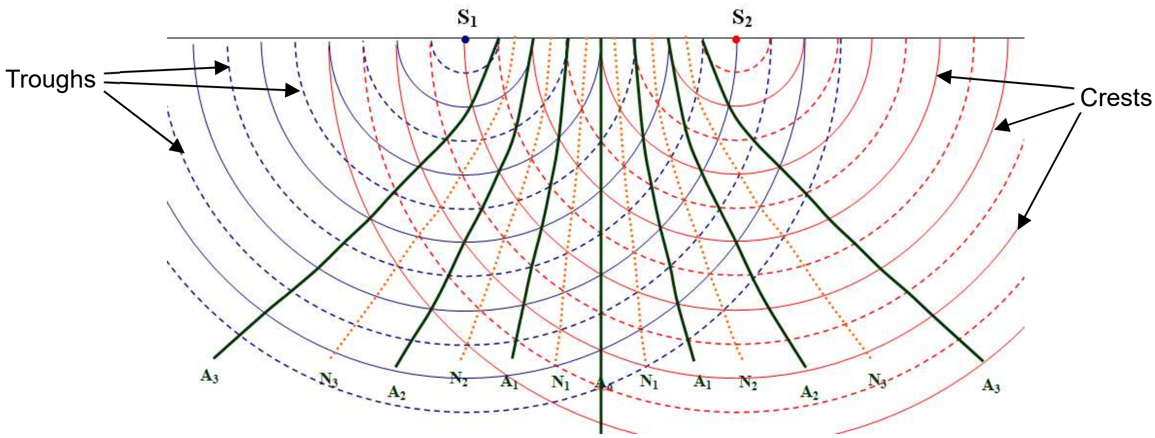



**Fig. 10.4.1.** Interference pattern of 2 waves sources oscillating in phase. Solid lines represent the crests while dotted lines represent the troughs of the waves produced by the two sources

### Path Difference between the Two Wave Sources

Before we derive the general conditions for *constructive interference* or *destructive interference* to be observed at any point **P**, we first define what we mean by *path difference*:

**Path difference** $\delta$ between the two waves,

$$\delta = \left|\text{Distance travelled by wave from } S_1 \text{ to P} - \text{Distance travelled by wave from } S_2 \text{ to P}\right|$$

$$= \left|S_1P - S_2P\right|$$

$A_o$

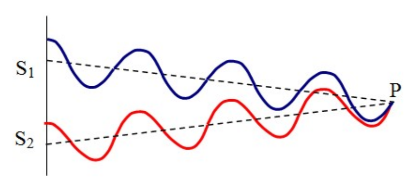



**Fig. 10.4.2.** Path difference between waves from $S_1$ and $S_2$ is the difference between the distances travelled by wave from $S_1$ and wave from $S_2$.

## Conditions for Constructive Interference

Consider the lines **$A_o$, $A_1$, $A_2$** and **$A_3$.**

Along these lines, the crests of the wave from $S_1$ (See Fig. 10.4.1) will always meet with crests from $S_2$. Similarly the troughs of the wave from $S_1$ will meet with the troughs from $S_2$.

From the Principle of Superposition, at the points where a crest meets another crest, we will get a resultant crest that is twice as high of that of the component waves. Similarly, at the points where a trough meets another trough, the resultant wave will have a trough that is twice as low as that of each component wave.

Therefore along the lines **$A_o$, $A_1$, $A_2$** and **$A_3$**, **the waves will always meet in phase** and hence **constructive interference** occurs. **$A_o$, $A_1$, $A_2$** and **$A_3$** are also known as the **antinodal lines**.

**For a point P on the antinodal line $A_o$**, two waves from $S_1$ and $S_2$ travel to P over the same distance, therefore they meet in phase. We see that for all points on line $A_o$

Path difference between the 2 waves at P: $\delta = |S_1P - S_2P| = 0$

Phase difference between the 2 waves at P: $\Delta\phi = 0$

**For any point P on the antinodal line $A_1$,** we see that the path (distance) travelled by one wave to P is always longer than the other wave by **one wavelength**.

Path difference between the 2 waves, $\delta = |S_1P - S_2P| = \lambda$

Phase difference between the 2 waves at P: $\Delta\phi = 2\pi$

**Similarly for antinodal lines $A_2$,** for all points P on the line, the path travelled by one wave is always longer than the other wave by **two wavelengths.**

Path difference between the 2 waves, $\delta = |S_1P - S_2P| = 2\lambda$

Phase difference between the 2 waves at P: $\Delta\phi = 4\pi$

In general,

**Conditions for Constructive Interference:**

1) **Looking at Phase difference (independent of phase difference at the sources):**

   **Phase difference** between the two waves **meeting at point P** is

$$\Delta\phi = 0, 2\pi, 4\pi, 6\pi, ... = 2m\pi$$

2) **Looking at Path difference (dependent on phase difference at the sources):**

   For **two waves sources $S_1$ and $S_2$** that are oscillating **in phase**,

   **Path difference** between the two waves travelling from the sources to the point P is:

$$\delta = |S_1P - S_2P| = 0, \lambda, 2\lambda, 3\lambda, ... = m\lambda$$

   where $m$ = 0,1,2,3,... and $\lambda$ is the wavelength of the individual waves.

**Note:**

1. The points of **constructive interference** are also known as points of **maxima**.
2. $m$ is also commonly known as the *order*.

- The central maximum is known as the *zeroth order maximum.* ($m$ = 0)
- The two maxima on either side of the zeroth order maximum are known as the **1st order maxima** ($m$ = 1). The next maxima further away are then known as the **2nd order maxima** and so on.
- Note that sometimes, we assign signs (+ or - ) to distinguish the "directions" of the order e.g. the orders observed to the right of the zeroth order may be given a positive sign (+) and therefore those orders observed to left of the zeroth order may be given a negative (-) sign.

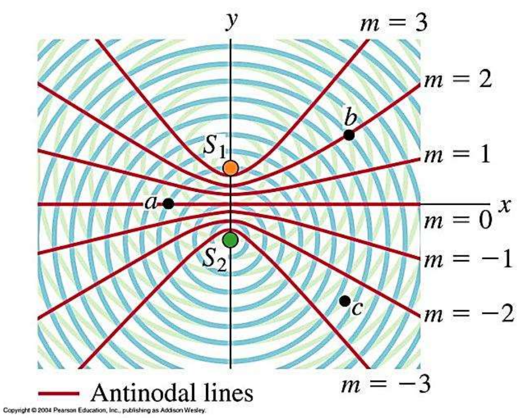



**Fig. 10.4.3.** Two wave sources interfere with each other to produce points of constructive and destructive interference in a two dimension area. Point *b* is a point where constructive interference is observed. Point *c* is a point where destructive interference is observed. The antinodal lines (lines that join points of constructive interference of the same order) are indicated as well with the value of the order *m* shown. Here orders which lie above the zeroth order maximum are given a (+) sign and orders which lie below the zeroth order maximum are given a (-) sign.

## Destructive Interference

Consider each point along lines $\mathbf{N_1}$, $\mathbf{N_2}$, and $\mathbf{N_3}$ (See Fig 10.4.1). We notice that, along these lines, the crest of a wave from $S_1$ will always meet with a trough from $S_2$.

Hence, the two waves will always meet in **antiphase** and therefore **destructive interference occurs.** $\mathbf{N_o}$, $\mathbf{N_1}$, $\mathbf{N_2}$ and $\mathbf{N_3}$ are also known as **nodal lines.**

We follow the same procedure to derive the general condition for a destructive interference to be observed at any point **P** from the two sources by examining the nodal lines.

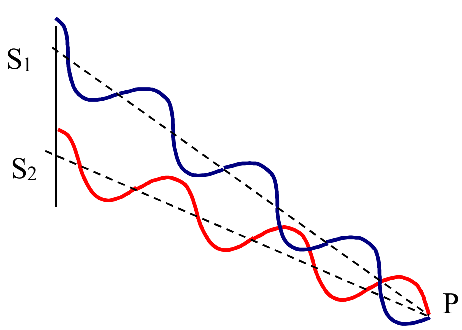



**Fig. 10.4.4**

For a particular nodal line $N_1$, the wave from $S_1$ travels one-half of a wavelength ($\lambda/2$) more than the wave from $S_2$ to P, so if initially the waves were produced at the source to be in phase (phase difference is zero), this additional $\lambda/2$ will cause an equivalent phase difference of 180° and hence the two waves will meet in anti-phase. Therefore a destructive interference will be observed at P.

Now if P is a point on $N_1$,

Path difference between the 2 waves,

$$\delta = |S_1P - S_2P| = \frac{\lambda}{2}$$

If P is a point on $N_2$,

Path difference between the 2 waves,

$$\delta = |S_1P - S_2P| = \frac{3\lambda}{2}$$

Finally, if P is a point on $N_3$,

Path difference between the 2 waves,

$$\delta = |S_1P - S_2P| = \frac{5\lambda}{2}$$

**Conditions for Destructive Interference:**

1) **Looking at Phase difference (independent of phase difference at the sources):**

**Phase difference** between the two waves meeting at point P is given by:

$$\Delta\phi = \pi,\ 3\pi,\ 5\pi,\ 7\pi, ... = (m - \frac{1}{2})\ 2\pi$$

2) **Looking at Path difference (dependent on phase difference at the sources):**

For **two waves sources $S_1$ and $S_2$** that are oscillating **in phase**,

**Path difference** between the two waves travelling from the sources to the point P is:

$$\delta \ = \ |S_1P - S_2P| \ = \ \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, ... \ = (m - \frac{1}{2})\lambda$$

where the order, $m$ = 1,2,3,... and $\lambda$ is the wavelength of the individual waves.

**Note :**

1. The points of **destructive interference** are also known as points of **minima**.
2. Notice that because there is no destructive interference in the central line, there is **no zeroth order minimum observed**.

### 10.4.2 Accounting for Phase Difference at the Sources

**Question:**

The conditions for constructive and destructive interference stated above are for the case that ***the sources started out in phase.*** If the **original sources** ***are 180º out of phase*** (i.e. in anti-phase or equivalent to an additional path difference of $\frac{\lambda}{2}$), will the **conditions for path difference** change and why?

**Answer:**

(The phase difference between the 2 wave trains arriving at P)

= (Phase difference at the source) + (Phase difference due to the equivalent path difference)

All the points where the waves meet in phase previously, the waves will now meet in antiphase.

All the points where the waves meet in antiphase previously, the waves will now meet in phase.

Hence, the conditions for constructive and destructive interference **looking at path difference** are interchanged

| For Constructive Interference : | For Destructive Interference : |
|---|---|
| For **two waves sources $S_1$ and $S_2$** that are oscillating **in anti-phase**,<br><br>**Path difference** between the 2 waves trains arriving at P,<br><br>$\delta = \lvert S_1P - S_2P \rvert = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots$<br><br>$= (m-1/2)\lambda$ where m = 1, 2, 3, 4,....<br><br>$m$: $m$th order maxima<br><br>**Note: Phase difference** between the two waves **meeting at point P** is still<br><br>$\Delta\phi = 2\pi, 4\pi, 6\pi, \ldots = 2m\pi$ | For **two waves sources $S_1$ and $S_2$** that are oscillating **in anti-phase**,<br><br>**Path difference** between the 2 wave trains arriving at P,<br><br>$\delta = \lvert S_1P - S_2P \rvert$ = 0, λ, 2λ, 3λ, …<br><br>$= m\lambda$ where m = 0, 1, 2, 3, 4,....<br><br>$m$: $m$th order minima<br><br>**Note: Phase difference** between the two waves **meeting at point P** is still given by:<br><br>$\Delta\phi = \pi, 3\pi, 5\pi, 7\pi, \ldots = (m - \frac{1}{2})\, 2\pi$ |

Note : In this case, the zeroth order is a minimum and there is no zeroth order maximum.

**Example 10.4.1: Two Source Interference**

The figure (Fig. 10.4.5) shows two sources X and Y which emit identical sound waves of wavelength 2.0 m. The two sources emit in phase, and the waves emitted have equal amplitudes, each *A*.

What is the amplitude of the sound wave

(a) at R,

(b) at Q?

Suppose the source X is 180° out of phase with source Y. What does an observer hear

(c) at R,

(d) at Q?

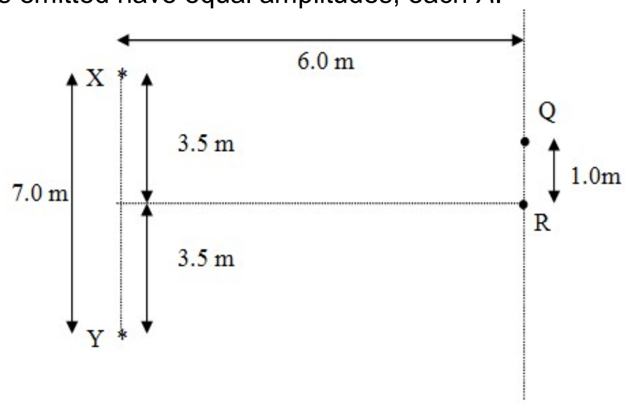



**Fig. 10.4.5**

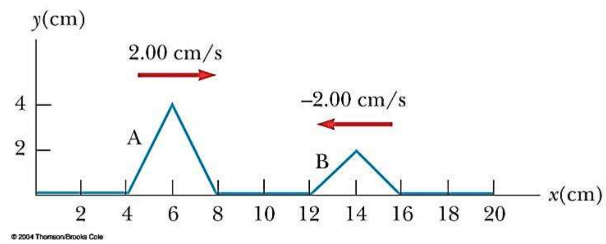
**Solutions:**

➢ **You may now complete Tutorial 10A on Superposition and Interference.**

## Tutorial 10A: Superposition & Interference

### Self-Practice Questions:

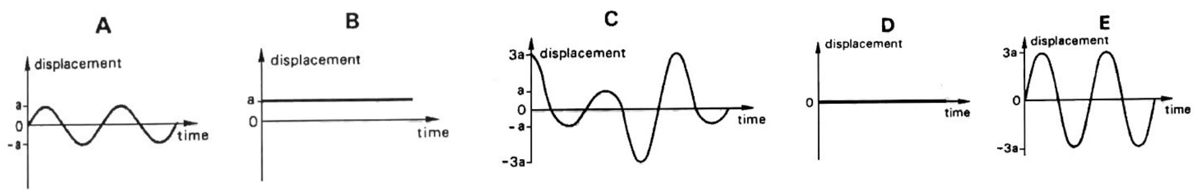
**S1 (Serway 6th Edition. P18.2)** Two pulses A and B are moving in opposite directions along a taut string with a speed of 2.00 cm $s^{-1}$. The amplitude of A is twice the amplitude of B. The pulses at $t$ = 0 are shown in the figure on the right. Sketch the shape of the string at $t$ = 1.0, 1.5, 2.0, 2.5 and 3.0 s.



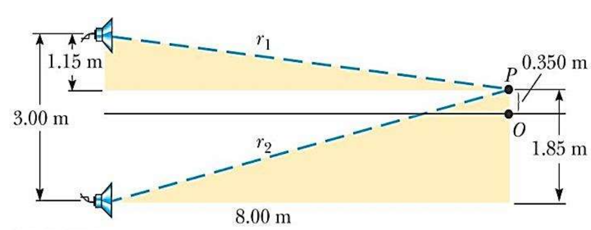

**S2 N77/II/9; J89/I/10; J83/II/10.**

Wave generators $S_1$ and $S_2$ generate waves of equal wavelength. At a point P, $S_1$ by itself produces an oscillation of amplitude 2$a$, and $S_2$ produces an oscillation of amplitude $a$, and there is a phase difference of $\pi$ between the oscillations.

Which graph best represents the resultant oscillation at P when both the generators are switched on?




**S3 Serway Example 18.1.** A pair of speakers placed 3.00 m apart are driven by the same oscillator. A listener is originally at point O, which is located 8.00 m from the centre of the line connecting the two speakers. The listener than walks to point P, which is a perpendicular distance 0.350 m from O, before reaching the *first minimum* in sound intensity. (Assume speed of sound in air is 330 m $s^{-1}$.)




(a) What is the frequency of the oscillator?

(b) What if the speakers were connected in antiphase? What happens at point P?

## Discussion Questions:

**D1** **Serway 6th Edition.** Two pulses move in opposite directions on a string and are identical in shape except that one has positive displacements of the elements of the string and the other has negative displacements. At the moment that the two pulses completely overlap on the string,

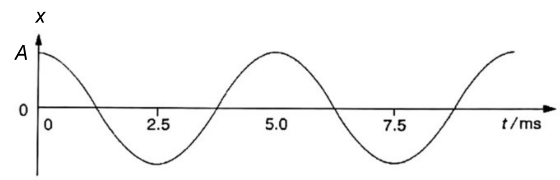
**A** the energy associated with the pulses has disappeared

**B** the string is not moving

**C** the string forms a straight line

**D** the pulses have vanished and will not appear

**D2** **J95/II/2.** A particle in a medium is oscillating because of a transverse wave $T_1$ of intensity $I$. The figure below shows the variation with time $t$ of the displacement of $x$ of the particle. The amplitude of oscillation is $A$.




A second, similar transverse wave $T_2$ has the same frequency, but the amplitude of the oscillation due to $T_2$ alone is $3A/2$.

**(a)** Calculate

**(i)** the frequency of the waves,

**(ii)** the intensity, in terms of $I$, of the wave $T_2$.

**(b)** State two conditions which are necessary for the waves $T_1$ and $T_2$ to interfere.

**(c)** **(i)** What additional condition must be satisfied when the resultant intensity is to be

**1.** a minimum, and

**2.** a maximum.

**(ii)** Calculate, in terms of $I$, this minimum and maximum intensities.

**D3** **J2000/II/4** Two microwave sources A and B are in phase with one another. They emit waves of equal amplitude and of wavelength 30.0 mm. They are placed 140 mm apart and at a distance 810 mm from a line OP along which the detector is moved.

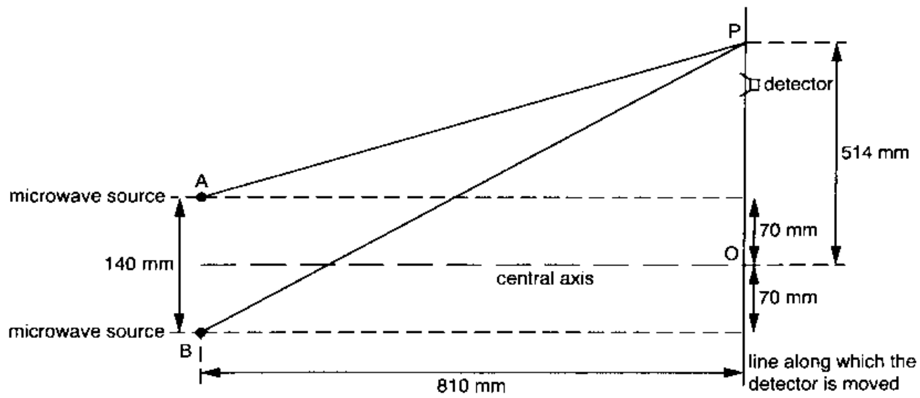



**Fig. 4.5 (not to scale)**

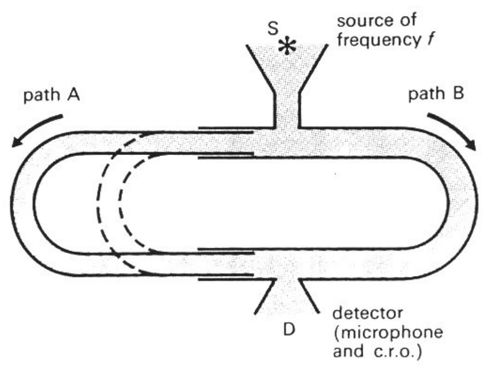
**(i)** Using Pythagoras' Theorem, it can be shown that the distance AP is 923.7 mm. Calculate the number of wavelengths between source A and point P.

