ANGLO-CHINESE JUNIOR COLLEGE MATHEMATICS DEPARTMENT

MATHEMATICS Higher 1

8865

JC 1 REVISION EXERCISE SET A Mar 2023

Topics	Start Page Number	
Exponential & Logarithmic Functions	1	Complete solutions
Graphing Techniques	2	Will be uploaded into Math Google Site
Equations & Inequalities	5	one week before CA1.
ACJC CA1 past year papers	8	

Name: _____

H1 Maths Group: 1MAX _____

Form Class: _____



Anglo-Chinese Junior College

EXPONENTIAL & LOGARITHMIC FUNCTIONS

1	(a)	Solve $lg(x-8) + lg\left(\frac{9}{2}\right) = 1 + lg\left(\frac{x}{4}\right)$.	[4]
	(b)	Solve the equation $\log_5(2x+1) - \log_5(3x-5) = 1$	[3]
2	SRJ	C 2016 Promo Q3	
	(a)	Given that $\ln p = k$, find $\ln (pe^{25})$ in terms of k.	[2]
	(b)	Given that $3e^{x} - 4e^{-x} = 11$, find the exact value of <i>x</i> .	[4]
3	CJC	2012 Promo Q3	
	(a)	Find the exact value(s) of the equation $2e^{3x} - 5e^x = 3e^{-x}$.	[4]
	(b)	Given that $\log_a \sqrt{x} = 3$, find $\log_a (ax^2)$.	[2]
4	RI 2 Solv	2012 Promo Q2 we the simultaneous equations $y = (1 + e^x)(1 - e^x),$	
		$x = \ln\left(y+1\right).$	[5]

5 YJC 2013 Prelim Q2

Solve the simultaneous equations

$$\frac{4^{3x}}{64} = 4^{y} \text{ and } \lg(y-x) = 1 - \lg(x-1).$$
[4]

6 TJC 2016 Promo Q3

Given that $\log_4 q^2 - \log_4 q - p = 0$ and	d $\sqrt{\log_p q} = 2$, show that $4^p = p^4$.	[3]
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- (i) On the same set of axes, sketch the graphs of $y = 4^x$ and $y = x^4$. [1]
- (ii) Hence find the solutions of p and corresponding values of q. [2]

7 JJC 2013 Promo Q4

In a research project, 5 000 ants are put in a man-made habitat for the population to grow. After t days, the population of the ants, P (in thousands) is given by

 $P = -45e^{At} + B$, for some constants A and B.

Given that 10 days later, the population has increased to 35 000.

- (i) Show that B = 50 and $A = k \ln 3$, where k is a constant to be found. [4]
- (ii) Find the population of the ants after 20 days.
- (iii) Find the population of the ants after a very long time. [1]
- (iv) Sketch the graph of *P* against *t*.

[2]

[1]

8 RVHS 2016 Promo Q11

The temperature, $\theta^{\circ}C$, of the water in a flask, *t* minutes after the start of heating is modelled by the equation $\theta = b + k \ln(2t+1)$, where *b* and *k* are constants. Initially the temperature of the water is

 $20^\circ C$. After 2 minutes of heating, the temperature is $\,65^\circ C$.

- (i) Find the values of b and k.
- (ii) Find the time it takes for the water to reach a temperature of 50° C.
- (iii) Sketch the graph of the temperature of the water against time for $0 \le t \le 8$, labelling clearly the end points of the graph. [3] (iv) State, with a reason, whether this model is suitable in the long term. [2]

Answers

1 (a) x = 18(b) - 2(a) k + 25(b) ln 4 2 (a) $x = \frac{\ln 3}{2}$ 3 (b) 13 x = 0, y = 04 x = 3.5, y = 7.55 (ii) p = 2, q = 16;6 p = 4, q = 216(i) $k = -\frac{1}{10}$ 7 (ii) 45 000 (iii) 50 000 (i) $k = \frac{45}{\ln 5}$ (ii) 0.962 min 8

(iv) No Suggested reason: Boiling point of water is 100° C but the graph is an increasing curve, so the model will be inaccurate once the temperature rises above 100° C as $t \rightarrow \infty$.

