Kent Ridge Secondary School
Secondary 4 Express/5 Normal Academic Preliminary Examination 2024
Add Math Prelim 2024 P1 Mark scheme

Qn	Solutions	Marks
1a	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$ $1 = \frac{2\tan A}{1-\tan^2 A}$ $1-\tan^2 A = 2\tan A$ $\tan^2 A + 2\tan A - 1 = 0$ $\tan A = \frac{-2 \pm \sqrt{4+4}}{2}$ $= -1+\sqrt{2}$	M1 A1 M1 A1
1b	$\sec^2 A = 1+\tan^2 A$ $= 1+\left(-1+\sqrt{2}\right)^2$ $= 1+1-2\sqrt{2}+2$ $= 4-2\sqrt{2}$	M1 A1
2a	Gradient $L_1 = -2$ Gradient $L_2 = \frac{1}{2}$ $-3 = \frac{1}{2}(2)+c$ $c = -4$ $B(0,-4)$	M1 M1 A1
2b	$(4-2)^2+(k+3)^2 = 25$ $(k+3)^2 = 21$ $k = \pm\sqrt{21}-3$	M1 M1 or apply quad formula to general eqn A1
3a	Min -2 and max 2	B1
3b	Min -3 + 1 = -2, max = 3+1 = 4	B1
3c	f(x) period or shape correct g(x) period or shape correct f(x) fully correct g(x) fully correct	B1 B1 B1 B1
3d	3	B1
4a	$(a-x^2)^6 = a^6 - 6a^5x^2 + \binom{6}{2}a^4x^4 + \ldots$ $= a^6 - 6a^5x^2 + 15a^4x^4 + \ldots$	B1,B1,B1
4b	$\left(\frac{1}{x^2}+2+x^2\right)(a-x^2)^6$ $\left(\frac{1}{x^2}+2+x^2\right)(a^6-6a^5x^2+15a^4x^4)$ Term independent of x: $-6a^5+2a^6 = 0$ $-6+2a = 0$ $a = 3$ Term in x^2: $b = 15a^4 - 12a^5 + a^6 = -972$	M1 M1 A1 M1,A1
5a	$f'(x) = \frac{(x-3)-(x+1)}{(x-3)^2}$	M1

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	$$f'(x) = \frac{-4}{(x-3)^2}$$ Since $(x-3)^2 > 0, x \neq 3$ **$f'(x)$ is always negative or** **Gradient of f is always negative**, so it is a decreasing function	A1 B1 B1
5b	$$f(x) = 1 + \frac{4}{x-3}$$ $$\int 1 + \frac{4}{x-3}\ dx = x + 4\ln(x-3) + c$$	M1 M1, M1, A1
6a	75°C	B1
6b	$75e^{-0.02t} = 65$ $e^{-0.02t} = \frac{65}{75}$ $-0.02t = \ln\left(\frac{65}{75}\right)$ $t = \frac{\ln\left(\frac{65}{75}\right)}{-0.02} = 7.16$ min	 M1 M1 A1
6c	$63e^{15k} = 54.2$ $k = \frac{\ln\left(\frac{54.2}{63}\right)}{15} = -0.01003 = -0.01$	M1 M1, A1
6d	$$75e^{-0.02t} = 63e^{-0.01t}$$ $$\frac{e^{-0.02t}}{e^{-0.01t}} = \frac{63}{75}$$ $$e^{-0.01t} = \frac{63}{75}$$ $t = \frac{\ln\left(\frac{63}{75}\right)}{-0.01} = 17.4$ minutes	M1 M1 A1
7a	$$x\left(\frac{1}{x}\right) + \ln x$$ $$= 1 + \ln x$$	M1 – diff ln x correctly A1
7b	$$f(x) = \ln x - x + c$$ $$0 = \ln 1 - 1 + c$$ $$c = 1$$ $$f(x) = \ln x - x + 1$$	M1 M1 A1
8a	$2x^2 - 8x = -x^2 - 4x - 3$ $3x^2 - 4x + 3 = 0$ Discriminant = $(-4)^2 - 4(3)(3) = -20$ Since **discriminant < 0, there are no real roots** to the simultaneous equations. The 2 curves do not intersect	M1 M1 A1
8b	$$2(x^2 - 4x) = 2[(x-2)^2 - 4]$$ $$= 2(x-2)^2 - 8$$ $$-(x^2 + 4x + 3) = -[(x+2)^2 - 1]$$ $$= -(x+2)^2 + 1$$ Sketch min curve with TP (2,-8) passing through O Sketch max curve with TP (-2,1) passing through (0,-3) Non intersecting	M1 M1 A1 B1 B1