**(ii)** Show that there are 33.3 wavelengths between source B and point P.

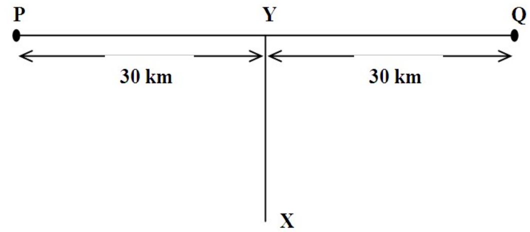
**(iii)** 1. State what intensity of microwaves will be received by the detector at P.

2. Describe how the intensity of reception varies as the detector is moved from P to point O on the central axis.

**D4** **The Quincke tube.** Refer to the diagram below. In this method for measuring the speed of sound in air, a source of frequency 1.7 k Hz was used. The sliding tube had to be moved 0.10 m between consecutive positions of minimum intensity. Calculate the speed of sound.



**D5** A ship at X is equidistant from two shore-based radio transmitters P and Q as shown in the figure below. Both transmitters operate on a wavelength of 300 m and radiate signals of equal amplitude.




**(a)** In the figure, the ship at X detects zero signal amplitude. What information does this give about the signals from P and Q?

**(b)** The ship moves in a straight line from X to Y. Throughout the journey the amplitude of the signal detected by the ship is zero. Explain this.

**(c)** The ship moves in the direction YQ until the signal detected has amplitude twice that from either transmitter alone. How far has the ship moved?

**(d)** When the ship sails from Y to the harbour at Q, the detected signal rises and falls in amplitude, calculate how many dips in intensity will be passed.

**<u>Numerical Answers to Select Questions in Tutorial 10A: Superposition & Interference</u>**

**S3** 1280 Hz

**D2** (a)(i) 200 Hz (ii) $\frac{9}{4}I$ (c) (ii) $\frac{1}{4}I$ , $\frac{25}{4}I$

**D3** (i) 30.8 (ii) 33.29

**D4** 340 m $s^{-1}$

**D5** (c) 75 m (d) 200

## 10.5 Interference of Light Waves - Young's Double Slit Experiment

The interference pattern produced by two *water waves* of the same wavelength can be readily seen in a ripple tank[1]. However, interference pattern is generally not easily observable when two light waves overlap.

**So can light waves interfere with each other?**

When two independent light sources are placed side by side, no interference pattern is observed because the light waves emitted by each of the sources are not **coherent** (i.e. they do not maintain a constant phase relationship with each other over the time of observation).

Light from an ordinary source undergoes random phase change about once every $10^{-8}$ s. Therefore, conditions for constructive interference, destructive interference, or some intermediate state last for times of the order of $10^{-8}$ s.

Since the eye cannot follow such short-term changes, no interference pattern is observed.

**Conditions for two waves to produce an Interference Pattern**

1. The sources must be **coherent**; i.e. they must maintain a *constant phase difference* with respect to each other.
2. The two wave sources must also emit waves of roughly the same amplitude.
3. For transverse waves, they must be unpolarised, or share a common direction of polarisation.

**Note:**

**1 Coherence:**

Two waves are said to be **coherent** if they have a **constant phase difference** between them.

- When two sources are coherent, it does not necessarily imply that they are in phase with each other.
- The constant phase difference may be non-zero.
- Therefore, if 2 sources are coherent, this necessarily implies that the frequencies and wavelengths (and hence speeds) of the waves emitted must be the same.

[1] Read more about the Ripple Tank in Appendix C (Pg. 48).

### 10.5.1 Description on Young's Double Slit Experiment

It was Thomas Young who first demonstrated that light can also interfere with each other to produce an interference pattern as predicted by wave theory and hence demonstrating that light is a wave. A schematic diagram of the apparatus that Young used is shown in Fig. 10.5.1.

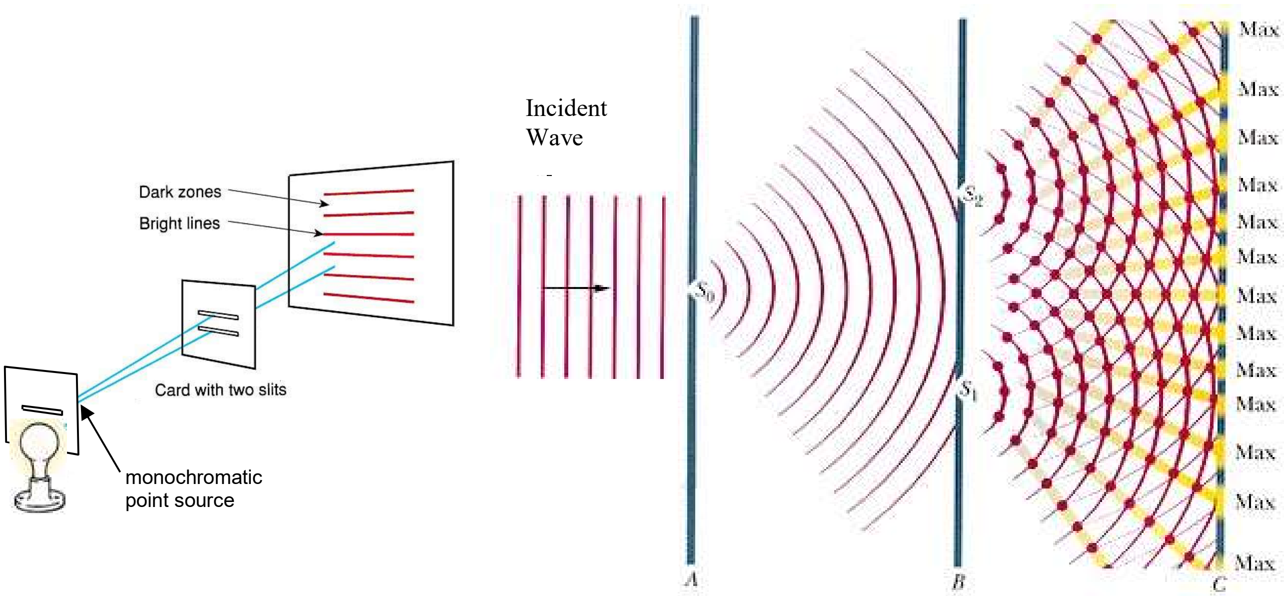



**Fig. 10.5.1** Setup for Young's Double Slit Experiment

- To produce light sources which are coherent in Young's interference experiment, a point source of monochromatic light is incident on 2 slits. As light reaches the two slits, it is diffracted[2] by slits $S_1$ and $S_2$, which then act as 2 point sources. Since there is only one primary point source, the light waves emerging from $S_1$ and $S_2$ are always coherent. If $S_1$ and $S_2$ are equidistant from $S_0$, then the 2 sources are in phase.

- The light waves travelling from $S_1$ and $S_2$ overlap and undergo interference, forming an interference pattern of **maxima** and **minima**.

- On the screen, points of visible bright rows called *bright bands, bright fringes,* or *maxima* are formed. The dark regions are called *dark bands, dark fringes,* or *minima.*

- The pattern of bright and dark fringes on the screen is called an **interference pattern**. (Fig.10.5.2)

- A point source is obtained usually by placing another slit $S_o$ between the double-slit and an extended light source.

[2] Diffraction will be discussed in more detail in Section 10.7

## Conditions for Maxima and Minima

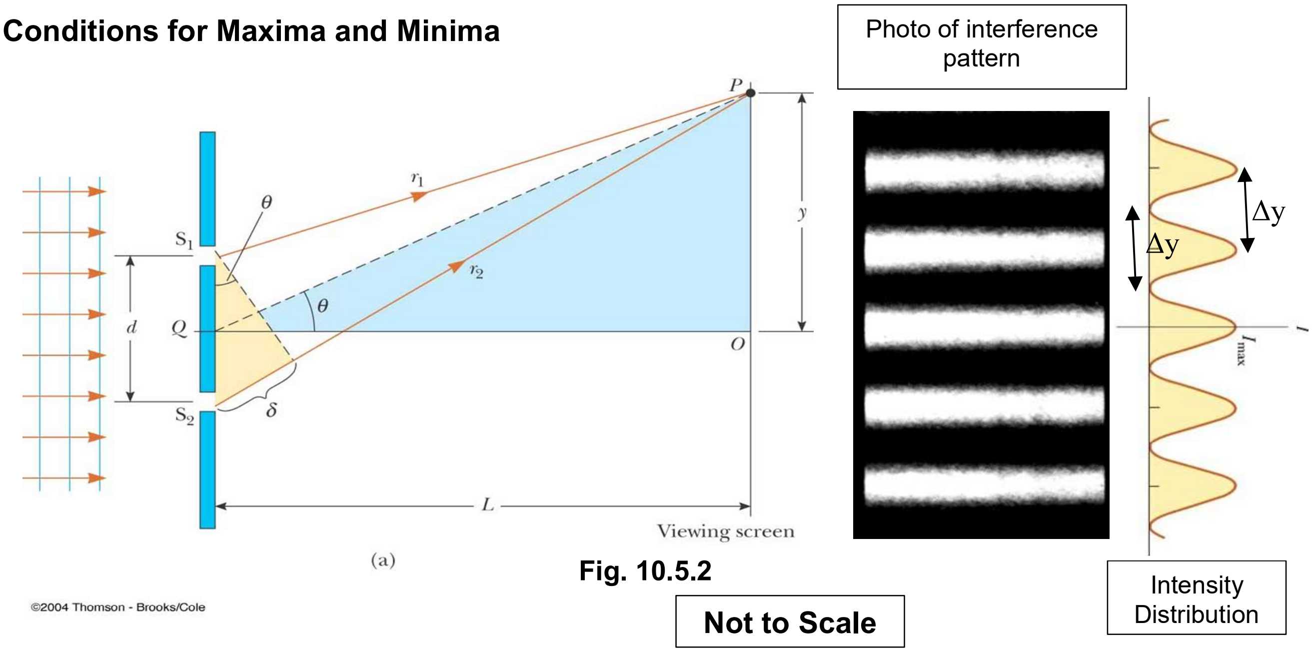



**Fig. 10.5.2**

Let us now derive the conditions for the positions of the maxima and minima of the Young's Double Slit experiment.

In Fig. 10.5.2, a screen is placed at a perpendicular distance *L* from the double-slits. The double slits are separated by a ***slit separation* of *d***. Because the wave through $S_1$ originate from the same wavefront as the wave through $S_2$, the two waves at $S_1$ and $S_2$ are always **in phase**.

Consider an arbitrary point **P** on the viewing screen,

- For **a maximum to be observed at P**, the wave from $S_1$ needs to **constructively interfere** with the wave from $S_2$. Hence, the path difference $\delta$ between the two waves when they meet at P needs to be:

$$\delta = |r_1 - r_2| = 0, \lambda, 2\lambda, 3\lambda, ... = m\lambda \qquad \text{where } m \text{ : order } (m = 0, 1, 2, 3, ...)$$

- Similarly, for **a minimum to be observed at P**, the wave from $S_1$ needs to **destructively interfere** with the wave from $S_2$. Hence, the path difference $\delta$ between the two waves when they meet at P needs to be:

$$\delta = |r_1 - r_2| = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2} = (m - \frac{1}{2})\lambda \qquad \text{where } m \text{ : order } (m = 1, 2, 3, ...)$$

However, for the experiment, the **separation between the slits *d* is often negligible compared to the distance *L* between the screen and the slits**, i.e. *L* >> *d*. Therefore the rays from $S_1$ and $S_2$ to *P* are almost parallel to each other. (Fig. 10.5.3)

***Point to Note:*** *What are the typical orders of magnitude for slit separation, d, slit widths $S_1$ and $S_2$ and distance L for Young's double slits experiment with light?*

Approximating the rays to be parallel to each other, the ***path difference*** $\delta$ may thus be *approximated* by:

$$\delta = d \sin\theta$$

where $d$ is ***the separation between the slits***, and $\theta$ is the ***angular displacement of the position of interference fringe observed from the principal axis QO***.[3]

The conditions that define whether *bright* or *dark* fringes are observed at point P is thus given by

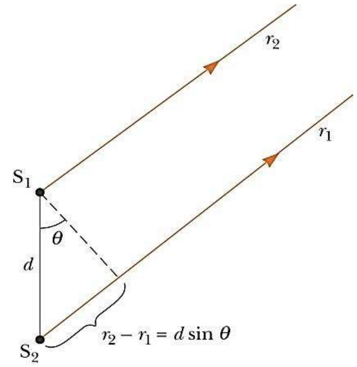



**Fig. 10.5.3**

## 1 Conditions for Constructive or Destructive Interference

**Condition for Constructive Interference (Maxima or Bright Fringes):**

$$d\sin\theta = m\lambda \qquad \text{where } m \text{ : order number } (m = 0, 1, 2, 3, \ldots)$$

**Condition for Destructive Interference (Minima or Dark Fringes):**

$$d\sin\theta = (m - \frac{1}{2})\lambda \qquad \text{where } m \text{ : order number } (m = 1, 2, 3, \ldots)$$

**The assumption made during the derivation of these two equations is *L* >> *d***

**Note:**

- Similar to the two source interference introduced earlier, $m$ is called the **order number.**
- $m$ = 0 corresponds to $\theta$ = 0. In this case, a central bright fringe is observed and hence it is known as the *zeroth order maximum.* The first maximum on either side is called the *first-order maximum,* and so forth.
- Sometimes, a sign is assigned to the order to help distinguish the positions of the maxima or minima. For example, in this case, maxima located at one side (suppose above) of the zeroth order maximum may have values of $m$ = 1, 2, 3, 4.... while maxima located at the other side (below) the zeroth order maximum may have values of $m$ = -1, -2, -3, -4, ...

## 2 Positions *y* of Bright and Dark Fringes

It is often useful to obtain expressions for the positions $y$ along the screen of the fringes measured vertically from O to P.

In a typical Young's Double Slit experiment, constructive interference occurs when

$$d \sin\theta = m\lambda \Rightarrow \sin\theta = m\lambda/d$$

**If $\lambda \ll d$, the value of $\theta$ would be very small for low order fringes (i.e. small $m$)**, hence we can use small angle approximation ($\sin\theta \approx \tan\theta$) to arrive at $\tan\theta = m\lambda/d$

From triangle OPQ (Fig. 10.5.2), we have $\tan\theta = \dfrac{y}{L} \Rightarrow \dfrac{m\lambda}{d} = \dfrac{y}{L}$

[3] The *principal axis* is a line joining the midpoint of the two slits perpendicularly to the viewing screen.

Hence,

**Positions of Bright Fringes:** $y_{bright} = m\frac{\lambda L}{d}$ ($m$ = 0, ±1, ±2, ±3, ...)

**Positions of Dark Fringes:** $y_{dark} = (m - \frac{1}{2})\frac{\lambda L}{d}$ ($m$ = ±1, ±2, ±3, ...)

**The assumptions made during derivation of the above 2 equations are**:

- **$\theta$ is small. (Applies for low values of *m* where $m\lambda << d$)**
- $L >> d$

**3 <u>Fringe width or Fringe Separation, $\Delta y$:</u>**

The ***fringe width* or *fringe separation*** is defined to be the *distance between 2 consecutive bright fringes* or the *distance between 2 consecutive dark fringes*.

Hence, **Fringe separation**

$\Delta y$ = Position of $(m+1)^{th}$ order bright fringe – Position of $m^{th}$ order bright fringe

= $y_{m+1} - y_m$

= $(m+1)\frac{\lambda L}{d} - m\frac{\lambda L}{d}$

i.e. $$\boxed{\Delta y = \frac{\lambda L}{d}}$$

where λ is wavelength of light;

*L* is the distance of screen from slits;

*d* is the slit separation.

Notice that on the right hand side of the equation, $\lambda$, *L* and *d* are constants, hence we can make the following conclusion:

*For **a two-source interference, where d << L and $\lambda$ << d,** the fringe separations <u>close to</u> the central maximum* **($\theta$ is small)** *is approximately* ***constant.***

This conclusion is further confirmed by the photo in Fig. 10.5.2, which captures the fringe pattern formed on the viewing screen during an experiment. The corresponding plot of the **intensity distribution** is also given.

**Example 10.5.1 : Determination of wavelength of light using Young's Double Slit**

A screen is separated from the double-slit source by 1.2 m. The distance between the two slits is 0.03 cm. The second-order bright fringe ($m$ = 2) is measured to be 4.5 mm from the centre line

(i) What is the fringe separation between 2 neighbouring bright fringes formed on the screen?

(ii) Determine the wavelength of light. Ans: (i) 2.25 mm (ii) 5.63 x $10^{-7}$ m

**Solution:**

## Effects of Diffraction on Double Slit

The slits used in the double slit experiment in practice cannot be considered as point sources. As a result, each slit produces a diffraction pattern as shown in Fig. 10.7.5. Fig. 10.5.4 illustrates the interference pattern of two slits superposed on the intensity pattern of a single slit. The interference fringe pattern due to the double slit gets moderated by the diffraction pattern.

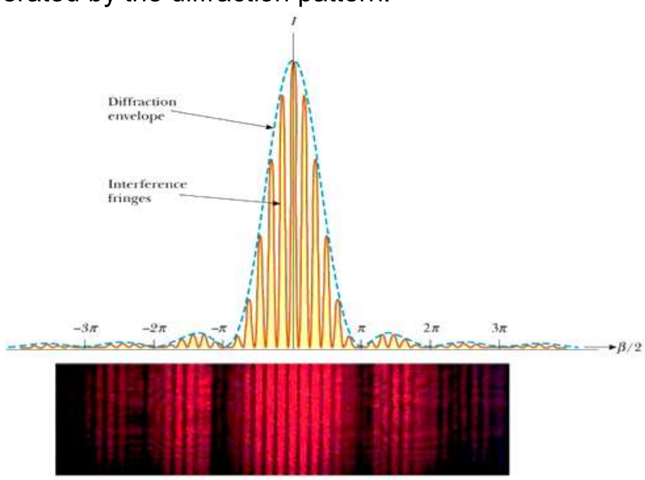



**Fig. 10.5.4**

This can sometimes result in missing orders in the interference fringe pattern. When the minima of the diffraction envelope coincide with the maxima of the interference pattern, the "maxima" of the interference pattern go missing because they are too dim to be seen.

## 10.6 Diffraction Grating

We have seen that the Young's double-slit experiment is able to allow us to determine the wavelength of light. However, a more commonly used tool is the ***diffraction grating.*** A diffraction grating has many fine, parallel, closely spaced and equidistant slits/rulings inscribed on a sheet of glass or metal. A typical diffraction grating has several hundreds to thousands of slits per mm.