GRAPHING TECHNIQUES

1 DHS 2014 Promo Q3

Sketch the graph of $y = x^3 - x^2 - x + 1$, stating clearly the coordinates of any points of intersection with the axes and the coordinates of the turning points. [2]

By sketching a suitable graph on the same diagram, find the number of real roots for the equation $2x^3 - 2x^2 - 2x + 2 + \ln x = 0.$ [3]

2 ACJC 2009 CT1 Q6 [modified]

Sketch the circle $x^2 + y^2 - 6x + 2y + 1 = 0$, stating the coordinates of its centre and radius. [4]

[4]

[3]

3 VJC 2009 Prelim Q5

- (i) Sketch, for $x \le 0$, the graphs of $y = \frac{4}{x-1}$ and $y = 12x^2 + 31x 4$ on the same axes. [2]
- (ii) The graphs intersect on the *y*-axis. Find, correct to 2 decimal places, the *x*-coordinate of the point of intersection for which x < 0. [1]

4 MJC 2008 Prelim Q4(i) [modified]

Sketch, for $x \ge 0$, the graphs of $y = 13 - x^2$ and $y = \frac{36}{x^2}$ on the same axes, showing clearly any axial intercepts. State the exact coordinates of any points of intersection for which $x \ge 0$. [4]

5 MI 2012 Promo Q3 [modified]

Sketch, on separate diagrams, the graphs of

(i)
$$y = 2e^x + 2$$
, [2]

(ii)
$$2x^2 + 3x + 2y^2 + 2y + 1 = 0,$$
 [4]

(iii)
$$y = \frac{4-x}{2x+1}$$
, [3]

labelling clearly the equations of any asymptotes and the coordinates of any intersections with the coordinate axes. State the equations of any axes of symmetry.

6 JJC 2009 Promo Q4

The diagram given shows the graph of the equation $y = \frac{ax+c}{x-b}$. The graph intersects the *x*-axis at

 $\left(-\frac{3}{2},0\right)$ and the lines x = 1 and y = 2 are asymptotes.

Find the values of *a*, *b* and *c*.

7 TJC 2014 Promo Q4

A curve *C* has equation $y = \ln(kx-3)$, where *k* is a positive constant. State in terms of *k*, the equation of the asymptote of *C* and the coordinates of the point where *C* crosses the *x*-axis. [2] Sketch the graph of $y = \ln(2x-3)$, showing clearly its asymptote and the point of intersection with the *x*-axis. By adding a suitable graph to the same diagram, solve the equation $e^x \ln(2x-3) = 1$. [5]



[3]

8 CJC 2012 Promo Q2 (Explore using GC)

A curve has equation $y = 3 + \frac{2}{x+1} - \frac{1}{x-1}$.

- (i) State the equations of the asymptotes of the curve. [2]
- (ii) Sketch the curve, indicating clearly the equations of the asymptotes, and the coordinates of any turning points and any points of intersection with the axes. [4]

9 ACJC 2009 CT1 Q3

Sketch the graph of $y = \frac{4x+1}{2x-2}$, showing clearly the asymptotes and the axial intercepts. [3] Given that the equation $x = \log_2(4x+1) - \log_2(2x-2)$ can be written in the form

 $\frac{4x+1}{2x-2} = f(x),$ [3]

determine the function f(x).

The equation $x = \log_2(4x+1) - \log_2(2x-2)$ can be solved graphically by drawing an additional curve in the same diagram as the graph of $y = \frac{4x+1}{2x-2}$. Sketch the additional curve and solve the equation $x = \log_2(4x+1) - \log_2(2x-2)$. [2]

10 TJC 2009 Prelim Q11 [modified]

(a) A rectangular hyperbola *H* is given by the equation $y = \frac{2x-a}{x+2a}$ where *a* is a positive real number. Sketch *H*, stating clearly the equations of any asymptotes and the coordinates of any intersections with the coordinate axes in terms of *a*. [3]

Verify that the point
$$\left(\frac{3a}{2}, \frac{4}{7}\right)$$
 lies on *H*. [2]

y = f(x)

0

----- y = 2

(b) The diagram shows the graph of y = f(x).



$$\mathbf{f}(x) = x^2 + 2x + (k+1)$$

will always have two real solutions for $k \ge 2$.