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9a	$$\frac{r}{x}=\frac{5}{25}$$ $$r=\frac{x}{5}$$ Volume of liquid $=\frac{1}{3}\pi(5)^2(25)-\frac{1}{3}\pi\left(\frac{x}{5}\right)^2(x)$ $=\frac{1}{3}\pi(625-\frac{x^3}{25})$	M1 B1
9b	$\frac{dV}{dx}=\frac{1}{3}\pi(-\frac{3x^2}{25})$ $\frac{dV}{dt}=\frac{dV}{dx}\times\frac{dx}{dt}$ $=\frac{1}{3}\pi(-\frac{3x^2}{25})\left(-\frac{1}{2}t\right)$ $$x=\int-\frac{1}{2}t\,dt$$ $$x=-\frac{1}{2}\left(\frac{t^2}{2}\right)+c$$ Sub t = 0, x = 25 $$c=25$$ $$x=-\frac{t^2}{4}+25$$ Sub x = 2 $$-\frac{t^2}{4}+25=2$$ $$t^2=92$$ $$t=\sqrt{92}$$ $\frac{dV}{dt}=\frac{1}{3}\pi\left(-\frac{3(2)^2}{25}\right)\left(-\frac{1}{2}\sqrt{92}\right)=2.41\ cm^3/s$	M1 M1 M1 A1 A1 A1
10a	$$BC=8\sin\theta$$ $$AC=8\cos\theta$$ Area $=\frac{1}{2}(8\sin\theta)(8\cos\theta)$ $=32\sin\theta\cos\theta$ $=16\sin 2\theta$	M1 – either BC or AC found M1 A1
10b	Max = 16	B1
10c	Perimeter = AB + BC + CA = $8+8\sin\theta+8\cos\theta$ $$=8(1+\sin\theta+\cos\theta)$$	B1
10d	$$\sin\theta+\cos\theta=\sqrt{2}\sin(\theta+45°)$$ Max perimeter = $8(1+\sqrt{2})$	B1, B1 B1
11(a)	$f(0)=-4+b=0$ $$b=4$$ $$f(3)=\frac{1}{3}(3)^3+a(3)^2-20(3)-4+4=-51$$ $$a=0$$	M1 A1 M1 A1
11(b)	$$f(x)=\frac{1}{3}x^3-20x$$ $$f'(x)=x^2-20=0$$ $$x=\pm\sqrt{20}=\pm2\sqrt{5}$$	 M1 A1

Qn	Solutions	Marks
	$f''(x) = 2x$ When $x = 2\sqrt{5}$, $f''(x) > 0$, it is a minimum point When $x = -2\sqrt{5}$, $f''(x) < 0$, it is a maximum point	M1 A1 A1
12(a)	$\frac{\sin A \cos B - \cos A \sin B}{\sin A \cos B + \cos A \sin B} = \frac{2}{3}$ $3 \sin A \cos B - 3 \cos A \sin B$ $= 2 \sin A \cos B + 2 \cos A \sin B$ $\sin A \cos B = 5 \cos A \sin B$ $\frac{\sin A \cos B}{\cos A \cos B} = \frac{5 \cos A \sin B}{\cos A \cos B}$ $\tan A = 5 \tan B$	M1 M1 B1
12(b)	$\frac{\sin(45° - \theta)}{\sin(45° + \theta)} = \frac{2}{3}$ $A = 45°, B = \theta$ $\tan 45° = 5 \tan \theta$ $\tan \theta = \frac{1}{5}$ $\alpha = \tan^{-1}\left(\frac{1}{5}\right) = 11.30°$ $\theta = 11.30, 180 + 11.30 = 11.3°, 191.3°$	M1 M1 A1 A1