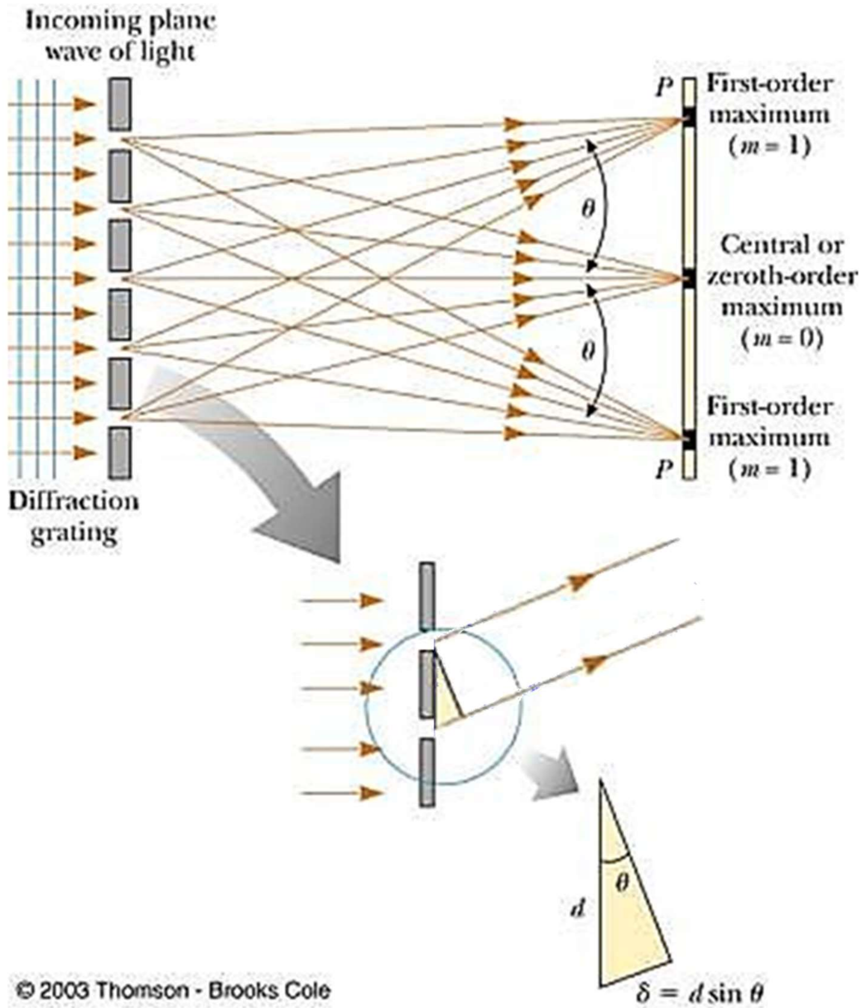



**Fig. 10.6.1.**

To see how a diffraction grating pattern differs from a Young's double-slit pattern and why it is so useful, we will analyse the diffraction pattern for various number of slits of the same slits-separation *d*.

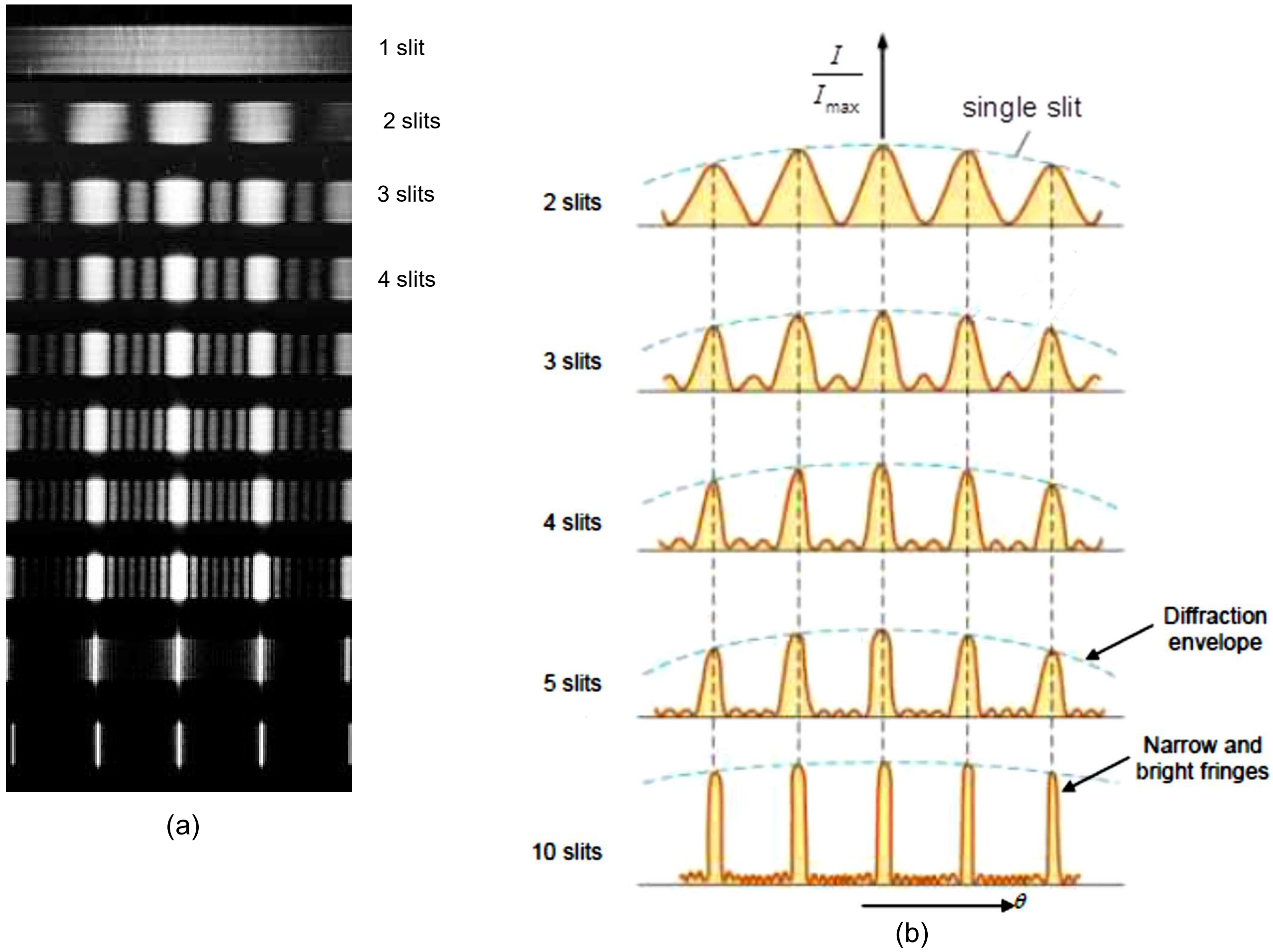



**Fig 10.6.2.** This shows the diffraction pattern of one slit, then two, then three, four, five, six, and more. When there are many slits, we call it a diffraction grating. The more slits, the narrower the fringes become. The pictures (a) above were taken with monochromatic light. The intensity distribution is shown in (b).

From Fig. 10.6.2(a) above, we make the following observations:

- For the same slit separation *d*, the **positions of the maxima remain the same**.
- As the number of slits increase, the maxima become narrower (i.e. the fringes become sharper on the screen).
- As the number of slits increase, the intensity of the maxima increases.

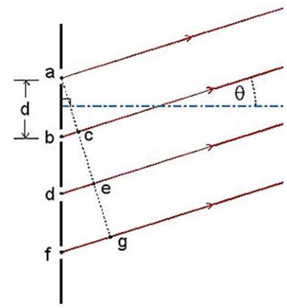
Therefore, we can infer that for a grating (no. of slits $\to \infty$) the **fringe pattern will have very good contrast** (narrower and brighter maxima separated by broad regions of dark minima).

## Condition for Maxima for a Diffraction Grating

- From Fig. 10.6.2(b) above, we notice that for same value of *d*, **the positions of the maxima (bright fringes) of a grating coincide with the positions of the maxima for a double slit:**
- We can therefore infer the equation that determines the positions of the maxima for a *diffraction grating pattern*.

**<u>Condition for  Constructive Interference (Maxima) for a Diffraction Grating</u>**

For the *nth order* ***maxima,***




$$d\sin\theta = n\lambda$$

where $d$ : separation between the slits
$\theta$ : angular position of the bright fringe from the principle axis
$n$ : order number ($n=0, 1, 2, 3,\ldots$)
$\lambda$ : wavelength of light passing through the grating

Commercially available gratings are labelled by the *no. of lines per unit length, N*. The relationship between *N* and *d* are given by

$$\boxed{N = \frac{1}{d}}$$

**<u>Note :</u>**

**1** In this case, we *do not define* the conditions for *destructive interference* as the width of the region between the bright fringes is very large.

**2** For most of the gratings in the laboratory, the slit separations are much smaller than that of the double slits. Hence, *you will notice that the positions of the first and higher orders of the bright fringes lie much further away from the central bright fringe and hence small angle approximation (that θ is small) does not hold*. Therefore, in most questions for diffraction grating it is not meaningful to talk about the *fringe separation* as the angular separation between consecutive fringes are not constant, in fact *they increase as the order increases*.

Although the formula for constructive interference for the double slit and the diffraction grating is the same, the formula for fringe separation for double slits, $\Delta y = \frac{\lambda L}{d}$, CANNOT be used in a grating, unless the fringes are shown to be regularly spaced.

**3** **Grating vs. Young's Double Slit**

The grating is often the preferred choice over the double-slit for the following reasons. For a diffraction grating,

- Fringes are sharper and positions are more precisely determined. There is therefore less uncertainty in measuring the position of the fringes, and less uncertainty in the computation of wavelength.

- Typically the angular displacements are large, so percentage uncertainty in measuring the angular displacement becomes relatively lower. This translates to a relatively lower percentage uncertainty in measuring wavelength as well.

## 4 Determination of Wavelength Using Grating[4]

- **Advantages and disadvantages of using the higher orders to determine the wavelength as compared to first order:**

  Using $d \sin\theta = m\lambda$, if we know the value of the slits separation *d*, we can generally determine the wavelength of the light used by measuring the value of $\theta$ for a particular order of fringe. There are both advantages and disadvantages in using the higher orders to determine this wavelength.

  - **<u>Advantages :</u>**
    - angular displacement $\theta$ is larger, therefore there is less percentage uncertainty in measuring the angle.
  - **<u>Disadvantages :</u>**
    - May not be as bright as the first order bright fringe.

**Example 10.6.1 : Diffraction Grating**

White light from a source passes through a filter which transmits only wavelengths of 400 nm to 600 nm. When the filtered light falls normally on a diffraction grating, light of wavelength 600 nm in one order of the spectrum is diffracted at the same angle, $30^{o}$, as the 400 nm light in the adjacent order.

Find the number of lines per mm for the grating. [Ans: 417 lines per mm]

**Solution:**

[4] The spectrometer is a piece of equipment that is commonly used with the grating to determine the wavelength of light. Do read more about it in Appendix D (Pg. 49).

**Example 10.6.2 : Maximum no. of Fringes Observable.**

A monochromatic source of 495 nm is incident normally on a diffraction grating which has 500 lines per mm. How many diffraction lines can be observed on the screen? . [Ans: 9]

**Strategy:** The maximum angular deviation from the principle axis is $90^0$. Using this condition, we can determine the maximum order and hence find the theoretical maximum number of fringes observable on the screen.

**Solution:**

- **You may now complete Tutorial 10B on Double Slits and Diffraction Grating.**

## Tutorial 10B: Young's Double Slits and Diffraction Grating

## Self-Practice Questions on Young's Double Slits

**S1** In a Young's double slit experiment, sodium light of wavelength $0.59 \times 10^{-6}$ m was used to illuminate a double slit with separation 0.36 mm. If the fringes are observed at a distance of 30 cm from the double slits, calculate the fringe separation.

**S2** When red monochromatic light of wavelength 0.70 $\mu$m is used in the Young's double slit arrangement, fringes with separation 0.60 mm are observed. The slit separation is 0.40 mm. Find the fringe separation if (independently)

**(a)** yellow light of wavelength 0.60 $\mu$m is used;
**(b)** the slit separation becomes 0.30 mm;
**(c)** the slit separation is 0.30 mm and slits to fringe distance is doubled.

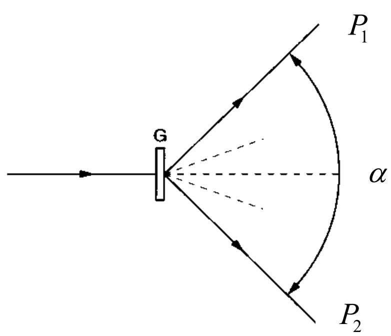
**S3** **J88/I/10; N92/I/11.** Under what conditions will the bright fringes of a double-slit light interference pattern be farthest apart?

| | *distance between slits* | *distance from slits to screen* | wavelength of source |
|---|---|---|---|
| **A** | small | small | short |
| **B** | small | large | short |
| **C** | small | large | long |
| **D** | large | small | short |
| **E** | large | small | long |

## Self-Practice Questions on Diffraction Grating

**S4** What is the wavelength of light which gives a first order maximum at an angle of 22.5° when incident normally on a grating with 600 lines mm$^{-1}$?

**S5** **N99/I/11**
A parallel beam of monochromatic light of wavelength $\lambda$ is incident normally on a diffraction grating G. The angle between the directions of the two second-order diffracted beams at P, and $P_2$ is $\alpha$, as shown.




What is the spacing of the lines on the grating?

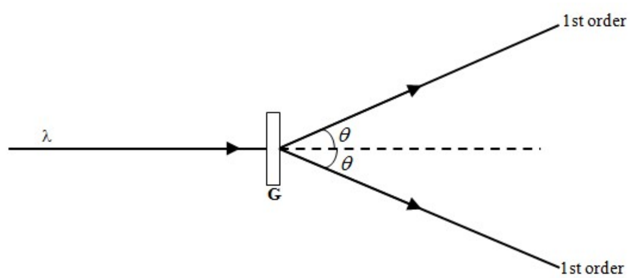
**A** $\dfrac{2\lambda}{\sin\alpha}$ **B** $\dfrac{\lambda}{\sin\alpha}$ **C** $\dfrac{2\lambda}{\sin(\alpha/2)}$ **D** $\dfrac{\lambda}{\sin(\alpha/2)}$

**S6** In a diffraction grating experiment, the first order image of the 435.8 nm blue light from a commercial mercury vapour discharge lamp occurred at an angle of 15.8°. A first order red line was also observed at 23.7°, thought to be produced by an impurity in the mercury.
The wavelengths of red lines of various elements are listed below. Which element is the impurity in the mercury lamp?

| | element | wavelength / nm |
|---|---|---|
| A | zinc | 636.0 |
| B | cadmium | 643.3 |
| C | hydrogen | 656.3 |
| D | neon | 670.8 |
| E | caesium | 697.8 |

**S7** A source emits spectral lines of wavelength 589 nm and 615 nm. This light is incident normally on a diffraction grating having 600 lines per mm. Calculate the angular separation between the first order diffracted waves. Find the maximum order for each of the wavelengths.

**S8** Monochromatic light of wavelength $\lambda$, in the visible light range, is incident normally on a diffraction grating of about 500 lines per mm.




If the angular displacement from the principal axis, of the first order diffraction fringe is found at $\theta$,

**(i)** Where would the angular displacement $\theta_2$ of the second order diffraction fringe be most likely found?

**A** $0 < \theta_2 < \theta$ **B** $\theta_2 < 2\theta$ **C** $\theta_2 > 2\theta$ **D** $\theta_2 = 2\theta$

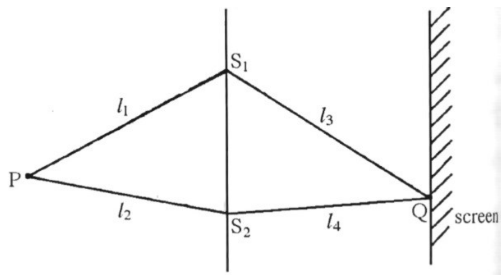
**(ii)** The light is now replaced by another monochromatic source of wavelength $\lambda' > \lambda$, still in the visible light range. Would the 1st order diffraction fringe be found at an angular displacement $\theta'$ such that

**A** $\theta' = \theta$ **B** $\theta' < \theta$ **C** $\theta' > \theta$

## Discussion Questions

### Young's Double Slits:

**D1 N80/II/9** Two identical narrow slits $S_1$ and $S_2$ are illuminated by light of wavelength $\lambda$ from a point source P.




If, as shown in the diagram above, the light is allowed to fall on a screen, and if *m* is a positive integer, the condition for destructive interference at Q is that

**A** $(l_1 - l_2) = (2m+1)\lambda / 2$

**B** $(l_3 - l_4) = (2m+1)\lambda / 2$

**C** $(l_3 - l_4) = m\lambda$

**D** $(l_1 + l_3) - (l_2 + l_4) = (2m+1)\lambda / 2$

**E** $(l_1 + l_3) - (l_2 + l_4) = m\lambda$

**D2** **N06/II/6**. Coherent light is incident normally on a double slit, as shown in Fig. 6.1.

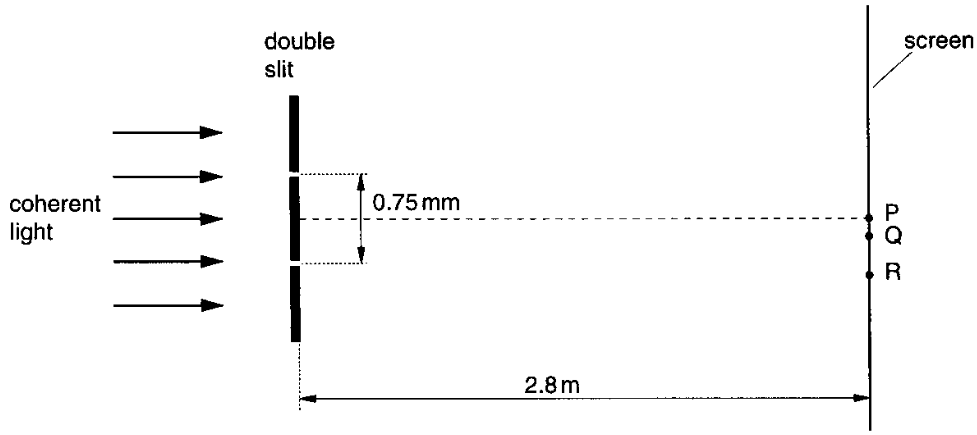



**Fig. 6.1** (not to scale)

The separation of the slits in the double-slit arrangement is 0.75 mm. A screen is placed parallel to, and at a distance of 2.8 m from, the double slit. P is a point on the screen that is equidistant from the two slits. Fig. 6.2 (below) shows the variation with distance from P of the intensity I of the light on the screen.

(a) Calculate the wavelength, in nm, of the coherent light. [4]

(b) Points Q and R are points on the screen. Their positions are indicated on Fig. 6.2.
Determine the phase angle between the waves from the double slit when the waves meet at
(i) point Q, [1]
(ii) point R. [2]

(c) Suggest why the maxima on Fig. 6.2 are not all of the same intensity. [2]

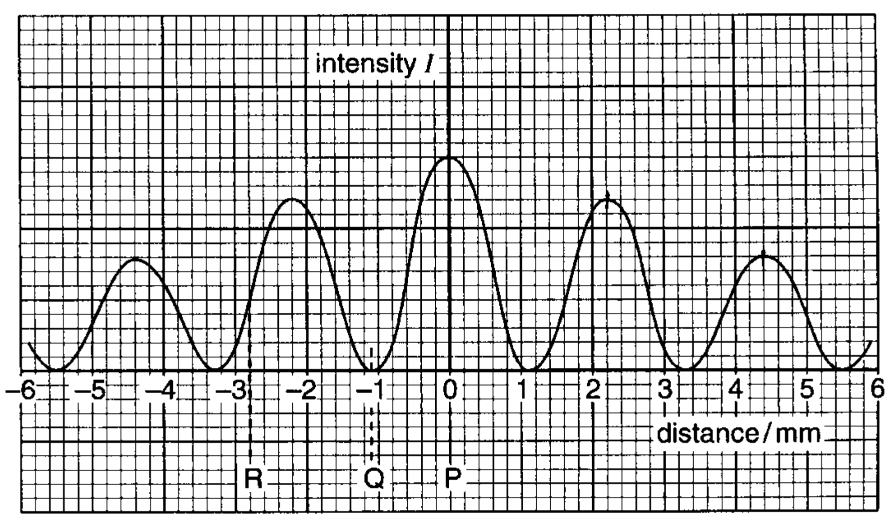



**Fig. 6.2**

**D3 N04/III/3**

**(b) (i)** Show how the principle of superposition of waves can be used to explain the formation of two-source interference fringes. [3]

**(ii)** Two-source interference fringes using light can only be obtained if light from the two sources is coherent. Explain

**1.** the meaning of the term *coherent,* [1]

**2.** why, in practice, interference fringes can be seen only if light from a single source is split into two. [2]

**(iii)** Coherent, monochromatic light from two narrow slits a distance 0.38 mm apart causes an interference pattern on a screen 1.20 m from the slits, as illustrated in Fig. 3.1.