4

[1]

[2]

Answers



EQUATIONS & INEQUALITIES

1 Two cubes of side x cm and y cm respectively, have a total surface area of 60 cm². The sum of the length of all the edges of the two cubes is 48 cm. Find the exact values of x and y, if y > x.

2 YJC 2013 Promo Q2(i)

It is given that $f(x) = ax^2 + bx + c$, where *a*, *b* and *c* are constants. Given that the curve with equation y = f(x) passes through the points with coordinates (0, 1.2), (2, 34.4) and (-3, -11.1), find the values of *a*, *b* and *c*. [3]

3 JJC 2013 Promo Q7b

A factory produces 3 brands of drinks, A, B and C. The total price of 1 litre of A, 1 litre of B and 2 litres of C is \$9. The total price of 1 litre of B and 1 litre of C is \$3.50. The total price of 2.5 litres of B and 2 litres of C is twice the price of 1 litre of A.

Write down and solve the equations to find the price of each litre of *A*, *B* and *C*. [4]

4 NJC 2012 Promo Q2

Find the set of values of k for which the line y = 3x + k intersects the curve $y = x^2 + kx + 11$. [4]

5 AJC 2012 Promo Q1

Find the range of values of k for which the line y = k - x does not intersect the curve $y = \frac{k}{2x}$. [4]

6 TJC 2012 Promo Q 10

- (a) Solve the inequality $5-x^2 \le 2-3x$, giving your answer in surd form. [4]
- (b) (i) By completing the square, show that $4x^2 24x + 39$ is always positive for all real values of x. [3]

(ii) Hence solve the inequality $\frac{4x^2 - 24x + 39}{(x+2)(x-1)} \le 0$, giving your answer in surd form. [2]

7 VJC 2012 Promo Q1

Show that the equation $x^2 + 2k = 4 - kx$ has real roots for all real values of k. [3]

8 VJC 2012 Promo Q7

Without using a calculator, solve the inequality $x^2 + x + 7 \le 2x^2 + 1$. [3] Hence solve the following inequalities:

(i)
$$(\ln x)^2 - \ln x - 6 \ge 0$$
, [3]

(ii)
$$e^{2x} - e^x - 6 \ge 0$$
. [2]

9 RVHS 2012 Promo Q2 [modified]

Sketch the graph of $y = \frac{7}{1+x^2}$. By adding the line y = x+5, find the set of values of x for which

$$\frac{7}{1+x^2} \le x+5$$
. Hence solve $\frac{7}{1+x^2} + x \le 5$. [3][3]

10 NYJC 2013 Prelim Q2

Find the range of values of k for which $kx^2 + x + k > 4 - 2x$ for all real values of x. [4]

11 SRJC2016 Promo Q1

A museum sells tickets at \$6.50 for adults, \$4 for students and \$2 for children under 7 years old. A total of \$1395 was collected from a sale of 280 tickets. The number of adult tickets sold was 10 less than twice the number of students' tickets sold.

Find the number of tickets sold for each category.

[4]

12 HCI 2016/Promo Q1

A store sells large, medium and small sizes of white, blue and yellow colour of shirts. The selling price of a large, medium and small sizes of any colour is x, y and z respectively. The number of shirts of each type and size that were sold in one day are given in the following table.

	Large	Medium	Small	Total amount
white	5	3	1	\$ 593
blue	2	7	5	\$ 829
yellow	6	4	2	\$ 778

Write down and solve the equations to find the selling price of a shirt for each of the size.Mr Tan bought 3 large white shirts, 1 medium blue shirt and 5 small yellow shirts.How much did he pay altogether? [3]

- 1 y = 3, x = 12 a = 2.5, b = 11.6, c = 1.23 A = \$4, B = \$2, C = \$1.50
- 4 $k \leq -5 \text{ or } k \geq 7$
- $5 \quad 0 < k < 2$

6
$$x \le \frac{3-\sqrt{21}}{2}$$
 or $x \ge \frac{3+\sqrt{21}}{2}$, $-2 < x < 1$

