Additional Mathematics Paper 1 (70 marks)

Qn. #	Solution	Mark Allocation
1	$54x^3+128$ $=2(27x^3+64)$ $=2\left[(3x)^3+4^3\right]$ $=2(3x+4)\left[(3x)^2-(3x)(4)+4^2\right]$ $=2(3x+4)(9x^2-12x+16)$	B1: $27x^3=(3x)^3$ B1: $64=4^3$ B1: All correct
2i	$x^2+4kx+1-3k=0$ For two real and distinct roots, discriminant >0 $(4k)^2-4(1)(1-3k)>0$ $16k^2-4+12k>0$ $4(4k^2+3k-1)>0$ $4k^2+3k-1>0$ (shown)	M1 A1
2ii	$4k^2+3k-1>0$ $(4k-1)(k+1)>0$ $k<-1$ or $k>\dfrac{1}{4}$	M1 A1
3i	Period $=\dfrac{2\pi}{\frac{1}{2}}$ $=4\pi$ Amplitude = 4	B1 B1

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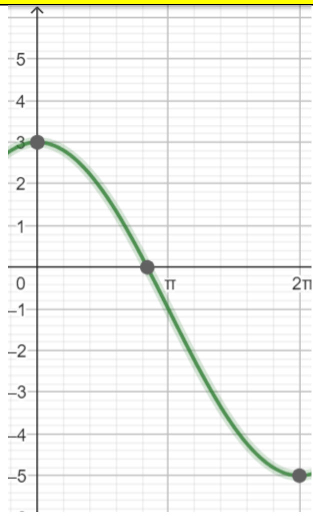
3ii		B1: Correct cycle B1: Correct $(\pi, -1)$ B1: Correct maximum and minimum points
4i	$\frac{dy}{dx} = 2(x^2+3)$ $y = \int 2(x^2+3)\ dx$ $= \frac{2}{3}x^3 + 6x + c$ At (1, 12), $12 = \frac{2}{3}(1)^3 + 6(1) + c$ $c = 5\frac{1}{3}$ $\therefore y = \frac{2}{3}x^3 + 6x + 5\frac{1}{3}$	M1: Attempt to integrate A1 M1: Substitution to obtain arbitrary constant A1
4ii	$\frac{dy}{dx} = 2x^2 + 6$ Since $x^2 \geq 0,\ 2x^2 \geq 0,\ 2x^2+6 > 0,\ \frac{dy}{dx} \neq 0$, therefore this curve has no stationary point.	B1: Show $\frac{dy}{dx} \neq 0$
4iii	$\frac{dy}{dx} = 2x^2 + 6$ Since $x^2 \geq 0,\ 2x^2 \geq 0,\ 2x^2+6 > 0,\ \frac{dy}{dx} > 0$, therefore $y = \mathrm{P}(x)$ is an increasing function.	B1: Show $\frac{dy}{dx} > 0$

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5a	$\sin 30° \cos 45° + \cos 30° \sin 45°$	
	$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$	M1
	$= \frac{\sqrt{2}+\sqrt{6}}{4}$	A1
5b	$\cos\left(2x+\frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$	
	Reference angle $= \cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$	M1
	$2x+\frac{\pi}{3} = \frac{\pi}{4},\ 2\pi-\frac{\pi}{4},\ \frac{\pi}{4}+2\pi,\ \left(2\pi-\frac{\pi}{4}\right)+2\pi$	
	$2x = -\frac{\pi}{12}$ (N.A.), $\frac{17\pi}{12},\ \frac{23\pi}{12},\ \frac{41\pi}{12}$ (N.A.)	M1
	$\therefore x = \frac{17}{24}\pi,\ \frac{23}{24}\pi$	A2
6i	$\frac{d}{dx}\left[\frac{1}{(1-x^2)^3}\right] = \frac{d}{dx}(1-x^2)^{-3}$	
	$= -3(1-x^2)^{-4}(-2x)$	M1
	$= \frac{6x}{(1-x^2)^4}$	A1
6ii	$\int_0^2 \frac{6x}{(1-x^2)^4}\,dx = \left[\frac{1}{(1-x^2)^3}\right]_0^2$	M1
	$6\int_0^2 \frac{x}{(1-x^2)^4}\,dx = \left[\frac{1}{(1-x^2)^3}\right]_0^2$	
	$\int_0^2 \frac{x}{(1-x^2)^4}\,dx = \frac{1}{6}\left[\frac{1}{(1-x^2)^3}\right]_0^2$	M1
	$= \frac{1}{6}\left[\frac{1}{(1-2^2)^3} - \frac{1}{(1-0^2)^3}\right]$	M1: Substitute limits