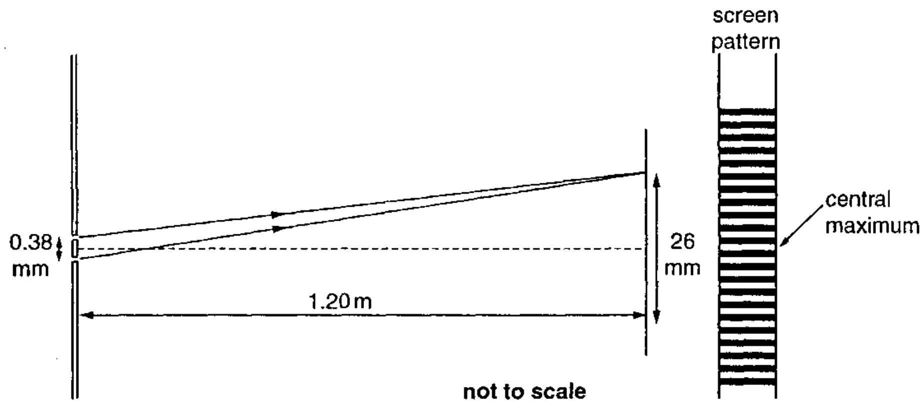



**Fig. 3.1**

The distance from the sixth bright fringe on one side of the pattern to the sixth bright fringe on the other side of the pattern is found to be 26 mm. Calculate the wavelength of the monochromatic light. [3]

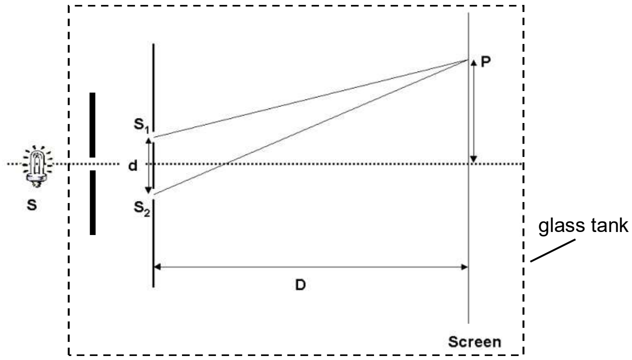
**(iv)** State the experimental advantage gained by determining the fringe width in the way that was used in **(iii)**. [1]

**(v)** Another way of obtaining fringes similar to those described in **(iii)** is illustrated in Fig. 3.2.

**Fig. 3.2**

A single slit is viewed both directly and by reflection from a mirror surface. Explain why this system produces a fringe pattern. [2]

**D4 N99/II/3 (modified)**




The wavelength of a monochromatic light source is to be determined by means of a double-slit interference experiment. Part of the setup is placed in a transparent glass tank as shown in the diagram above.

Initially, the glass tank is empty. State and explain what changes, if any, occur in the pattern of the fringes observed when each of the following changes is made separately:

(a) increasing the intensity of the light source incident on the double slits,

(b) increasing the distance between the double slit and the screen,

(c) reducing the intensity of light incident on one slit of the double slit.

(d) the light source is replaced with one of white light.

**Diffraction Gratings:**

**D5 J85/I/11**

Monochromatic light of wavelength $\lambda$ is incident normally on a diffraction grating consisting of alternate opaque strips of width x and transparent strips of width y. The angle between the emerging zero-order and first-order spectra depends on

**A** $x$, $y$ and $\lambda$ **B** $x$ and $\lambda$ only **C** $y$ and $\lambda$ only

**D** $x$ and $y$ only **E** $\lambda$ only

**D6 Hutchings Pg 231, Question 13.10**

The visible spectrum extends from 400 nm to 750 nm.

**(a)** If observed with a diffraction grating having 480 lines per mm, find the angular width of the first and third order spectra.

**(b)** Give two reasons why such a spectral will normally be viewed in the first order.

**D7 J97/III/3**

A diffraction grating with 250 lines per millimetre is placed in front of a monochromatic source of red light. A screen placed 200 cm beyond the grating has red light images measured at certain positions on a scale on the screen, as shown in Fig. 3.1.

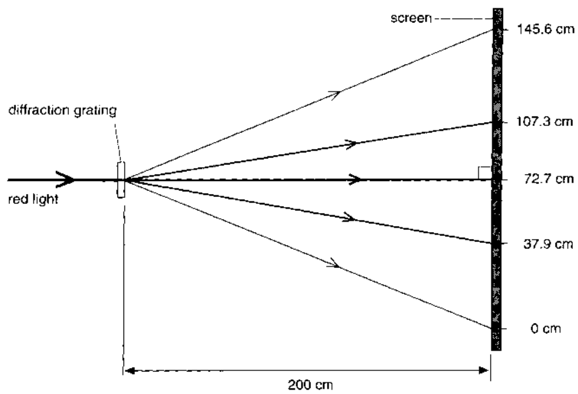



Fig. 3.1

**(a)** **(i)** Use the first order spectrum to deduce a value for the wavelength of the red light.

**(ii)** Make a check, using the second order spectrum, to show that your calculation is correct. [6]

**(b)** How would the pattern obtained be different if blue light were used in place of red light? You are not expected to make any calculations when answering this part of the question. [3]

**(c)** What main problem would arise if the experiment were repeated with infra-red radiation? Suggest how this problem could be overcome. [2]

**D8 N2000/III/3(part), J96/III/3 (part)- (modified)**

**(a)** An experiment is set up to demonstrate the behaviour of water waves in a ripple tank. Draw a diagram, one in each case, to show the diffraction of water waves through

**(i)** a narrow gap.

**(ii)** a wide gap. [3]

**(b)** Light from a low pressure sodium lamp consists mostly of two wavelengths, 588.99 nm and 589.59 nm. This light is allowed to fall normally on a diffraction grating with 500.00 lines per millimetre.

**(i)** Calculate the maximum angular separation between the light of the two wavelengths.

**(ii)** What problem would be likely to arise in observing the spectral lines in the order in (i)? [8]

**D9 TYS Pg 127 Qn 28 N2000/II/4. N94/III/2 (part)**

Light from a distant source of monochromatic light of wavelength 590 nm passes through a fine nylon mesh. The light is then incident on a screen, as illustrated in Fig. 4.1.

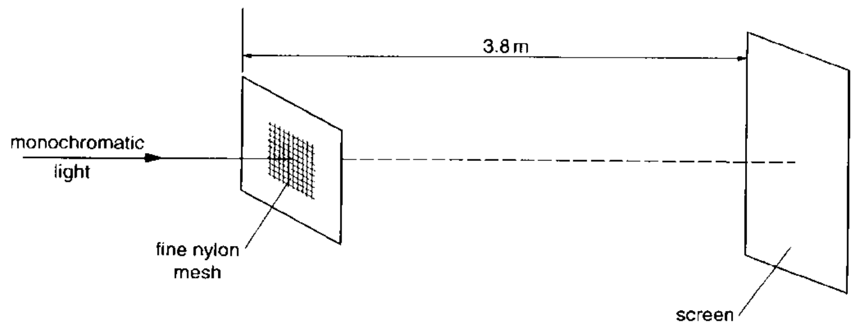



**Fig. 4.1**

The threads of the nylon mesh act as a diffraction grating with lines in the horizontal and in the vertical direction. **Part of the pattern of spots** of light on the screen is shown in Fig. 4.2.

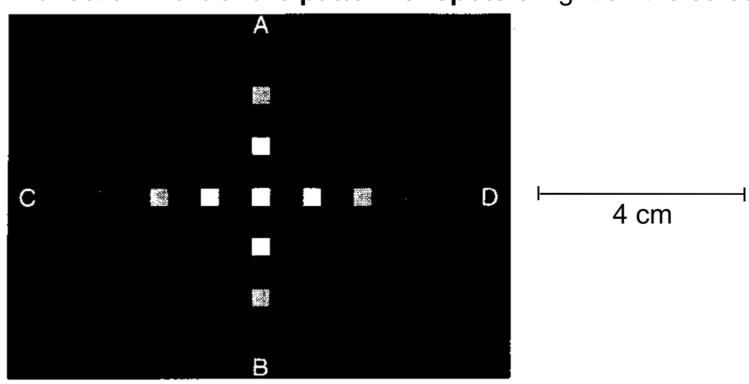



Fig. 4.2

(a) Explain what is meant by the *diffraction* of light. [2]

(b) State which line of spots of light, AB or CD, is produced by the *horizontal* nylon threads.

(c) Calculate the angle, in radians, between the orders of the diffracted light. [2]

(d) Using your answer to (c), determine the number of nylon threads per millimetre of the mesh. [4]

(e) The spots are observed to be almost equally spaced apart both horizontally and vertically. What does this suggest about the structure of the nylon mesh?

(f) Patterns such as this can be used to determine the spacing of atoms in crystals. Suggest one problem that occurs in carrying out such an experiment with the wavelength used. Indicate how a different wavelength of radiation can overcome this problem.

## Numerical Answers for Tutorial 10B: Young's Double Slit and Diffraction Grating

**S1** $4.92 \times 10^{-4}$ m **S2**(a) 0.51 mm (b) 0.80 mm (c) 1.60 mm
**S4** 638 nm **S7** 0.959°, 2, 2
**D2**(a) 589 nm (b)(i) $\pi$ rad (ii) 8.0 rad
**D3** (iii) $6.86 \times 10^{-7}$ m
**D6** **(a)** First at 10.0°, second at 23.5°. Incomplete third starting at 35.2°.
**D7** (a)(i) 684 nm **D8** (b)(i) 0.110° **D9** (c) $2.6 \times 10^{-3}$ rad (d) 4.5

## 10.7 Diffraction of Waves

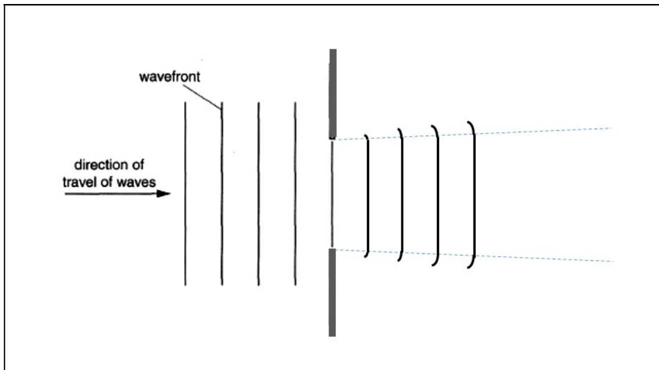
### What is diffraction?

**Diffraction** is the ***spreading*** of a wave (into the "geometrical shadow") after passing through a slit or around an obstacle.

Diffraction effects become more apparent when the width of the opening is small enough compared to the wavelength.

### 10.7.1 Diffraction of water waves

When waves meets a barrier in which there is an opening, if the width of the opening is much larger than the wavelength of the waves, the wave emerging from the opening continues to move in a straight line (apart from some small spreading at the edge). If the width of the opening is comparable or smaller than the wavelength, then the bending is a lot more obvious, and emergent wave becomes more circular.



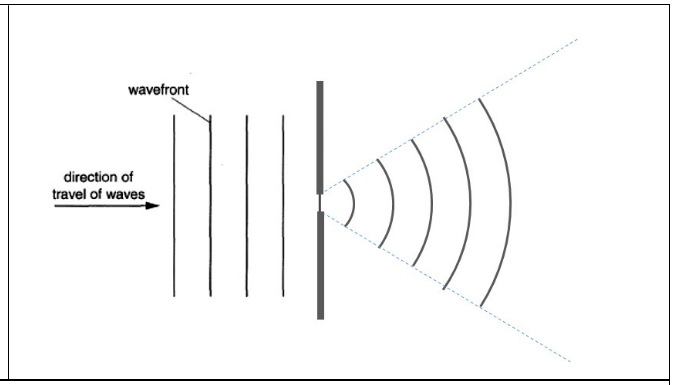




**Fig 10.7.1** Waves of identical wavelengths meet gaps of different widths. When the **width of the gap** is **comparable** to the **wavelength**, diffraction effects are more obvious.

**What is meant by "Geometrical Shadow"**

Consider a series of particles projected (diagram below) separately into an aperture and obstacle.

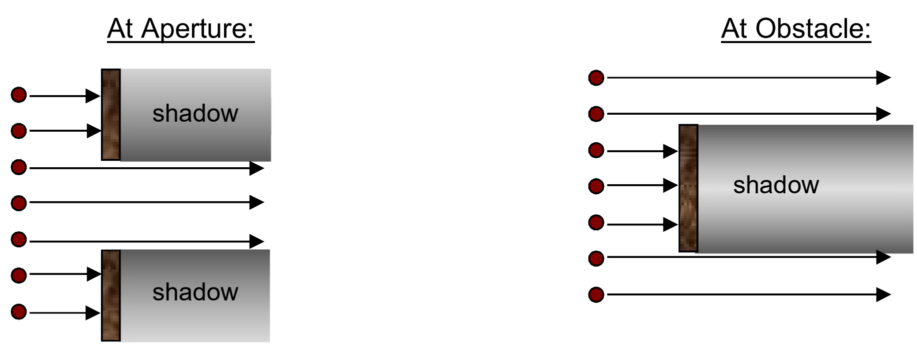



**Fig. 10.7.2**

As the particles are blocked by the barriers, there will be a sharp 'shadow' region where the particles do not reach. This shadow is known as the "geometrical shadow" of the barriers.

Diffraction is also observed for waves meeting an obstacle in its path. Again, **the relative size of the obstacle to the wavelength will determine the effect of diffraction** as shown in Figure 10.7.3. It is found that in order for the waves to spread around the edges of the obstacle to extend into the geometric shadow of the obstacle, the width of the obstacle should be small enough compared to the wavelength.

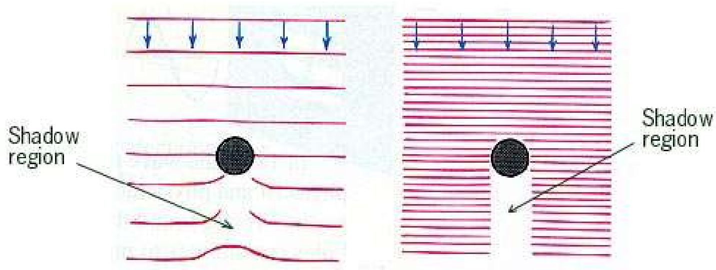



**Fig. 10.7.3** The waves will bend if the length of the obstacle is comparable to the wavelength of the waves.

Since light and sound demonstrate diffraction effects, they are said to behave like waves. This diffraction effect cannot be explained by particle theory (i.e. diffraction effect is unique to waves).

## 10.7.2 Single Slit Diffraction Pattern

### Diffraction Pattern of a Single Slit

We might expect that light passing through a small aperture would result in a broad region of light on a screen, due to the spreading of the light as it passes through the opening.

We actually find something more interesting.

A diffraction pattern consisting of light and dark areas is observed (see Figure 10.7.4), somewhat similar to the interference pattern discussed earlier.

This pattern consists of a broad intense central band (called the central maximum), flanked by a series of alternating bright and dark bands.

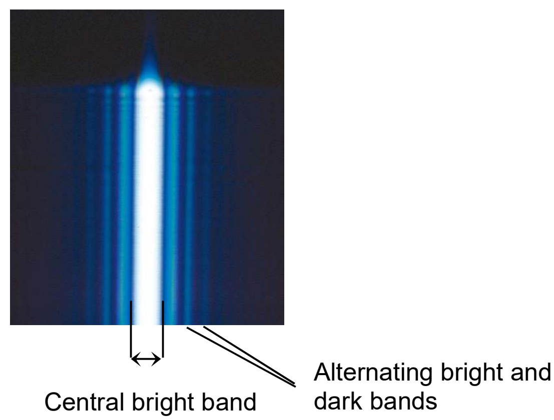



**Fig. 10.7.4**

**Single-slit Diffraction of light**

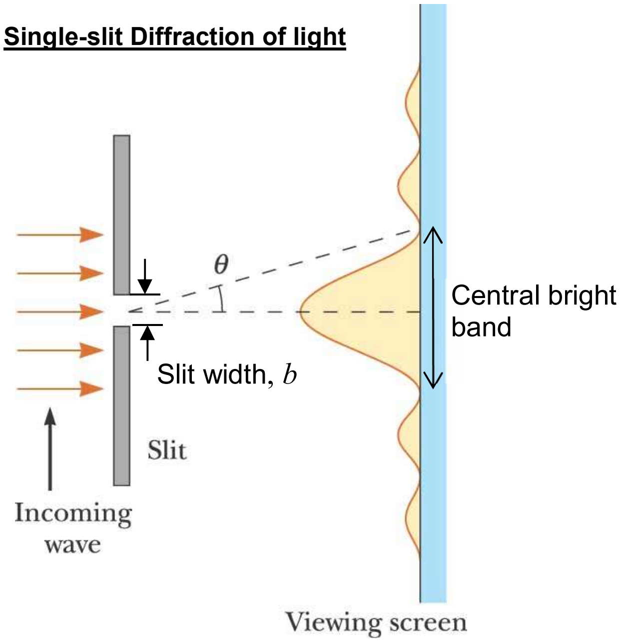



**Fig. 10.7.5**

In a single slit diffraction, **light waves of wavelength $\lambda$** passing through a **single slit of width *b*** undergoes diffraction to produce a broad intense central band with an intensity profile that is maximum at the centre, and tapers off towards both sides as shown in Figure 10.7.5.

You are not required to explain formally these observations for the A-level Physics syllabus.
Nonetheless, for those interested, these observations can be explained by means of Huygen's Principle, which explains how the slit can be modeled as row of secondary point sources, the waves from which interfere with one another to create an interference pattern.
A detailed explanation can be found in Appendix A.

The **first minimum intensity of the single slit diffraction pattern** occurs at **angle** $\theta$ from the central maximum where

$$\boxed{\sin\theta = \frac{\lambda}{b}}$$

This angle $\theta$ is important because it tells us the angular spread ($2\theta$) which contains most of the energy of the wave. It can be seen from this that as ***b* (the slit width) is reduced, $\theta$ gets larger and the pattern is more spreads out**. The **pattern also gets dimmer as less light energy passes through a smaller slit**. A more complete analysis of the diffraction pattern can be found in Appendix B.

## Width of the single slit pattern

For small angles the **angular spread of the diffraction pattern** is inversely proportional to the **slit width *b*** or, more precisely, to the **ratio of *b* to the wavelength $\lambda$.** Figure 10.7.6 shows graphs of intensity ***I*** as a function of the angle for three values of the ratio $b/\lambda$.