8
$$x \le -2 \text{ or } x \ge 3$$
 (i) $0 < x \le e^{-2} \text{ or } x \ge e^{3}$ (ii) $x \ge \ln 3$

- 9 { $x: x \in \mathbb{R}, -4.70 \le x \le -0.822 \text{ or } x \ge 0.518$ } 0.822 $\le x \le 4.70 \text{ or } x \le -0.518$
- **10** k > 9/2
- no. of adult tickets = 150, no. of student tickets = 80, no. of children tickets = 50
- 12 x = 72, y = 60, z = 53 Mr Tan pays \$541

ACJC 2015 CA1 [24 April 2015]

45 minutes

- 1 Find the values of k for which the equation $4x^2 + 3 = 2kx$ has two real distinct roots, giving your answer in surd form. [3]
- 2 The magnitude of an earthquake is defined to be

$$M = \lg\left(\frac{I}{S}\right)$$

where I is the intensity of the earthquake and S is a constant (known as the "intensity of a standard earthquake").

The value of *M* was 8.3 in an earthquake in Alaska.

- (a) Given that an earthquake in Iceland was four times the intensity of the earthquake in Alaska, find the magnitude of the earthquake in Iceland, giving your answer correct to one decimal place.
 [3]
- (b) Given that an earthquake in Italy had a magnitude of 7.1, find the ratio of the intensity of the earthquake in Italy to the earthquake in Alaska, given your answer in exact form. [3]
- 3 On a single diagram, sketch the graphs of $y = 3 \left(\frac{1}{2}\right)^x$ and $y = \frac{3x + 13}{x + 4}$, indicating clearly the

coordinates of all intersections with the axes and the equations of any asymptotes. [4] Hence find the range of values of *x* that satisfy the inequality

$$\frac{3x+13}{x+4} - 3 + \left(\frac{1}{2}\right)^x \ge 0.$$
 [3]

- 4 On a single diagram, sketch the graphs of y = x 2a and $y^2 = ax$, where a > 0. Indicate clearly the coordinates of any intersections with the axes on the diagram. [2]
 - (i) Given that the graphs of y = x 2a and $y^2 = ax$ intersect at the points A and B, show that the length of AB is $3a \sqrt{2}$ units. [5]
 - (ii) Given that *C* is the mid-point of *A* and *B*, find the equation of the circle which has centre *C* and diameter *AB*. [2]

- 1 $k < -2\sqrt{3} \text{ or } k > 2\sqrt{3}$
- 2 (a) 8.9 (b) Ratio is $I : I_A = 10^{-1.2} : 1 \text{ or } 1 : 10^{1.2}$
- 3 Intersections at (-1.58, 0), (0, 2), y = 3; Intersections at (-13/3, 0), (0, 13/4), x = -4 y = 3 $x \le -4.06$ or x > -4

4 (ii)
$$\left(x - \frac{5}{2}a\right)^2 + \left(y - \frac{1}{2}a\right)^2 = \frac{9}{2}a^2$$

ACJC 2016 CA1 [21 April 2016]

45 minutes

- 1 The curve *C* has equation $y = \frac{2x-3}{x+3}$. Sketch the graph of *C*, stating the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]
- 2 Given that $e^{2x} + 2e^2 = 3e^{x+1}$, use the substitution $u = e^x$ to find the exact value(s) of x. [4]
- **3** Find the range of values of k for which $kx^2 + k + 3 > 4x$ for all real values of x. [4]
- 4 Concert tickets are sold at three different prices, depending on the age of the customer. The age categories are under 12 years, between 12 and 55 years, and over 55 years. Four extended families Tan, Chan, Lee and Lim, go to the concert on the same day. The numbers in each age category for each family, together with the total cost of the tickets for each family, except the Lim family, are given in the following table.

Family	Under 12 years	Between 12 and 55 years	Over 55 years	Total cost
Tan	10	4	5	\$320.00
Chan	9	6	4	\$352.50
Lee	7	5	3	\$282.50
Lim	5	10	5	?