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	$=\frac{1}{6}\left[-\frac{1}{27}-1\right]$ $=-\frac{14}{81}$	A1
7i	$a=\frac{2.7-0.1}{2}=1.3$ $b=\frac{2\pi}{12}=\frac{\pi}{6}$ $c=\frac{0.1+2.7}{2}=1.4$	B1 M1, A1 B1
7ii	$y=1.3\sin\left(\frac{\pi}{6}t\right)+1.4$ $y=1.3\sin\left(\frac{\pi}{6}\times 16\right)+1.4$ $=2.53\text{ m}$	M1 A1
7iii	$0\le t\le 6$	B1
8a	$\frac{\sqrt{3}+2}{5\sqrt{3}-1}=\frac{\sqrt{3}+2}{5\sqrt{3}-1}\times\frac{5\sqrt{3}+1}{5\sqrt{3}+1}$ $=\frac{\left(\sqrt{3}+2\right)\left(5\sqrt{3}+1\right)}{\left(5\sqrt{3}\right)^2-1^2}$ $=\frac{15+\sqrt{3}+10\sqrt{3}+2}{\left(5\sqrt{3}\right)^2-1^2}$ $=\frac{17+11\sqrt{3}}{74}$	M1 B1: Correct numerator B1: Correct denominator A1
8b	$\sqrt{7}\left(x+2\right)=1-x$ $\sqrt{7}x+x=1-2\sqrt{7}$ $x\left(\sqrt{7}+1\right)=1-2\sqrt{7}$ $x=\frac{1-2\sqrt{7}}{\sqrt{7}+1}$ $=\frac{1-2\sqrt{7}}{\sqrt{7}+1}\times\frac{\sqrt{7}-1}{\sqrt{7}-1}$	M1: Rearrange to make *x* the subject M1: Attempt to rationalise

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	$=\dfrac{\sqrt{7}-1-14+2\sqrt{7}}{\left(\sqrt{7}\right)^2-1^2}$ $=\dfrac{-15+3\sqrt{7}}{6}$ $=-\dfrac{5}{2}+\dfrac{1}{2}\sqrt{7}$	M1: Numerator or denominator correct A1
9i	$\dfrac{d^2y}{dx^2}=-3$ $\dfrac{dy}{dx}=\int -3\ dx$ $=-3x+c$ At $(2,-1)$, $\dfrac{dy}{dx}=2$ $2=-3(2)+c$ $c=8$ $\dfrac{dy}{dx}=-3x+8$ $y=\int -3x+8\ dx$ $=\dfrac{-3x^2}{2}+8x+d$ At $(2,-1)$, $-1=\dfrac{-3(2)^2}{2}+8(2)+d$ $d=-11$ $y=-\dfrac{3}{2}x^2+8x-11$	M1 M1: Substitution to obtain arbitrary constant A1 M1 M1: Substitution to obtain arbitrary constant A1
9ii	Discriminant $=8^2-4\left(-\dfrac{3}{2}\right)(-11)$ $=-2<0$ Since discriminant is less than zero, $y=-\dfrac{3}{2}x^2+8x-11$ has no real roots, therefore the curve does not intersect the x-axis.	M1: Find discriminant A1

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10i	$A=8^2-\frac{1}{2}\times 8\times(8-k)-\frac{1}{2}\times 8\times(8-k)-\frac{1}{2}\times k\times k$ $=64-2\left[4(8-k)\right]-\frac{1}{2}k^2$ $=64-64+8k-\frac{1}{2}k^2$ $=8k-\frac{1}{2}k^2$ (shown)	M1: Find area of triangle *PQX* or *PSY* M1: Area of square subtract 3 triangles A1
10ii	$\frac{\mathrm{d}A}{\mathrm{d}k}=8-k$ For stationary value of *A*, $\frac{\mathrm{d}A}{\mathrm{d}k}=0$ $8-k=0$ $k=8$ $A=8(8)-\frac{1}{2}(8)^2$ $A=32$	M1 M1 A1 M1 A1
10iii	$\frac{\mathrm{d}^2A}{\mathrm{d}k^2}=-1$ Since $\frac{\mathrm{d}^2A}{\mathrm{d}k^2}<0$, *A* is a maximum value.	B1
11i	$x^2+y^2-6x+8y=39$ $x^2+y^2-6x+8y-39=0$ $2g=-6$ $g=-3$ $2f=8$ $f=4$ Centre of circle: $(3,-4)$ Radius $=\sqrt{(-3)^2+4^2-(-39)}$ = 8 units	 M1 A1 M1 A1
11ii	At $A(-5,-4)$, $LHS=(-5)^2+(-4)^2-6(-5)+8(-4)$ $=39$ $=RHS$	B1

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	Therefore, $x=-5$ and $y=-4$ satisfy the equation of circle and point $A(-5,-4)$ lies on the circle. At $B(11,-4)$, $LHS=(11)^2+(-4)^2-6(11)+8(-4)$ $=39$ $=RHS$ Therefore, $x=11$ and $y=-4$ satisfy the equation of circle and point $B(11,-4)$ lies on the circle.	B1
	Length of $AB=\sqrt{[11-(-5)]^2+[-4-(-4)]^2}$ $=\sqrt{256}$ $=16$ units	M1
	Since the length of *AB* is $2\times$ the radius of the circle, and the points $A(-5,-4)$ and $B(11,-4)$ lie on the circle, *AB* is a diameter of the circle	A1