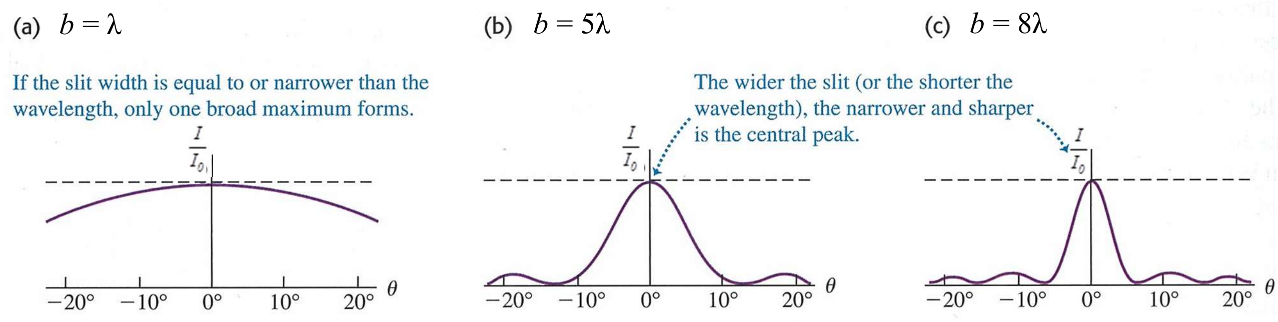



**Fig. 10.5.6** The single slit diffraction pattern depends on the ratio of the slit width $b$ to the wavelength $\lambda$.

**Example 10.7.1: Single slit diffraction**

The figure (Fig. 10.7.7a) shows light from a He-Ne laser of 633 nm is incident on a 2.0 x $10^{-4}$ m wide slit and the resulting diffraction pattern on the screen as shown in Fig 10.7.7b. What is the width of the central maxima (the distance between the dark fringes on either side of the central maxima) on a screen 2.0 m away? . [Ans:12.7 mm]

**Solution:**

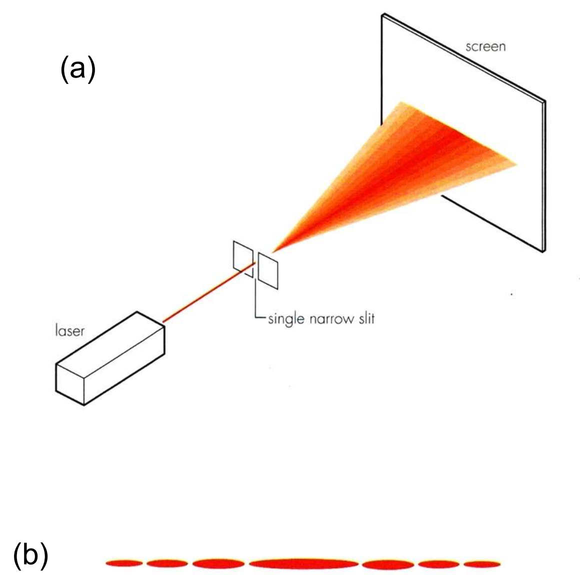


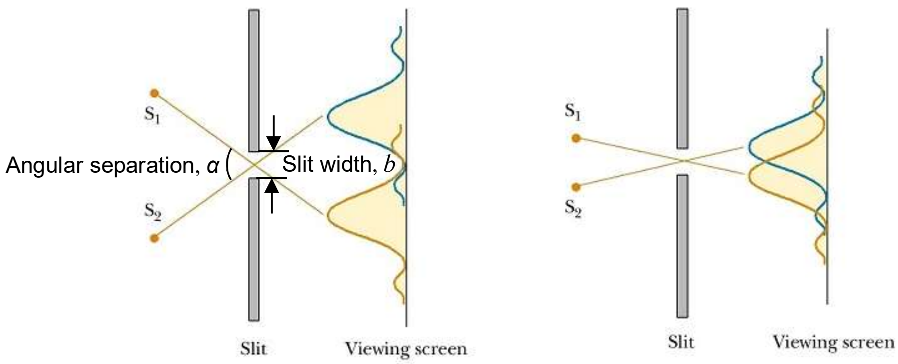

**Fig. 10.7.7**

### 10.7.3 Resolving Power and Rayleigh's Criterion

The resolving power of an optical instrument is the ability of the instrument to differentiate clearly between the images of two close objects, i.e., the two images are seen as two distinct images. The resolving power is measured in terms of angular separation (units in radians) of the two images.

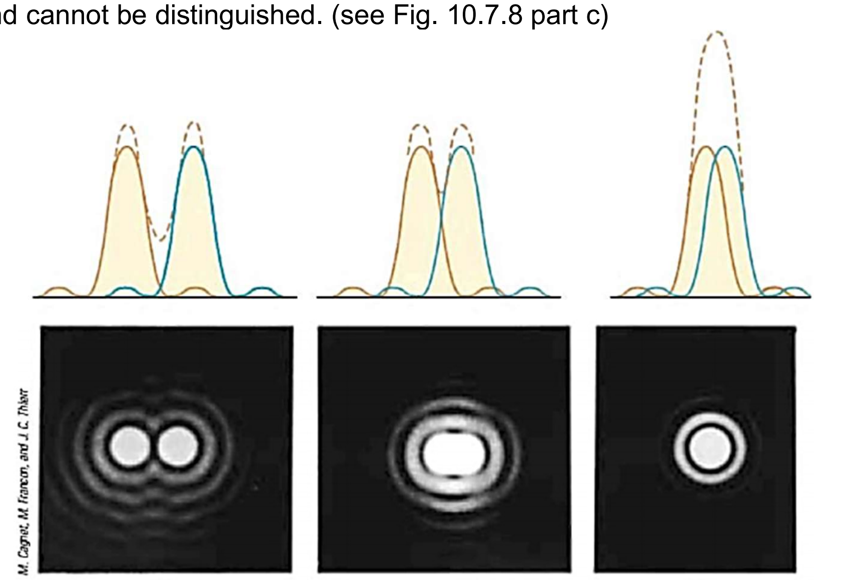
As most images are viewed through an aperture as in the case of microscopes and telescopes, it is identical to viewing the diffraction pattern produced. Hence, the diffraction of light affects greatly or limits the resolving power of an optical instrument.

To understand this limitation, let us consider two light sources far away from a narrow slit of **slit width *b***, with angular separation of $\alpha$.




(a) Little overlap of diffraction patterns (b) Significant overlap of diffraction patterns

Since the two sources are usually incoherent (such as two distant stars), let's assume that no interference occurs between the two light passing through the slit. So each light source simply produces its own diffraction pattern on the screen. What is observed on the screen will be the summation of two diffraction patterns, one from each source. If the sources are close together, their diffraction patterns overlap; if their angular separation is small enough, their diffraction patterns overlap almost completely and cannot be distinguished. (see Fig. 10.7.8 part c)



| (a) sources are far apart and **patterns are well resolved.** **($\alpha > \theta_{min}$)** | (**b) The sources are closer together and the patterns are just resolved.** **($\alpha \approx \theta_{min}$)** | (c) the sources are so close that the **patterns are not resolved**. **($\alpha < \theta_{min}$)** |
|---|---|---|

**Fig 10.7.8**
Individual diffraction patterns of two point sources (solid curves) and the resultant patterns (dashed curves) for various separation of sources as light passes through a circular aperture. In each case, the dashed curve is a summation of the two solid curves.

The generally accepted criterion for whether two images are distinguishable is the **Rayleigh criterion**, which states that **two images are just resolved or distinguishable when the central maximum of one image coincides with the first minimum of the other image** (see Fig. 10.7.8 part b).

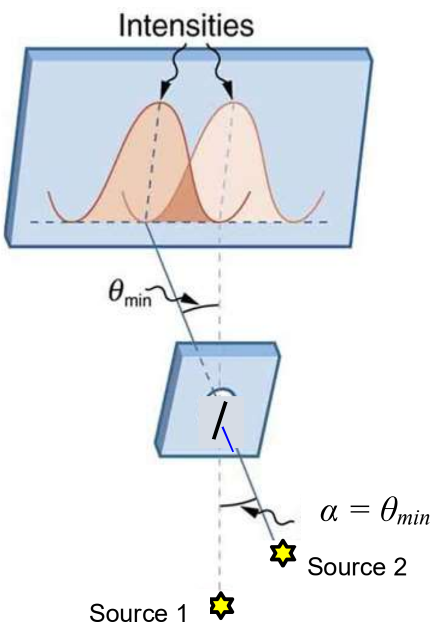



**Fig. 10.7.9**

*(Image from http://cnx.org/contents/9ANhisjh@5/Limits-of-Resolution-The-Rayle)*

Hence according to the Rayleigh criterion, **two images are just resolved or distinguishable** when the **angular separation between the two sources, *α*** is (approximately) equal to the **angular separation $\theta_{min}$** between the central maximum and the first minimum of the images (See Fig. 10.7.9).

From section 10.5.2 (single slit diffraction pattern), we learnt that the **first minimum** for a single slit diffraction occurs at an angle $\sin\theta_{\text{min}} = \dfrac{\lambda}{b}$ where *b* is the slit width.

When $\theta_{min}$ is small, we can use the approximation $\sin\theta_{\text{min}} \approx \theta_{\text{min}}$. The **Rayleigh criterion for the resolving power of a single aperture** is thus given by

$$\boxed{\theta_{\text{min}} \approx \frac{\lambda}{b}}$$

where ***b* is the slit width and $\theta_{min}$ is in radians**.

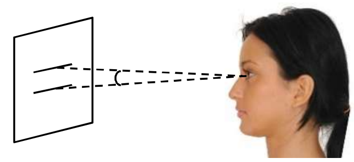
Hence we can also say that **the angle subtended by the two sources, *α* must be greater than $\dfrac{\lambda}{b}$ if the images are to be resolved**.

**Note:**

1. The resolving power of a telescope can be improved by using an objective lens of wide aperture (i.e. large diameter). Besides reducing the diffraction effects, more light is also transmitted through the telescope and a brighter image is observed.
2. In microscopes, the resolving power is improved by using radiation of much smaller wavelengths than visible light, such as ultra-violet. In electron microscopes, the wavelength of the electron beam is $10^5$ times smaller than visible light, resulting in much higher resolution in the images. The wavelength associated with the electron beam will be covered in Chapter 17 Quantum Physics.
3. For circular lens aperture (e.g. telescope), the formula becomes $\sin\theta_{\text{min}} = \dfrac{1.22\lambda}{D}$, where *D* is the diameter of the circular lens. **Not required for A Level, for information only.**

**Example 10.7.2**

The resolving power of the eye may be determined by drawing two parallel lines at a distance of say 2 mm apart on a piece of card. The card is slowly moved away from the eye until the eye just cannot see the two lines as separate lines. The distance of the card from the eye is measured. Suppose for a particular person, the distance of the card from his eye is 5 m when his eye just fail to see the two lines distinctly. [Ans: (a) 0.0004 rad (b) 1.25 mm]



(a) Determine the angle subtended at the eye by the two lines.

(b) By approximating the wavelength of the visible light to be 500 nm, estimate the aperture of the pupil of the eye.

**Solution:**

➢ **You may now complete Tutorial 10C on Single Slit and Rayleigh's Criterion.**

## Tutorial 10C: Single Slit and Rayleigh's Criterion

### Self-Practice Questions

**S1** Calculate the width of the central maximum in the single-slit diffraction pattern of yellow light of = 589.0 nm by a slit 0.250 mm wide viewed on a screen 2.50 m away.

**S2** A single-slit pattern is formed by light passing through a narrow slit 0.050 mm wide. If the width of the central maximum is 3.8 cm on a screen 1.5 m away from the slit, what is the wavelength of the light?

**S3** The images of two sources are just resolved. Which of the following is a correct statement of the Rayleigh criterion for this situation?

**A** The central maximum of the diffraction pattern of one source must coincide with the central maximum of the diffraction pattern of the other source.

**B** Light from the sources must pass through a circular aperture.

**C** Light from the sources must be coherent.

**D** The first minimum of the diffraction pattern of one source must coincide with the central maximum of the diffraction pattern of the other source

**S4** Two binary stars emit radio waves of wavelength 6.0 x $10^{-2}$ m. The waves are received by a radio telescope whose collecting dish has a diameter of 120 m. The two stars are just resolved if their **minimum** angular separation in radians is of the order of

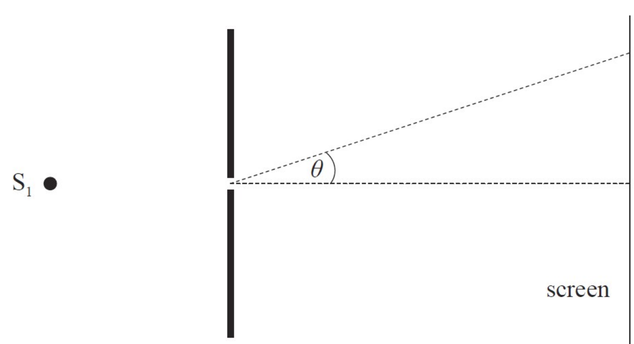
**A** $10^{4}$ **B** $10^{2}$ **C** $10^{-2}$ **D** $10^{-4}$

### Discussion Questions

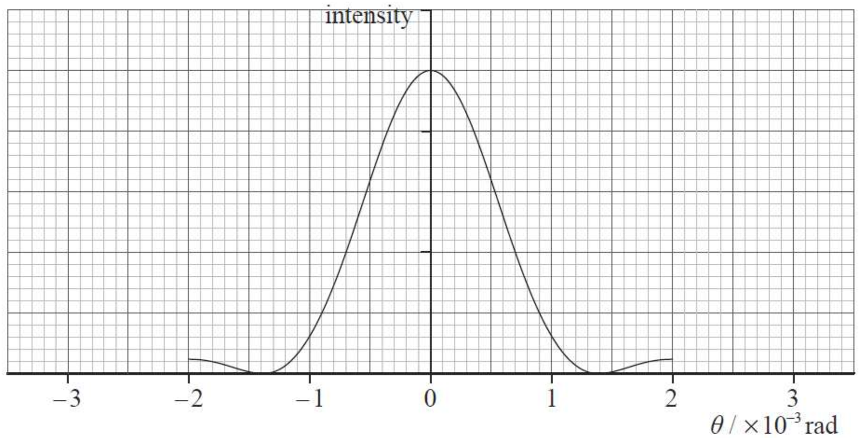
**D1** **IB 2010**

This question is about diffraction and resolution.

(a) Light from a monochromatic point source S1 is incident on a narrow rectangular slit.

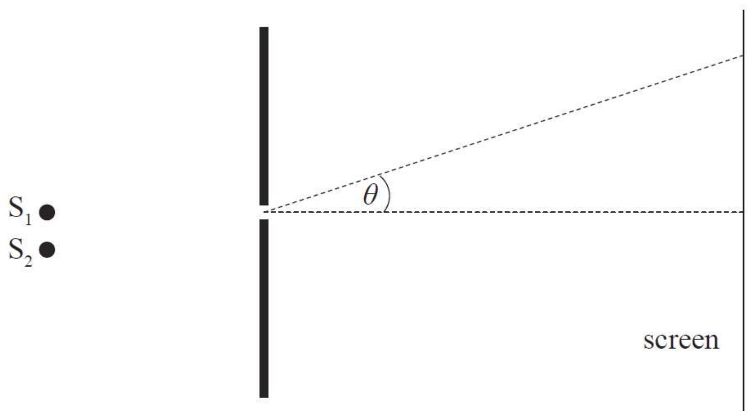



After passing through the slit, the light is incident on a screen some distance away from the slit. The graph shows how the intensity distribution on the screen varies with the angle θ shown in the diagram.




(i) The width of the slit is 4.0 ×10$^{-4}$ m. Use data from the graph to calculate the wavelength of the light. [2]

(ii) An identical source S2 is placed close to S1 as shown.

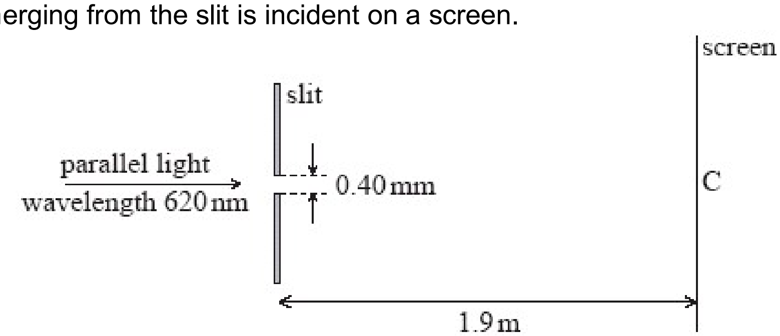



The images of the two sources on the screen are just resolved according to the Rayleigh criterion. On the graph above, draw the intensity distribution of the second source. [1]

**D2 IB**

(a) (i) Explain what is meant by the diffraction of light. [2]

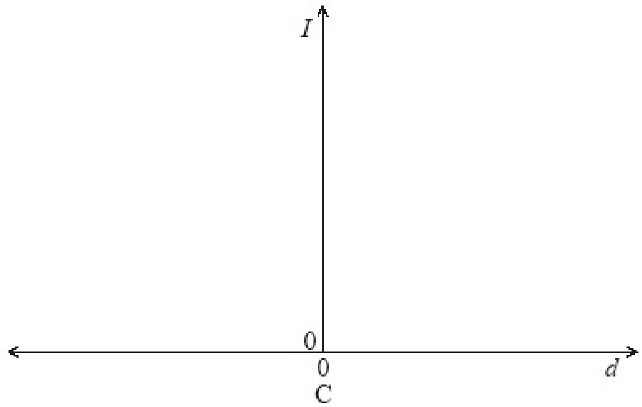
(ii) A parallel beam of monochromatic light from a laser is incident on a narrow slit. The diffracted light emerging from the slit is incident on a screen.




*(not to scale)*

The slit width is 0.40 mm and it is 1.9 m from the screen. The wavelength of the light is 620 nm. Determine the width of the central maximum on the screen. [3]

(iii) The centre of the diffraction pattern produced on the screen is at C. On the axes sketch a graph to show how the intensity $I$ of the light on the screen varies with the distance $d$ from C. [3]

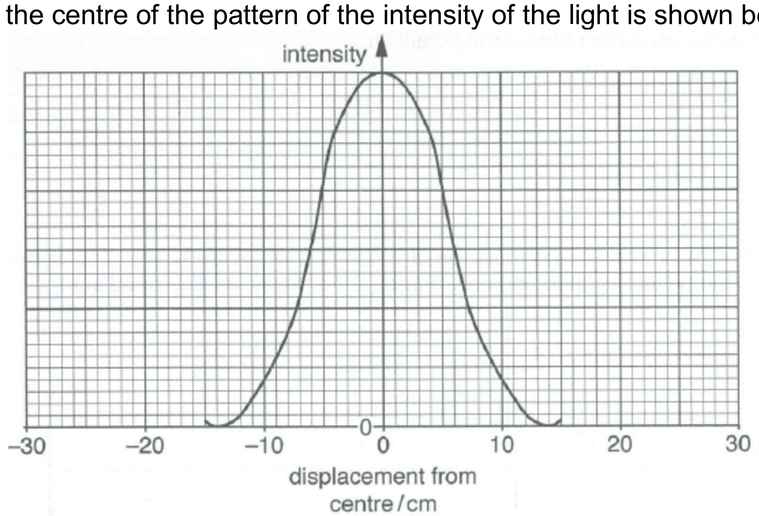



(b) (i) When two separate lasers are used as sources, the images of the slit formed by the light from each laser are resolved. State what is meant by the term resolved in this context. [1]

(ii) On the same axes, sketch a graph to show how the intensity $I$ of the light from the second laser varies with distance along the screen if the two images are just resolved. Label this curve B. [2]

(iii) A car, with its two headlights switched on, is approaching an observer who has good eyesight. Outline why, at a long distance from the observer, the images of the headlights of the car are not resolved by the observer. [4]

**D3** **Modified N2018/P2/Q3.** When laser light is incident normally on a single slit of width 12 μm, the light diffracts and a diffraction pattern is observed on a screen 2.7 m away. The screen is parallel to the plane of the single slit.
A diffraction pattern is formed on the screen. Part of the diffraction pattern showing the variation with displacement from the centre of the pattern of the intensity of the light is shown below.