Write down and solve equations to find the total cost of the tickets for the Lim family. [4]

- 5 In a bacteria culture *t* hours after the start of an experiment, the number of bacteria, *n* units, in the culture is given by $n = Ae^{1.5t}$. Find
 - (i) the number of bacteria in the culture at the start of the experiment; [1]
 - (ii) the value of t when the bacteria colony is 50 times its initial size. [3]
- 6 (i) On a single diagram, sketch the graphs of $y = 2 (0.5)^x$ and $y = \ln(x-2)$, stating clearly the equations of any asymptotes and the coordinates of any points of intersection with the axes. [4]
 - (ii) Find the x-coordinate of the point of intersection of $y = 2 (0.5)^x$ and $y = \ln(x-2)$. [1]
 - (iii) Hence solve $2 (0.5)^x < \ln(x-2)$.

Answers

2 x = 1 or $x = 1 + \ln 2$

- **3** k > 1
- **4** \$437.50
- **5** (i) *A* (ii) 2.61 (3 sf)
- 6 (iii) x > 9.38 (3 sf)

[1]

<u>ACJC 2017 CA1 [21 April 2017]</u>

45 minutes

[4]

[1]

[1]

[2]

[1]

1 Find algebraically, the set of values of *k* for which

$$kx^2 + (k-2)x + k > 0$$

for all real values of *x*.

2 Without using a calculator, solve the following inequality exactly

$$(\ln x)^2 + \ln x^2 - 3 \ge 0.$$
 [5]

3 In a business, it is often important to find a company's break-even point as it is the point where a company's total cost is equal to its total revenue. A company that sells closed-circuit television (CCTV) finds that if it sells x CCTV per day, then its total cost per day, in dollars, will be C(x) = 120x + q. Its total revenue per day, in dollars, will be $R(x) = -2x^2 + 400x$. Find the set of values of *a* such that the company will never break even. [3]

4 In Singapore, the acceptable limit of blood alcohol content is 0.08%, beyond which a driver can be convicted of drink driving. The function

 $A(x) = 0.003631x^3 - 0.03746x^2 + 0.1012x + 0.009$ for $0 \le x \le 5$,

- gives the blood alcohol content in a man, x hours after drinking alcohol on an empty stomach.
- Sketch the graph of y = A(x) for $0 \le x \le 5$, with coordinates of the endpoints clearly **(i)** shown. [1]
- Find the time when the blood alcohol content is at a maximum. (ii)
- (iii) Estimate the interval of time for which this man exceeded the legal blood alcohol content. [2]
- 5 In a chemical reaction, the mass *m*, in grams, of a chemical after *t* minutes is modelled by the equation $m = (2 - e^{-0.1t})^2$.
 - What is the mass of chemical in the long term? **(i)**
 - **(ii)** Find the time when the mass is 2.56 grams exactly.
 - Sketch the graph of *m* against *t* stating the equations of any asymptote(s) and the (iii) coordinates of any point(s) of intersection with the axes. [2]
- Sketch the curve with equation $y = \frac{0.7x}{100 x}$ stating the equations of any asymptotes and **(a)** 6
 - the coordinates of any point(s) of intersection with the axes. [3] The cost y, in millions of dollars, to remove x percent of a given pollutant is given by **(b)** the model $y = \frac{0.7x}{100 - x}$ for $x \ge 0$. Using this model, explain whether it is possible to
 - remove all the pollutant.

Answers

 $k > \frac{2}{2}$ 1 $0 < x \le e^{-3}$ or $x \ge e$ 2 5(iii) q > 98003 m^{\prime} m = 44 (ii) 1.85 h (iii) 1.11 < x < 2.735 (i) 4 g (ii) 10 ln (2.5) 6 No (0.1)

t

ACJC 2018 CA1 [26 April 2018]

45 minutes

[3]

- 1 Solve the equation $3^{x} 6(3^{-x}) = 5$, giving your answer in exact form. [4]
- 2 The quadratic curve with equation $y = (k+2)x^2 + kx + 5$ has a minimum point and intersects the line y = x+4 at two distinct points. Find the range of values of k. [4]
- 3 A dessert shop is promoting its Chocolate, Strawberry and Vanilla ice cream at \$16, \$14 and \$12 per tub respectively. Susan is planning an event with a budget of \$860 to buy 60 tubs of ice cream. The number of tubs of Vanilla ice cream should exceed that of Strawberry ice cream by one-third the number of tubs of Chocolate ice cream.
 - (i) Find the number of tubs of each flavour that Susan should order.
 - (ii) Susan is offered a membership at a one-time cost of \$10. As a member, she is entitled to buy the Chocolate, Strawberry and Vanilla ice cream at a 20%, 10% and 5% discount per tub respectively. Calculate the amount that she saves if she decides to purchase the membership and order the same number of tubs of each flavour as your answer in (i). [2]
- 4 A curve C has equation $y = \frac{x+1}{2x-1}$.
 - (i) Sketch the graph of *C*, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]
 - (ii) By sketching a suitable graph on the same diagram, solve $\frac{x+1}{2x-1} \le x$, leaving your answer in exact surd form. [4]
- 5 A pot of soup is left to cool after being heated to a temperature of 98° C. The temperature T° C of the pot of soup, *t* minutes after being removed from the heat, is given by

$$T=30+a\mathrm{e}^{-bt},$$

- where a and b are positive constants.
- (i) Calculate the value of a. [1]
 (ii) If the pot of soup cooled to 63°C after 12 minutes, find the value of b. [2]
- (iii) Sketch the graph of T against t, stating the equation(s) of any asymptote(s) and coordinates of any intersection with the axes. [2]
- (iv) What is the temperature of the pot of soup in the long run? [1]

1
$$x = \frac{\ln 6}{\ln 3}$$

2 $-2 < k < -1 \text{ or } k > 7$
3 $c = 30, s = 10, v = 20, \112
4 $\frac{1 - \sqrt{3}}{2} \le x < \frac{1}{2} \text{ or } x \ge \frac{1 + \sqrt{3}}{2}$
5 (i) $a = 68$ (ii) $b = -\frac{1}{12} \ln \frac{33}{68} \text{ or } 0.0603 \text{ (3sf)}$ (iv) 30°C

ACJC 2019 CA1 [9 May 2019]

45 minutes

[3]

[2]

1 Without using a calculator, solve the equation

$$2\log_2(x+3) - \log_2(1+x) = 3.$$
 [3]

2 Find, algebraically, the greatest integer value of *k* for which

$$-3x^2 + kx - 4 < 0$$

for all real values of x.

3 Alice, Mariam and Chandra went separately to the Popular book store near their school to purchase supplies for making decorations for their respective CCA booths during the school open house. All purchases at the store are subjected to 7% goods and services tax (GST).

Alice purchased three sheets of craft paper, four markers and five glue sticks. Her final bill was \$26.75 inclusive of GST. Mariam bought six sheets of craft paper, five markers and two glue sticks. She was given a 20% discount on her total purchase as she holds a Gold membership card. Her final bill was \$26.12 inclusive of GST. Chandra bought three sheets of craft paper, two markers and one glue stick. He enjoyed a 10% discount due to his Ordinary membership card. He paid a total of \$12.91 inclusive of GST. Determine the unit cost of each of the items purchased. [4]

4 The curve *C* has equation $y = \frac{a(1-2x)}{x+b}$, where *a* and *b* are constants.

The lines y = 4 and x = 3 are asymptotes of the curve *C*.

- (i) Find the values of *a* and *b*.
- (ii) Sketch the graph of *C*, stating the coordinates of any points of intersection with the axes and the equations of any asymptotes. [2]
- (iii) By sketching a suitable graph on the same diagram, solve $\frac{a(1-2x)}{x+b} < x$. [2]
- 5 A line, *L* has gradient *m* and passes through the point (0, -3). Given that *L* is tangential to the curve with equation $y = -x^2 + 3x 28$, find the possible values of *m*. [4]
- 6 The concentration of a certain drug in a patient's body can be modelled by an exponential decay model given by

$$F(t) = 2e^{-0.5t}$$

5

6

where F(t) represents the concentration of the drug in mg/ml (milligrams per milliliters) at time *t* hours after the drug is administered to the patient.