(i) State and explain the changes to the observed diffraction pattern above when visible light of a longer wavelength is used.
(ii) When white light is incident on a single slit, the central fringe is coloured at the edges and has a white central region. Explain this observation.

**D4 HCI 2019 Prelim P3 Q9(part)**

Light of wavelength 650 nm is incident normally on a double slit such that the waves emerge from X and Y in phase, and reach a screen 1.5 m away, as shown in Fig. 9.4.

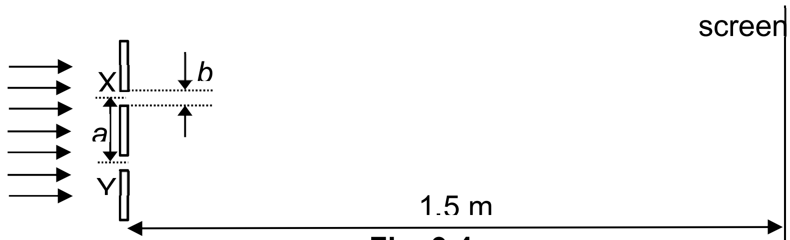



**Fig. 9.4**

The variation of intensity with distance along the screen is shown in Fig. 9.5.

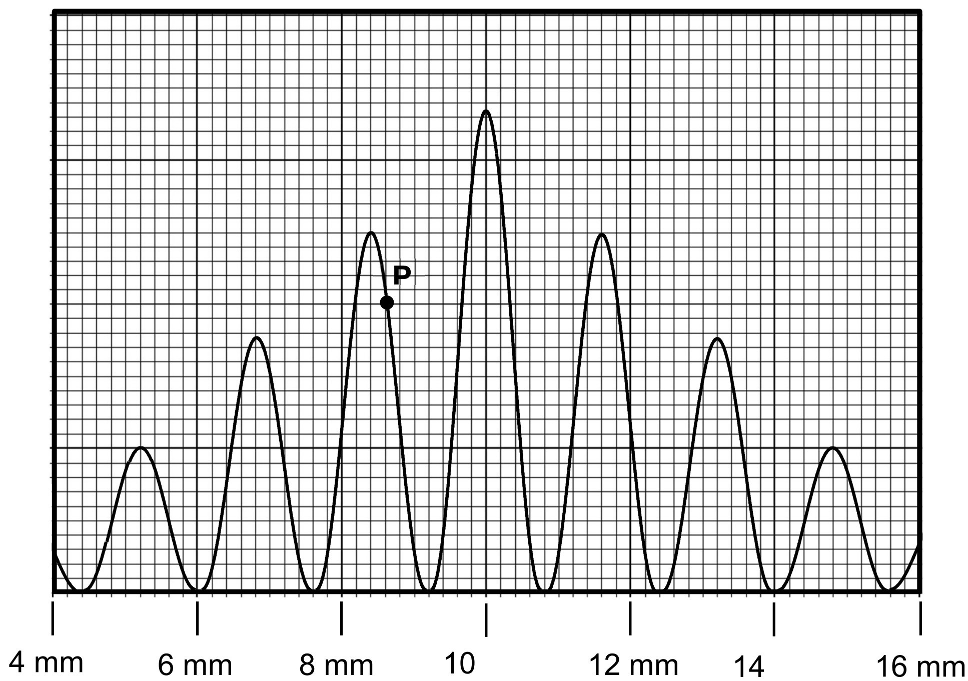



**Fig. 9.5**

**(i)** Explain how it can be deduced from Fig. 9.5 that the waves from the two slits are coherent. [1]

**(ii)** Determine the phase angle between the waves from the slits when the waves meet to produce the intensity at point **P** on the pattern of Fig. 9.5. [2]

**(iii)** Calculate the separation *a*, between the slits. [2]

**(iv)** Given that the 6th order bright fringe is the first missing order due to the diffraction envelope, calculate the width *b*, of each slit. [3]

**<u>Numerical Answers for Tutorial 10C: Single slit and Rayleigh's Criterion</u>**

**S1** 0.012 m **S2** 633 nm

**D1 (a)(i)** 5.6 x $10^{-7}$ m

**D2 (a)(ii)** 5.89 x $10^{-3}$ m

**D4 (ii)** 5.5 rad

**(iii)** 0.61 mm

**(iv)** 0.10 mm

## 10.8 Stationary Waves

Consider two identical waves moving in opposite directions overlapping in the same region of space. Figure 10.8.1 shows wave P moving towards the right and wave Q moving towards the left. By applying the principle of superposition, we know that during the time the two waves overlap, the resultant displacements is given by the vector sum of the displacements of the primary waves at that location.

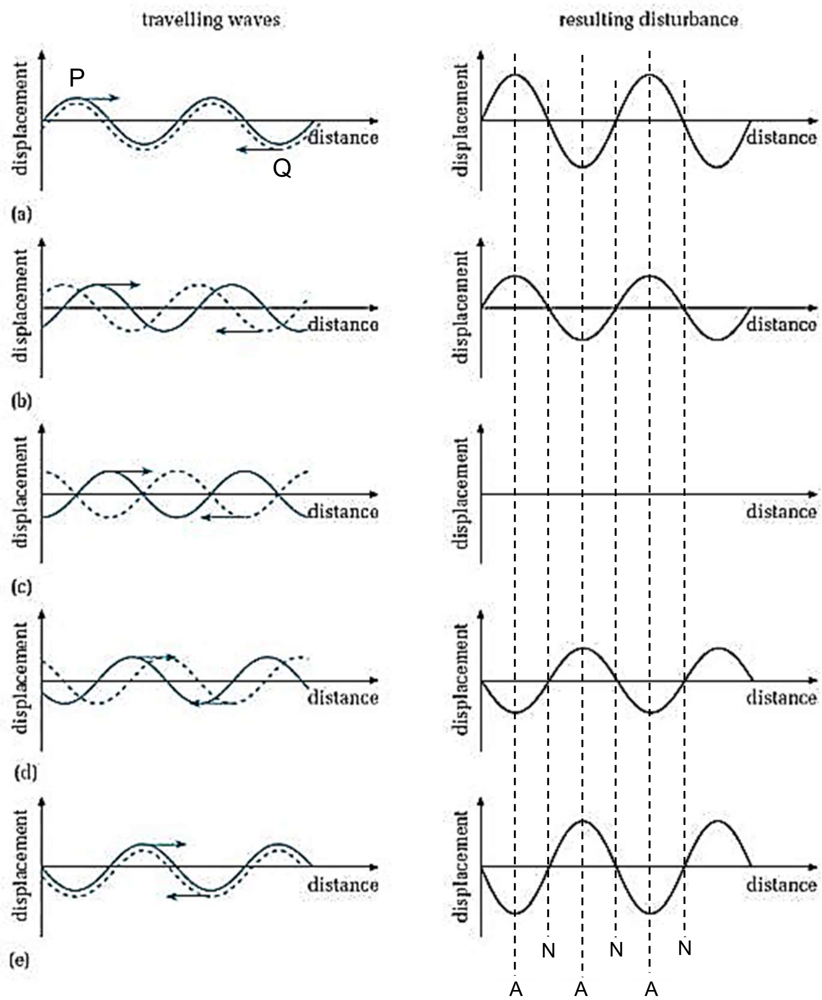



**Fig. 10.8.1**

The diagram shows how the superposition of 2 identical waves moving in opposite directions overlap to form a stationary wave.

Because the wave pattern does not appear to move in either direction along the wave, the resultant waveform is known as a **standing wave** or **stationary wave**.

We can observe from the diagram that amplitude of vibration for every point of the resultant wave varies.

- There are particular points called **nodes** (*N*) that never move, i.e. the amplitude of vibration is zero.
- Midway between the nodes are points called **antinodes** (*A*), where the amplitude of vibration is greatest.

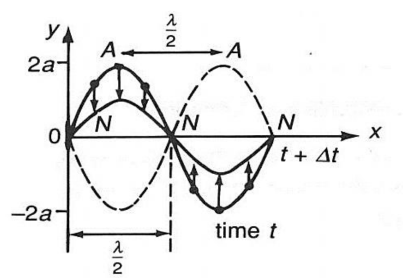
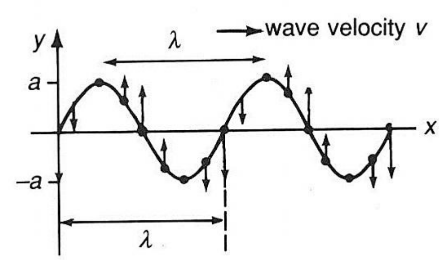
## Formation of Stationary Wave (or Standing Wave)

**<u>Stationary Waves</u>**

When two identical waves of the **same amplitude**, **frequency** and **speed** but travelling in **opposite directions** are superposed together, the resultant wave obtained is called a stationary wave.

### 10.8.1 Characteristics of Stationary Wave

Let us examine the properties of the stationary wave by comparing it with a travelling wave.

| | Standing Wave | Progressive Wave |
|---|---|---|
| **Diagram** |  |  |
| **Waveform** | Does not advance. | Waveform advances with the velocity of the wave. |
| **Amplitude** | Varies according to position from zero at the nodes (permanently at rest) to a maximum of 2 $A$ at the antinodes.<br>($A$ is the amplitude of each of the component travelling waves that form the stationary wave.) | Apart from attenuation, it is the same for all particles in the path of the wave. |
| **Frequency** | All the particles vibrate in SHM with the same frequency as the component waves (except for those at the nodes which are at rest). | All particles vibrate in SHM with the frequency of the wave. |
| **Phase** | A segment (loop) refers to points between two adjacent nodes.<br>Phase of all particles within a segment is the same (in phase). Particles in adjacent segments have a phase difference of $\pi$ rad (in antiphase). | All particles within one wavelength have different phase. |
| **Wavelength** | 2 x distance between a pair of adjacent nodes or antinodes. | Distance between two adjacent particles that are vibrating in phase. |
| **Energy** | No propagation of energy, but there is energy associated with the wave. | Energy propagates in the direction of the wave travel. |

**Example 10.8.1 : Stationary Waves**

Fig 10.8.2 shows a car driven at a speed of 30 m $s^{-1}$ along a straight road between 2 radio transmitters. The transmitters $T_1$ and $T_2$ are sending out the same programme, using a frequency of 1.50 MHz. The radio is heard to fade and strengthen regularly. [Ans: 3.3 s]

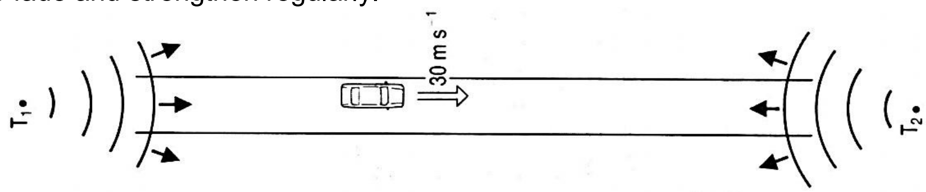


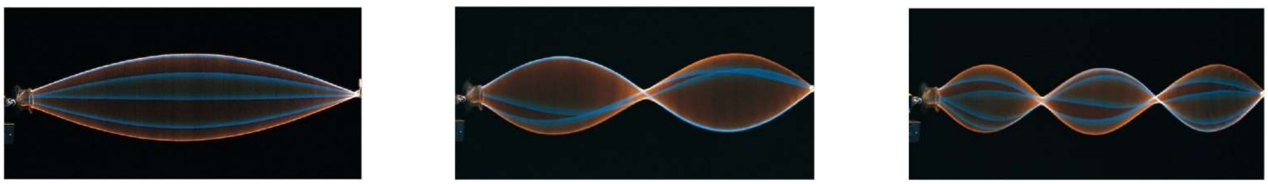

**Fig. 10.8.2**

What is the period of this regular fading? .

**Solution:**

### 10.8.2 Stationary Waves in Strings

Consider a string, fixed at one end, and the other end attached to an oscillator. If the oscillator moves up and down in simple harmonic motion, it will produce a continuous wave-train down the string. As the wave-train reaches the fixed end, it gets reflected back. The reflected wave-train will interfere with the wave-train from the source to form a resultant waveform on the string. It is observed that at certain frequencies of vibration of the source, we will be able to see some unique waveforms on the string as illustrated in Fig. 10.9.3(a)-(c).



(a) (b) (c)

**Fig. 10.8.3**

Time exposures of standing waves in a stretched string. At certain driven frequencies of vibration, the amplitude of vibration of the string become very large and the resultant waveform does not appear to progress along the string. (a) to (c) show standing waveforms at increasing frequencies of vibration.

The frequencies at which standing waves are produced are known as the **natural frequencies** or **resonant frequencies** of the string.

The different standing wave patterns shown in Fig. 10.8.3 are the different **"resonant modes of vibration"**. When the driving frequency matches that of the natural frequency of the string, the amplitude of oscillation generated are the largest.

This principle is very important as it is used in the generation of music in many stringed instruments like guitar and violin.

### Resonant modes of vibrations in a stretched string fixed at two ends

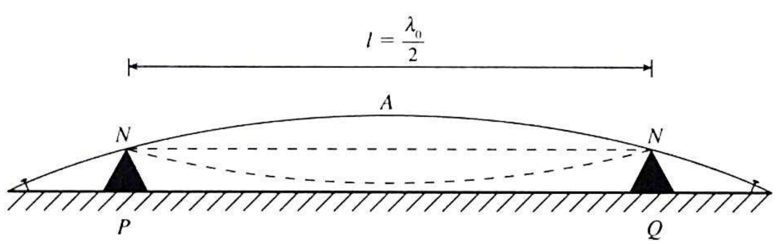


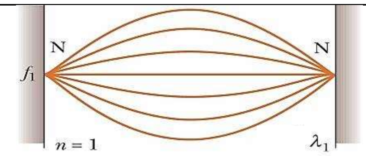
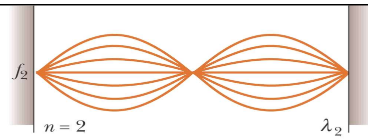
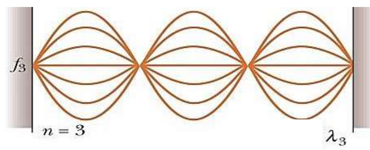

**Fig. 10.8.4**

Consider a wire stretched between 2 points and plucked in the middle, a transverse wave travels along the wire and is reflected at the ends. The reflected and re-reflected waves from the ends will continually interfere with each other to produce a stationary wave on the string.

**The two *ends of the strings are fixed and thus must have zero displacement* and hence must be <u>displacement nodes.</u>**

This additional constraint or boundary condition results in the string being allowed to have only certain wavelengths and frequencies of oscillations.

Fig. 10.8.4 shows the largest possible wavelength of stationary wave that can be produced on the wire - corresponding to the smallest frequency. This mode of oscillation is known as the **fundamental mode** and the lowest frequency of vibration $f_1$ is known as the **fundamental frequency.**

| Mode of Vibration | Wavelength | Frequency | Harmonic | Overtone |
|---|---|---|---|---|
|  | $\lambda_1 = 2L$ | $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ | 1st | – |
|  | $\lambda_2 = L$ | $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$ <br> $f_2 = 2f_1$ | 2nd | 1st |
|  | $\lambda_3 = \frac{2}{3}L$ | $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$ <br> $f_3 = 3f_1$ | 3rd | 2nd |

Generally the *nth* harmonic is given by: $f_n = n\left(\frac{v}{2L}\right)$ , where $n$ = 1, 2, 3, 4, ...

**Note:**

**1. Fundamental Frequency and Harmonics**

All resonant frequencies of vibrations that can be generated are called harmonics. The harmonics are numbered according to the ratio of their frequencies to the first natural frequency $f_1$. Thus if the 1st harmonic is 110 Hz, then 220 Hz, 330 Hz, 440 Hz etc. are the 2nd, 3rd , 4th etc harmonics respectively.

- The first natural frequency is the lowest possible frequency and is known as the *fundamental frequency* (or 1st harmonic)
- The frequencies of the harmonics are related to the fundamental frequency $f_1$ by $f_n = nf_1$ where $f_n$ is the frequency of the $n^{th}$ harmonic.
- Note that all harmonics are present for the string shown above.

**2. Overtones**

Overtones are the frequencies (excluding the fundamental) that can be produced by an instrument accompanying the 1st harmonic that is played.

The tuning fork and frequency generator are the only examples that can produce a pure note.

The lowest overtone is known as the 1st overtone, the next overtone is the 2nd overtone. Hence, in this case, the 1st overtone of the string corresponds to the 2nd harmonic of the string.

**3. Speed of a wave on a string**

The speed of a wave on a string is given by $v = \sqrt{\frac{T}{\mu}}$ where $T$ is the tension in the string and $\mu$ is the mass per unit length of the string. **Not required for A Level, for information only.**

**Example 10.8.2: Resonant Wavelengths of a Wire**

A 1.0 m wire stretched between 2 points is plucked near one end. What are the three longest wavelengths present on the vibrating wire? [Ans: 2.0 m, 1.0 m, 0.67 m]

**Solution:**

### 10.8.3 Standing Waves in Air Column

Like standing transverse wave on a stretched string, a standing longitudinal wave is obtained by *the superposition of two trains of progressive longitudinal waves of equal amplitude, and frequency, travelling with the same speed in the opposite directions*.

Stationary transverse and longitudinal waves share similar properties except for the *direction of the displacement of the particles* where the displacement of air molecules are parallel to the direction of propagation of the component waves that form the stationary wave.

The formation of stationary waves in pipes and tubes is the basis of production of musical notes in most wind instruments. When stationary waves are set up in the tubes, **the air column resonates at its natural frequencies.**

The boundary conditions that determine the natural frequencies of the air column depends on whether that end is open or closed.

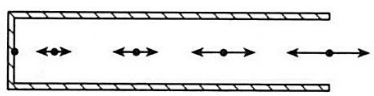


**Fig. 10.8.5**

Looking at Fig. 10.8.5,

**At the closed end**,

- the longitudinal movement of the air molecules is prevented and is always a **displacement node.**

**At the open end,**

- air molecules can vibrate freely, and is always a **displacement antinode**.

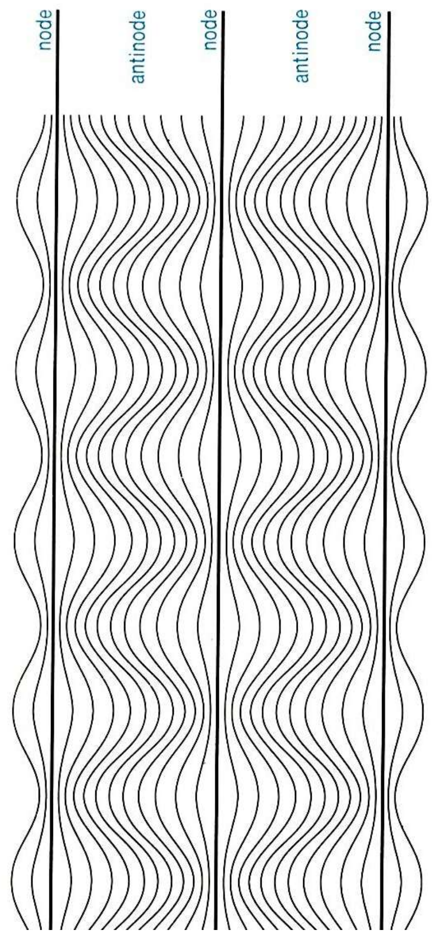



**Fig. 10.8.6**

The diagram can be used to show a longitudinal standing wave. To see the effect, move a piece of stiff card with a slot of no more than 1 mm cut in down over the diagram.