- (i) Find the initial concentration of drug in a patient's body and the time taken for this concentration to be halved. Give you answer to the nearest minute. [3]
- (ii) Sketch the graph of F(t) against t, stating the equation(s) of any asymptote(s) and coordinates of any intersection with the axes. [2]

- 1 x = 12 k = 63 x = \$1.65, y = \$3.70, z = \$1.05
- 4 (i) a = -2, b = -3(iii) 0.298 < x < 3 or x > 6.70



ACJC 2020 Quiz 1 [9 June 2020]

1 Find the range of values of *k* such that

45 minutes

$$x^2 + 2kx + k^2 > x.$$
 [3]

2 Using an algebraic method, solve the inequality $5-6x^2 < 7x$. [3] Hence find the exact range of values of x such that $5-6e^{2x} < 7e^x$. [2]

3 Sketch, on the same diagram, the graphs of
$$y = x+3$$
 and $y = \frac{5-2x}{x-2}$, stating the coordinates of any intersection with the axes, and the equation of any asymptote. [3]

Hence solve the inequality

$$\frac{1}{x-2} > x+5$$
. [2]

4 (a) A Cafe sells cakes in three flavours: original, chocolate and salted egg-yolk. The amount of sugar required to bake each original, chocolate and salted egg-yolk cake is 150 g, 80 g and 50 g respectively. Every day, the café uses 15 kg of sugar to bake a total of 150 cakes, of which only a maximum of 20 is of the salted egg-yolk flavour, while at least 50 of them are of the original flavour.

Find the number of chocolate cakes ACafe bakes every day. [4]

Given that each original, chocolate and salted egg-yolk cake is sold for \$2.80, \$3.50 and \$5 respectively, calculate ACafe's total cake sales when it sells out of cakes for the day. [2]

(b) The daily profit from ACafe's sale of cakes, *P* (in hundreds of dollars), can be modelled by the equation

$$P = 5 \ln (c+1) - 0.0001 c^2, c \ge 0$$

where c (in dollars) is the cost of fixed overheads like ingredients used to make the cakes, as well as rent.

- (i) Sketch the graph of P against c. [1]
- (ii) Suggest a reason why *P* initially increases with *c*. [1]
- (iii) State ACafe's maximum daily profit based on this model, and the corresponding cost of fixed overheads. [2]
- (iv) By adding a suitable graph on the same diagram in (i), find the range of c such that ACafé's daily profit is more than \$2100. [2]

1
$$k > \frac{1}{4}$$
 2 $x < -\frac{5}{3}$ or $x > \frac{1}{2}$, $x > -\ln 2$

- 3 x < -5.14 or 2 < x < 2.14
- **4** (a) 80, \$157.80 (b) (iii) \$22.80, c = \$157.60 (iv) 73.238 < c < 261.89

ACJC 2021 CA1 [11 May 2021]

45 minutes

- 1 The curve $y = (k-6)x^2 8x + 1$ cuts the x-axis at two distinct points and has a minimum point. Find the range of values of k. [3]
- 2 Solve the equation

$$\ln\left(\frac{e^{2x}-e^x}{6}\right)=0,$$

leaving your answer in exact form.

3 A bubble tea shop sells three flavours of tea: peach, milk and green. The table below shows the number of cups of each flavour of tea sold, and the total revenue, for two particular days.

	peach tea (cups)	milk tea (cups)	green tea (cups)	total revenue (\$)
Day 1	45	36	55	244.50
Day 2	55	46	50	276.50

On its anniversary, the shop gave a discount of 25%, 20% and 10% on the peach, milk, and green tea respectively. The total revenue from the sale of 85 cups of peach tea, 66 cups of milk tea and 60 cups of green tea that day was \$313.65.

Find the original price of each flavour of tea.

- 4 An air freshener that emits fragrance at a rate of $x \text{ m}^3$ per minute is installed in a new office unit with a space of 105 m³.
 - (i) Find the time for the fragrance to fill the entire room, leaving your answer in minutes, in terms of *x*.

If the air freshener is faulty, it emits fragrance at a rate of $(x-25)m^3$ per minute.

- (ii) Given that it takes 2 minutes longer for the fragrance to fill the entire room when it is faulty, show that $2x^2 50x 2625 = 0$. [2]
- (iii) Hence, find the time it takes for the fragrance to fill the entire room when the air freshener is faulty, giving your answer to the nearest minute. [1]

[3]

[4]

- 5 The curve C_1 has equation $y = \frac{16x-1}{1-4x}$.
 - (i) Sketch C_1 , stating the coordinates of any points of intersection with the axes and the equation(s) of any asymptote(s).



(ii) By adding a suitable curve C_2 on the same diagram as (i) above, solve the equation

$$\ln(4-x) = \frac{16x-1}{1-4x}.$$
[3]

6 A scientist is investigating a certain species of birds with a population of n thousand at t days. The scientist proposes that n can be modelled by the equation

$$n = \frac{65}{e^{-0.5t} + 1}.$$

- (i) Find the initial size of the population.
- (ii) Sketch the graph of *n* against *t*, stating the coordinates of any point(s) of intersection with the axes. [1]
- (iii) What is the approximate size of the population for large values of t? [1]
- (iv) Find the number of days for the population to first exceed 64000. [2]

Answers
1
$$6 < k < 22$$

2 $x = \ln 3$
3 $P = \$2.20, M = \$1.75, G = \$1.50$
4 (i) $\frac{105}{x}$ (iii) 4 (nearest minute)
5 (i) $y = \ln(4 - x)$
 $y = \frac{16x - 1}{1 = 4x}$
(ii) 0.110 and 3.99
(iii) 0.110 and 3.99
2 $x = \ln 3$
4 (i) $\frac{105}{x}$ (iii) 4 (nearest minute)
6 (i) 32500
(ii) n
(0, 32.5)
0
(0, 32.5)
0
(iii) 65000 (iv) 9 days

[2]

ACJC 2022 CA1 [9 May 2022]

45 minutes

[1]

[3]

[2]

- 1 A manager ordered three different types of set meals A, B and C for his employees. He paid a total of \$616 for 12 sets of A, 10 sets of B and 8 sets of C. He found that twice the amount he spent on sets C was \$248 less than three times the amount he spent on sets A. He also realised that he would have spent another \$63 if he were to order one more of each of the 3 set meals A, B and C. Find the price of set meal A. [4]
- 2 Find the set of values of k for which the line y=3x-4k does not intersect the curve $v = kx^2 + kx$. [4] Find the set of values of k for which $kx^2 + kx > 3x - 4k$ for all real values of x. [1]
- 3 A glass of boiling water at 100°C is placed in a room where the temperature is constant at 25° C. The temperature, θ° C, of the water after *t* minutes is given by $\theta = 25 + Ae^{-kt}$.
 - (i) Show that A = 75.
 - After 10 minutes, the water temperature is 50°C.
 - (ii) Find the exact value of *k*.
 - (iii) Sketch the graph of θ against t, stating the exact coordinates of the points of intersection with the axes and the equations of any asymptotes. [2]
- The curve C defined by $y = \frac{4x+5}{ax+b}$ has asymptote x = -2 and passes through the point with 4

coordinates (1, 1).

- (i) Show that b = 2a and hence show that a = 3.
- Sketch the curve C, stating the coordinates of the points of intersection with the axes (ii) and the equations of the asymptotes. [3]

Another curve C' has equation $y = 2x + \ln(3-x)$.

- (iii) Sketch C' on the same diagram, stating the coordinates of the points of intersection with the axes and the equation of the asymptote. [3]
- (iv) By considering the graphs of C and C', find the positive root of the equation $4x+5=2x(3x+6)+(3x+6)\ln(3-x)$, giving your answer correct to three decimal places. [2]

Answers

1 \$18

$$k < -1$$
 or $k > \frac{3}{5}, k > \frac{3}{5}$.

3 (ii)
$$k = -\frac{1}{10} \ln\left(\frac{1}{3}\right) \left(\operatorname{or} \frac{1}{10} \ln 3\right)$$

2
$$k < -1$$
 or $k > \frac{5}{5}, k > \frac{5}{5}$
4 (iv) 2.992