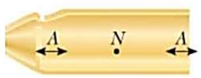
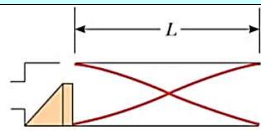
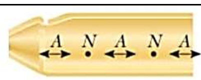
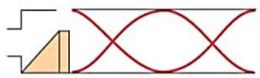
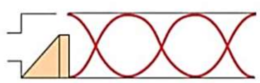
## Resonant Frequencies of Air Columns

There are generally 2 types of air columns we want to concern ourselves with:

- open pipe
- closed pipe ( closed at one end)

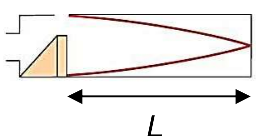
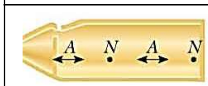
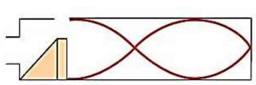
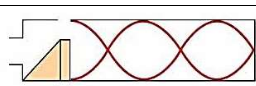
Recall that the open end of the pipe, we have a displacement antinode, and at the closed end of the pipe, we have a displacement node.

### Resonant Vibrations in an Open Pipe

| | (Displacement) Wave Profile | Wavelength | Frequency | Harmonic | Overtone |
|---|---|---|---|---|---|
|  |  | $\lambda_1 = 2L$ | $f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$ | 1st (fundamental) | – |
|  |  | $\lambda_2 = L$ | $f_2 = \frac{v}{\lambda_2} = \frac{v}{L}$<br>$f_2 = 2f_1$ | 2nd | 1st |
|  |  | $\lambda_3 = \frac{2}{3}L$ | $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2L}$<br>$f_3 = 3f_1$ | 3rd | 2nd |

The *nth* harmonic can be expressed as: $f = n\left(\frac{v}{2L}\right)$ , where *n* = 1, 2, 3, 4, ...

### Resonant Vibrations in a Closed Pipe

| | (Displacement) Wave Profile | Wavelength | Frequency | Harmonic | Overtone |
|---|---|---|---|---|---|
|  |  | $\frac{\lambda_1}{4} = L$<br>$\lambda_1 = 4L$ | $f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$ | 1st (fundamental) | – |
|  |  | $\frac{3\lambda_3}{4} = L$<br>$\lambda_3 = \frac{4L}{3}$ | $f_3 = \frac{v}{\lambda_3} = \frac{3v}{4L}$<br>$f_3 = 3f_1$ | 3rd | 1st |
|  |  | $\frac{5\lambda_5}{4} = L$<br>$\lambda_5 = \frac{4L}{5}$ | $f_5 = \frac{v}{\lambda_5} = \frac{5v}{4L}$<br>$f_5 = 5f_1$ | 5th | 2nd |

The *nth* harmonic is given by: $f_n = n\left(\frac{v}{4L}\right)$ , where *n* = 1, 3, 5,...

**Note:**

1. The fundamental frequency of an open pipe is twice that of a closed pipe of the same length.
2. Only odd harmonics are present in closed pipe.
3. Assuming that we have a closed pipe and an open pipe with characteristics the same in all aspects inclusive of length, the following table presents a possible situation for the combination of frequencies emitted.

| Harmonics | Frequency emitted from a Closed Pipe /Hz | Frequency emitted from an Open Pipe / Hz |
|---|---|---|
| 1 | 256 | 512 |
| 2 | - | 1024 |
| 3 | 768 | 1536 |
| 4 | - | 2048 |
| 5 | 1280 | 2560 |

The quality of sound emitted is a mixture of these harmonics for each system. Hence we see that the "note" from an open pipe is "richer" than that from a closed pipe.

**Example 10.8.3 Serway p18.37.**

Calculate the length of a pipe that has a fundamental frequency of 240 Hz if the pipe is
(a) closed at one end and
(b) open at both ends.

Take the speed of sound to be 343 m $s^{-1}$. [Ans: (a) 0.357 m (b) 0.715 m]

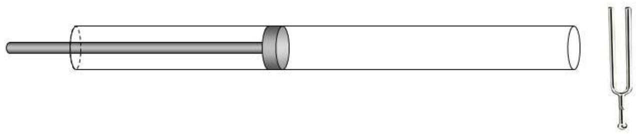
**Solution:**

**Example 10.8.4 Serway P18.45.**

An air column in a glass tube is open at one end and closed at the other by a movable piston. The air in the tube is warmed above room temperature, and a 384-Hz tuning fork is held at the open end. Resonance is first heard when the piston is 22.7 cm from the open end and again when it is 68.2 cm from the open end. [Ans: (a) 349 m $s^{-1}$ (b) 1.14 m]

(a) What speed of sound is implied by these data?

(b) How far from the open end will the piston be when the next resonance is heard?

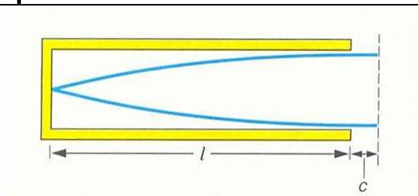
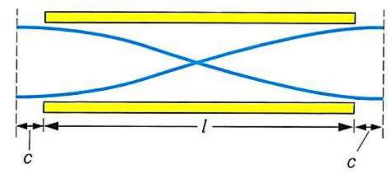


**Solution:**

**End Corrections[5]**

In practice, the air just outside the open end of a pipe is set into vibration and the displacement antinode of a standing wave occurs a distance $c$ – called the **end correction** beyond the end of the tube.

The effective length of the resonant air column is larger than the length of the pipe. The following figure shows the respective corrected lengths for the fundamental frequencies of a closed and open pipe.

<table>
<tr><th>Closed Pipe</th><th>Open Pipe</th></tr>
<tr><td><u>Making Amendments for End Correction</u><br>$$\frac{\lambda}{4}=L+c$$</td><td><u>Making Amendments for End Correction</u><br>$$\frac{\lambda}{2}=L+2c$$</td></tr>
</table>

- Note that the end correction for a cylindrical pipe is approximately $0.6r$ where $r$ is the radius of the pipe. Hence, for the larger pipes, it is necessary to take end correction into consideration.

[5] The knowledge of end correction is not stipulated in the syllabus. But this can be found in many questions in A-levels especially in the earlier years.

### 10.8.4 What about the pressure?

Earlier in Waves topic, we discussed the pressure variations in a progressive sound wave; now we can consider how pressure varies in a stationary sound wave.

Fig. 10.8.7(a) shows the two extreme curves of the displacement-position graph for a stationary wave for times when the displacements are greatest. As before, we will take displacements of air molecules to the right to be positive.

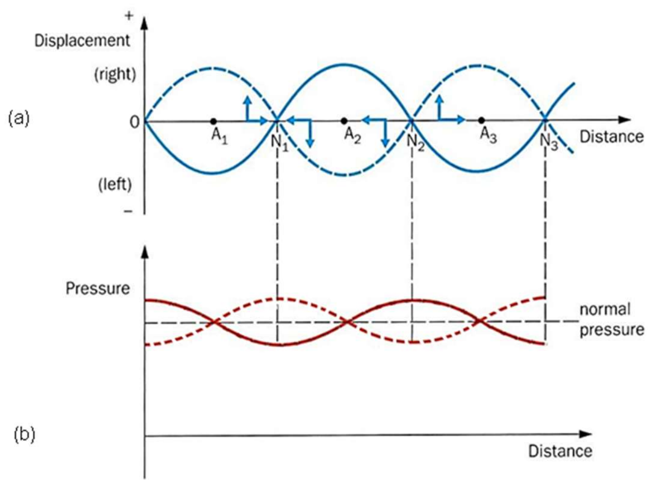



**Fig. 10.8.7**

Pressure variation for a stationary sound wave

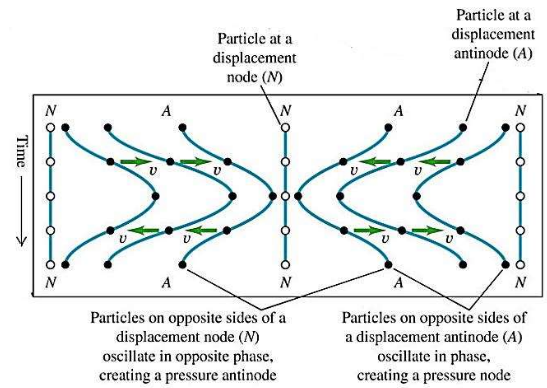



**Fig. 10.8.8**

In a standing longitudinal wave, a displacement node *N* is a pressure antinode (a point where the pressure fluctuates the most) and a displacement antinode A is a pressure node (a point where pressure does not fluctuate).

How does the pressure vary at displacement node $N_1$?

- Consider the instant in time when the wave corresponds to the dashed line in Fig. 10.8.7 (a).
- At position node $N_1$, air molecules to the left of $N_1$ are displaced positively and hence to the right towards $N_1$; air molecules to the right of $N_1$ has a negative displacement and hence to the left toward $N_1$ as well; air molecules at $N_1$ has zero displacement and remain at their equilibrium positions. **At this instant in time, the air at $N_1$ is compressed and the pressure is greater than normal.**
- Half a period later, the wave would correspond to the solid line in Fig. 10.8.7 (a),
- At position node $N_1$, air molecules to the left of $N_1$ are displaced negatively and hence to the left away from $N_1$; air molecules to the right of $N_1$ has a positive displacement and hence to the right away from $N_1$ as well. Air molecules at $N_1$ has zero displacement and remain at their equilibrium positions. **At this instant in time, the air at $N_1$ is rarefied and the pressure is lower than normal.**
- This illustrates the fact that **pressure variation is greatest at displacement nodes.**
- **Displacement node** is subjected to large pressure changes and hence it is a **pressure antinode.**

How does the pressure vary at displacement antinode $A_1$?

- At position antinode $A_1$, air molecules to the left of $A_1$, to the right of A1 and at A1 are always displaced in the same direction. This means that at antinode $A_1$, the separation between air molecules is maintained constant and **the pressure remains at normal all the time**.
- **Displacement antinode** is subjected to zero pressure changes and hence it is a **pressure node.**

**Example 10.8.5: End Corrections**

A source of sound of frequency 250 Hz is used with resonant tube, closed at one end, to measure the speed of sound in air. The two shortest resonant lengths are found to be 0.30 and 0.96 m. Calculate

(a) the speed of the sound, and

(b) the end correction of the tube. [Ans: (a) 330 m $s^{-1}$ (b) 3.0 cm]

**Solution:**

## 10.8.5 Investigating the speed of sound using stationary waves

We can determine the speed of sound wave by setting up a stationary wave and measure the corresponding frequency and wavelength as described by the experiment below. A stationary sound wave is set up by reflection from a reflector.

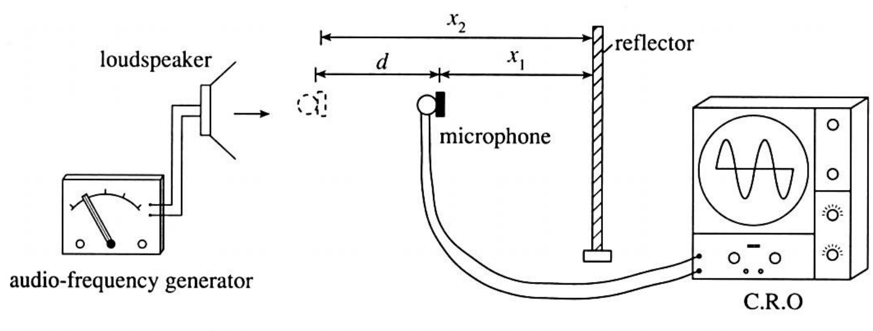



**Fig. 10.8.9** Measurement of speed of sound

The experimental setup is shown in Fig. 10.8.9.

- The signal generator (See Appendix E, Pg. 49) connected to a speaker delivers a note of frequency *f* towards a reflector (e.g. a metal sheet).
- The sound wave gets reflected off the reflector and interference occurs between the forward and reflected waves and a stationary wave pattern with nodes and antinodes is established between the speaker and reflector.
- The positions of antinodes are determined using a microphone connected to the oscilloscope (See Appendix A of waves lecture notes for the write-up on oscilloscopes). The microphone is slowly moved away from the reflector until a trace of maximum amplitude is obtained. This denotes the position of a pressure antinode. The distance $x_1$ of the microphone from the reflector is measured using a metre rule.
- The microphone is next moved slowly further away from the reflector until the amplitude of the trace on the C.R.O. is once again maximum. The distance of the microphone from the reflector, $x_2$, is again measured.

  Therefore, Distance between 2 successive pressure antinodes, $d = x_2 - x_1 = \lambda/2$

  Wavelength of the sound wave, $\lambda = 2d$
- If the frequency *f* of the sound wave is known, the speed of sound *v* can be determined from

$$v = f\lambda$$

➢ **You may now complete Tutorial 10D on Stationary Waves.**

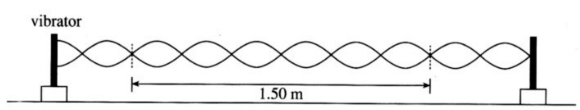
## Tutorial 10D: Stationary Waves

### Self-Practice Questions

**S1 J84/II/12**. Which one of the following correctly compares characteristics of travelling and stationary plane waves?

| | *Travelling Wave* | *Stationary wave* |
|---|---|---|
| **A** | no medium required | requires a material medium |
| **B** | separation between two adjacent points of corresponding phase is one wavelength | separation between a node and the adjacent antinode is half a wavelength |
| **C** | the amplitude of vibration is the same at all points | amplitude of vibration varies with position |
| **D** | energy at any point is always kinetic | energy at any point changes from kinetic and potential and back again. |
| **E** | energy is transported at a speed given by the frequency divided by the wavelength | No net transport of energy |

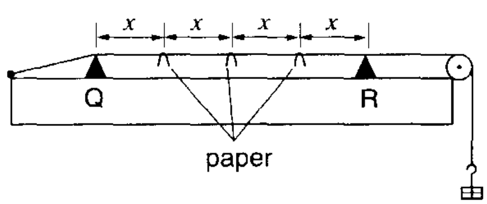
**S2** A vibrator produces a stationary wave on a stretched string as shown in the figure below. If the frequency of the vibrator is 20 Hz, what is the speed of the wave?




**A** 3.0 m s$^{-1}$ **B** 10.0 m s$^{-1}$ **C** 15.0 m s$^{-1}$ **D** 22.5 m s$^{-1}$

**E** 43.0 m s$^{-1}$

**S3 J97/I/12.** A wire is stretched over two supports, Q and R, a distance 4$x$ apart. Three light pieces of paper rest on the wire, as shown.




When the wire is made to vibrate at one particular frequency, the middle piece of paper stays on, but the others fall off the wire.

What is the wavelength of the vibration produced on the wire.

**A** 2$x$ **B** 3$x$ **C** 4$x$ **D** 8$x$

**S4** A tall vertical cylinder is filled with water and a tuning fork of frequency 512 Hz is held over its open end. Calculate the position of the water level below the open end when (a) the first resonance and (b) second resonance are heard. (Assume speed of sound in air is 330 m s$^{-1}$)

**S5 N84/II/12.**

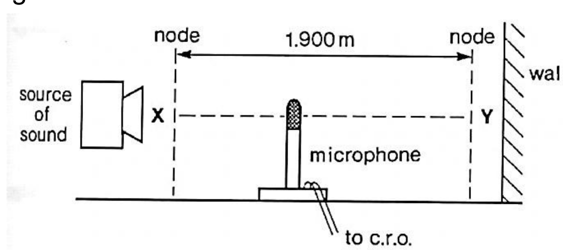
A stationary wave in the gas in a resonance tube can be described in terms either of amplitude $\Delta x$ of oscillation of the particles of the gas from their mean positions or of the fluctuation of pressure $\Delta p$ above and below the average. Which one of the following correctly describes the situation at resonance in a tube which is closed at one end? (Neglect end correction at the open end.)

| | At closed end | | At open end | |
|---|---|---|---|---|
| | $\Delta x$ | $\Delta p$ | $\Delta x$ | $\Delta p$ |
| **A** | zero | maximum | zero | maximum |
| **B** | zero | maximum | maximum | zero |
| **C** | maximum | zero | maximum | zero |
| **D** | maximum | zero | zero | maximum |
| **E** | zero | zero | maximum | maximum |

**Discussion Questions:**

**D1 N89/I/7; J95/I/10**

A source of sound of frequency 2500 Hz is placed several metres from a plane reflecting wall in a large chamber containing a gas. A microphone, connected to a cathode-ray oscilloscope, is used to detect nodes and antinodes along the line **XY** between the source and the wall.




The microphone is moved from one node through 20 antinodes to another node, a distance of 1.900 m.

What is the speed of sound in the gas?

**A** 238 m s$^{-1}$ **B** 250 m s$^{-1}$ **C** 330 m s$^{-1}$ **D** 475 m s$^{-1}$

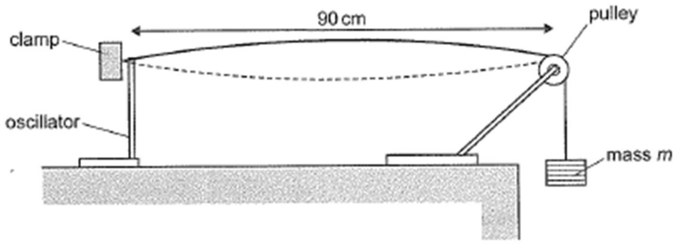
**D2 N79/II/14; J85/I/12.**

A suspension bridge is to be built across a valley where it is known that the wind can gust at 5 s intervals. It is estimated that the speed of transverse waves along the span of the bridge would be 400 m s$^{-1}$. The danger of resonant motions in the bridge at its fundamental frequency would be greatest if the span had a length of

**A** 2 000 m **B** 1 000 m **C** 400 m **D** 80 m **E** 40 m

**D3 N2017/I/17**

The speed of a wave on a stretched string is proportional to the square root of the tension in the string. The string is held under tension between a clamp and a pulley 90 cm apart by a mass *m* of 400 g hanging from the end of the string.




When the frequency *f* of the mechanical oscillator close to the clamped end is 15 Hz, the stationary wave shown is set up between the clamp and the pulley. The hanging mass *m* and frequency *f* are varied until another stationary wave is formed.

Which combination of *m* and *f* gives a stationary wave?

| | *m* / g | *f* / Hz |
|---|---|---|
| **A** | 500 | 30 |
| **B** | 600 | 40 |
| **C** | 800 | 60 |
| **D** | 900 | 90 |

**D4 J99/III/4 (part).**

**(a)** One end of a horizontal string is attached to the oscillating plate. The string passes over a pulley and the string is kept under tension by means of a weight, as illustrated in the figure below.

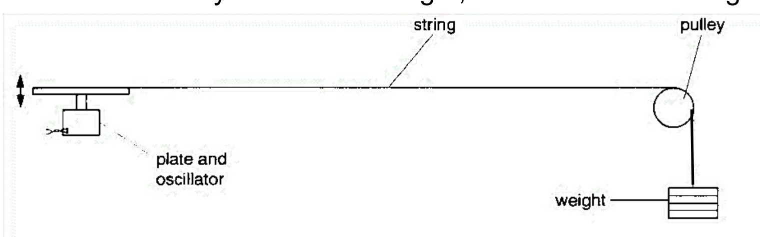



Fig. 4.2

The frequency of oscillation of the plate is increased and at certain frequencies, stationary waves are produced on the string.

(i) Copy the above figure and on your diagram show the stationary wave on the string when the frequency is such that the distance between the plate and the pulley corresponds to two wavelengths of the wave on the string.

(ii) On your diagram, label the position of a node on the string.

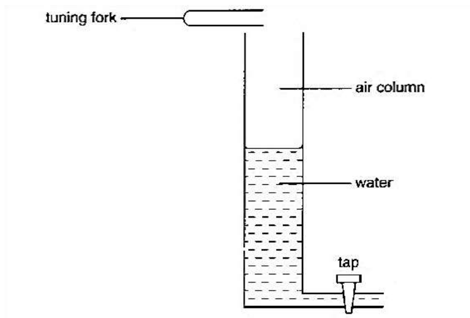
(iii) Briefly explain why a stationary wave is observed on the string only at particular frequencies of vibration of the plate. [4]

**(b)** Some musical instruments rely on stationary waves on strings in order to produce sound.

Suggest why strings made of different materials or with different diameters are sometimes used. [2]

**D5 N2000/I/10**

The diagram shows an experiment to produce a stationary wave in an air column. A tuning fork, placed above the column, vibrates and produces a sound wave. The length of the air column can be varied by altering the volume of the water in the tube.



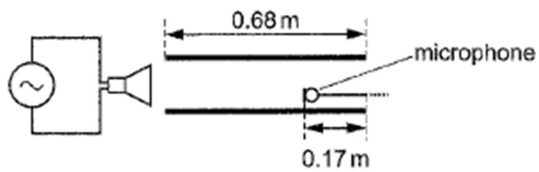

The tube is filled and then water is allowed to run out of it. The first two resonances occur when the air column lengths are 0.14 m and 0.46 m.

What is the wavelength of the sound wave?

**A** 0.32 m **B** 0.56 m **C** 0.60 m **D** 0.64 m

**D6 N2018/I/18**

A stationary sound wave is formed inside an open tube of length 0.68 m. A small micophone is inserted into the tube. It detects the first node at a distance of 0.17 m from the end. The microphone is then fixed in this position. The speed of sound in the tube is 340 m $s^{-1}$.

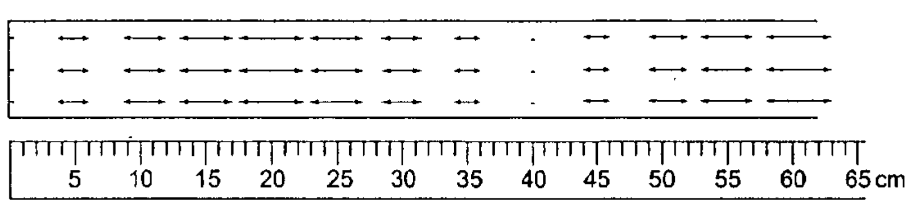



The frequency of the signal generator is now increased until the microphone again detects a node. What is this new frequency?

A 750 Hz B 1000 Hz C 1500 Hz D 3000 Hz

**D7 N2004/I/18.**

A stationary sound wave is set up in a pipe using an oscillator of frequency 440 Hz. The extent of the vibration of the molecules in the pipe is illustrated. A centimetre scale is shown alongside the pipe.




What is the speed of sound in the pipe?

**A** 176 m $s^{-1}$ **B** 328 m $s^{-1}$ **C** 337 m $s^{-1}$ **D** 352 m $s^{-1}$

**D8 (Jun 07 P2/5)** Light reflected from the surface of smooth water may be described as a polarised transverse wave.

**(a)** By reference to the direction of propagation of energy, explain what is meant by
**(i)** a *transverse wave,* [1]
**(ii)** *polarisation.* [1]

**(b)** A glass tube, closed at one end, has fine dust sprinkled along its length. A sound source is placed near the open end of the tube, as shown in Fig. 5.1.

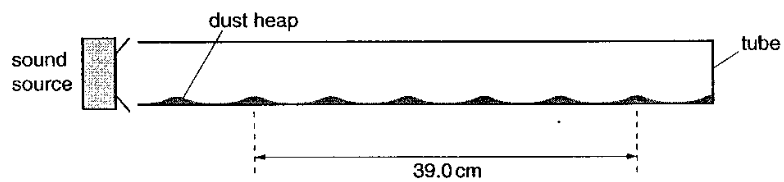



**Fig. 5.1**

The frequency of the sound emitted by the source is varied and, at one frequency, the dust forms small heaps in the tube.

**(i)** Explain, by reference to the properties of stationary waves, why the heaps of dust are formed.[3]

**(ii)** One frequency at which heaps are formed is 2.14 kHz. The distance between six heaps, as shown in Fig. 5.1, is 39.0 cm. Calculate the speed of sound in the tube. [3]

**(c)** The wave in the tube is a stationary wave. Explain, by reference to the formation of a stationary wave, what is meant by the speed calculated in **(b)(ii)**. [3]

## <u>Numerical Answers to Selected Questions in Tutorial 10D: Stationary Waves</u>

**S4**(a) 0.161 m (b) 0.483 m
**D8** (b)(ii) 334 m $s^{-1}$

## Appendices

## Appendix A: Huygens' Principle

But how can one explain the bending of waves due to diffraction? Why does the spreading of waves become increasingly more obvious with the width of the aperture decreases to about the same order as the wavelength? This phenomenon may be explained using *Huygens' Principle.*

The *Huygens' Principle* is a geometrical method for finding, from the known shape of a wave front at some instant, the shape of the wave front at some later time. Huygen assumed that

> ***every point of a wave front may be considered the source of secondary wavelets that spread out in all directions with a speed equal to the speed of propagation of the wave.***

The new wave front at a later time is found by constructing a surface *tangent to the secondary wavelets,* called the *envelope* of the wavelets.

The following diagram shows how Huygens' construction is used to determine the new wavefronts for a plane wave and for a spherical wave.

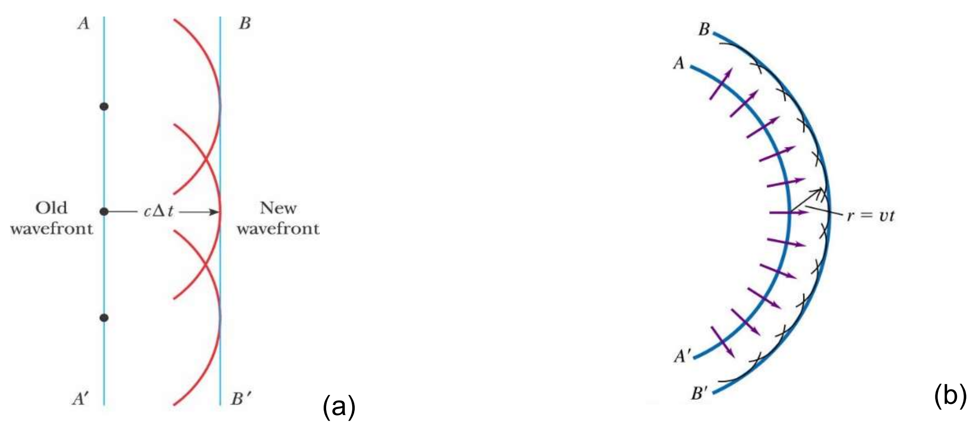
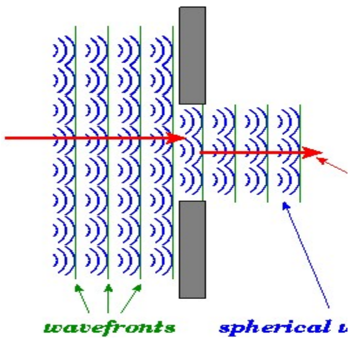
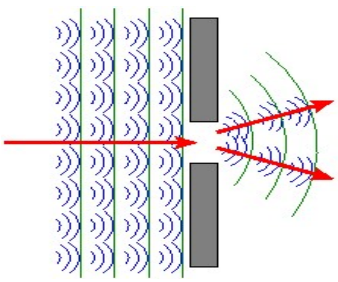
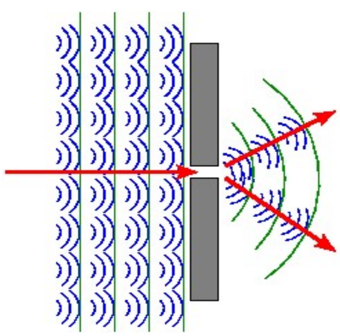



**Fig. 1:** Huygens' construction for (a) a plane wave propagating to the right and (b) a spherical wave propagating to the left.

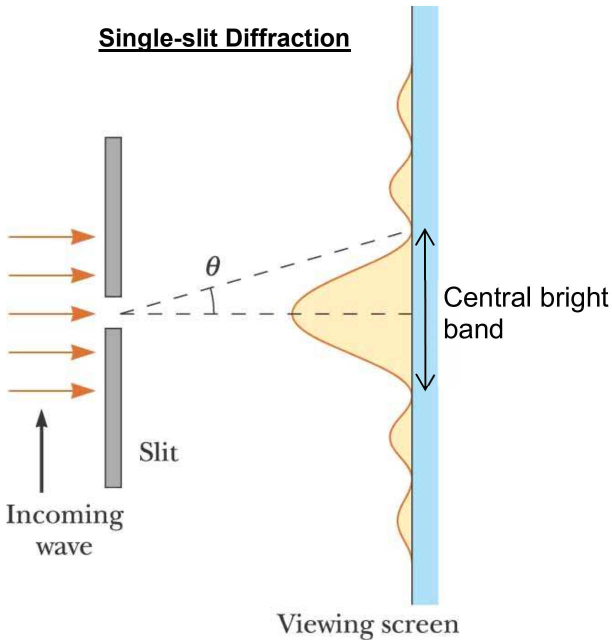
Through Huygens' construction, we can now see how the subsequent wavefronts 'reconstruct' themselves when incident plane waves pass through a slit.

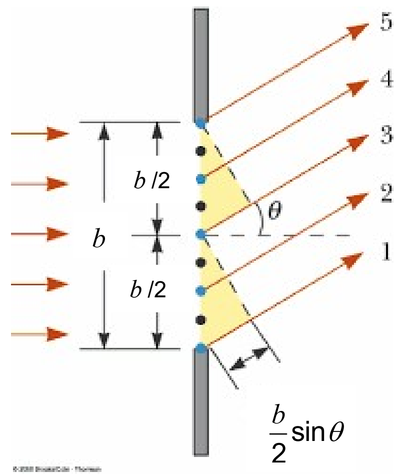
Let us now apply Huygens' construction to explaining diffraction.

| | |
|---|---|
|  Fig. 2 | Consider first the case where the obstacle or opening is much larger than the wavelength of the waves (Fig. 2), we find that the wavelets emerging forms wavefronts that is essentially straight. Diffraction is not very apparent. |
|  Fig. 3 | If, however, the size of the opening becomes comparable to the wavelength, the waves proceed to "bend through" or around the opening or obstruction as shown in Fig. 3. |
|  Fig. 4 | Finally, as the obstruction or opening equals the wavelength, the diffraction becomes quite pronounced as shown in Fig 4. |

## Appendix B: Single Slit Diffraction Pattern

To analyze the diffraction pattern of the single slit, let us divide the slit into two halves as shown.




Consider ray 1 and 3. As the two rays travel towards the screen, ray 1 travels an additional path difference of $\frac{b}{2}\sin\theta$ where $b$ is the width of the slit and $\theta$ is the angle with the principle axis. Similarly, the path difference between rays 2 and 4, as well as between rays 3 and 5, is also $\frac{b}{2}\sin\theta$.

If this path difference is exactly half a wavelength, then destructive interference occurs. This cancellation occurs for any two rays that originate at the points separated by half a slit width because the phase difference between 2 such points is 180°. Therefore the waves from the upper half of the slit interfere destructively with the waves from the lower half when

$$\frac{b}{2}\sin\theta = \pm\frac{\lambda}{2}$$

$$\text{or } b\sin\theta = \pm\lambda$$

Dividing the slit into 4 equal parts and using the same reasoning, we find that destructive interference also occurs when $\frac{b}{4}\sin\theta = \pm\frac{\lambda}{2}$ or $b\sin\theta = \pm 2\lambda$.

This same argument can be applied when the slit is divided into 6, 8 and so on equal parts, and we obtained the general condition for **destructive interference** (or **dark region**) to be

$$b\sin\theta = m\lambda \text{ where m = } \pm 1, 2, 3, \ldots$$

## Appendix C: Ripple Tank

One way to observe interference of water waves is in a ripple tank. Generally a ripple tank could be used to investigate various properties of water waves.

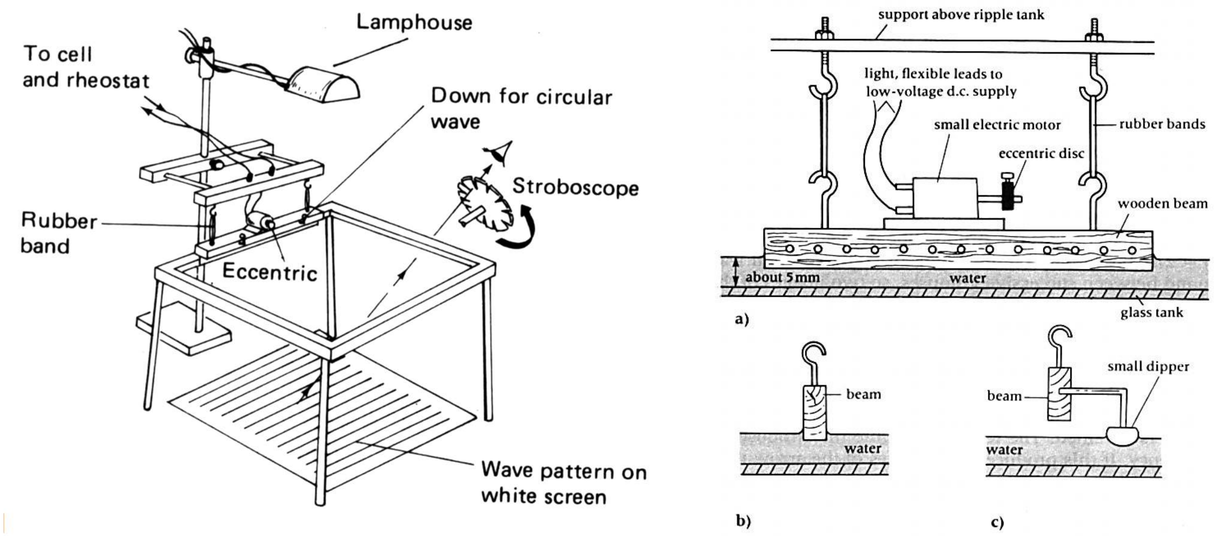



**Fig. 1** Set-up for the ripple tank

1. **<u>Eccentric:</u>** An eccentric (off-centre) metal disc on the axle of the motor causes the beam to vibrate at the same frequency as the revolutions of the motor. The speed of the motor is controlled by a rheostat in series with a low-voltage d.c. supply, or a dry battery (usually 3.0 or 4.5 V).

2. To generate continuous straight wave, a vibrating beam just touching the water is used.

3. To generate circular waves, a small spherical dipper is fixed to the beam instead.

4. The speed of the motor may be adjusted to vary the frequency and wavelength of continuous wave train.

5. The progressive waves move quite quickly across the water surface and any attempt to measure the wavelength or draw the wave diagrams from the moving water shadows would be very difficult. A standard method of freezing or slowing down rapid motion would be to use a stroboscope.

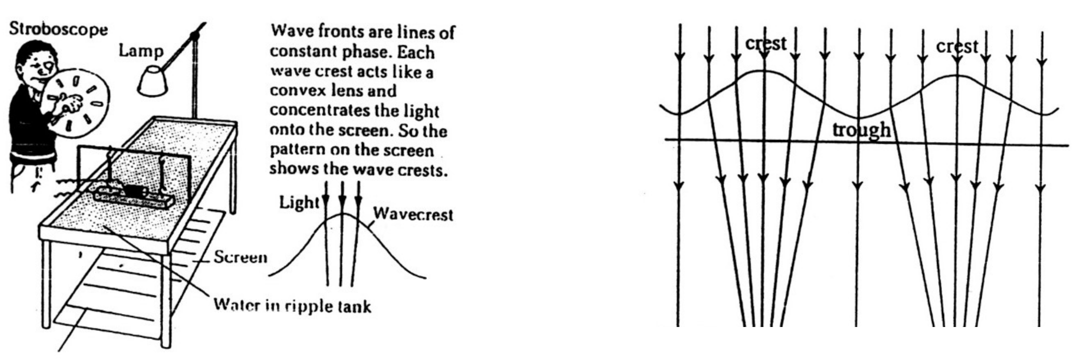



**Fig. 2** Observations of wavefronts in a ripple tank.

## Appendix D: Spectrometer

The grating or double slit is usually mounted on a device known as a *spectrometer* to allow us to easily take measurements of $\theta$ so that we can study about the light source. Fig. 1 below shows you a typical laboratory spectrometer and Fig. 2 shows a schematic sketch of the spectrometer.

**Fig. 1**
A laboratory spectrometer

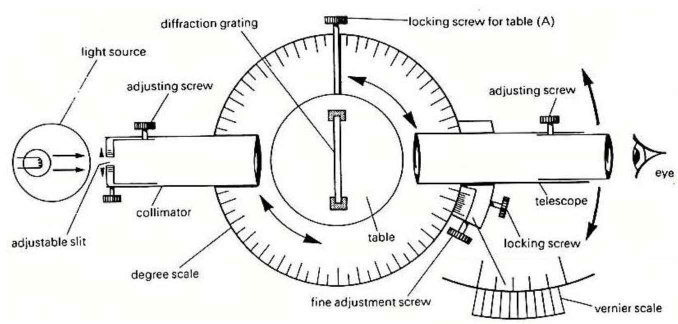



**Fig. 2**
The schematics of the spectrometer

The essential parts of a spectrometer are

(a) A fixed *collimator* with a movable slit of adjustable width to produce a parallel beam of light from the source illuminating the slit. The light source to be studied is placed between the collimator and the turntable.

(b) a *turntable* on which the prism, double slit or grating can be mounted, and

(c) a *telescope* attached to the turntable such that it can be rotated about same vertical axis as the turntable. Usually there is also vernier scale attached below the turntable so that you to read off the *angle* though which the telescope is turned through from this scale. The vernier scale allows you to read the angular position of the telescope to the nearest 0.1°.

When used with a grating,

- we usually start off by locating the position of the *zeroth order maximum*. It is the brightest fringe that has the brightest intensity. This provides a reference point for which angular displacement can be measured. The angular displacement of the $m^{th}$ *order* maximum,

  $\theta$ = |Angular position of zeroth order bright fringe - Angular position of $m^{th}$ order bright fringe|

  $$\Rightarrow \qquad \theta = |\theta_o - \theta_m|$$

  Alternatively, by noting the angular positions of the *mth* order bright fringe at either side of the central maximum ($\theta_{m,left}$ and $\theta_{m,right}$), we can also determine the angular displacement $\theta$ of $m^{th}$ order maximum from the central maximum. In this case, $2\theta = |\theta_{m,left} - \theta_{m,right}|$

- We also move the telescope to each side of this central maximum to check that several orders of *diffraction* can be seen and there is some symmetry about this central maximum. If not, the plane of the surface of the grating is not perpendicular to the plane in which telescope moves and tilt of the table needs to be adjusted.