(b) Given that $\log_4 xy = 7$ and $\dfrac{\log_4 x}{\log_4 y} = -8$, find the value of $\log_4 y$.

Hence, evaluate $\log_4 \dfrac{2x}{y^3}$.

[6]

$$\left[\begin{array}{l}\log_4 y = -1, \\ \log_4 \dfrac{2x}{y^3} = 11.5\end{array}\right]$$

2. The population of polar bears in the arctic is given by the formula $N = 8000(2+3e^{-\frac{t}{50}})$, where t is measured in years. Find

(i) the initial population, [1]

[40000]

(ii) the population after 50 years, [1]

[24800]

(iii) the least number of years it would take the population to exceed 20 000, [3]

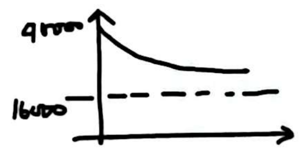
[90 years]

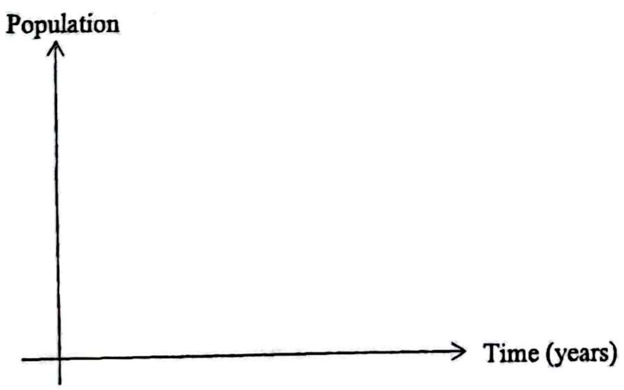
(iv) the rate at which the polar bears is decreasing when $t = 10$, [2]

[393]

(v) From the formula $N = 8000\left(2+3e^{-\frac{t}{50}}\right)$, explain why the population of the polar bears can never fall below 16000. [2]

(vi) Sketch the population-time curve in the grid below. [2]





3. Given that the expression $2x^3 + ax^2 + bx - 6$ is divisible by $(x^2 - 2)$, find the value of a and of b. [4]

$\left[\begin{array}{l} a = 3 \\ b = -4 \end{array}\right]$

Hence,

(i) solve the equation $2x^3 + ax^2 + bx - 6 = 0$, for the exact value(s) of x, [3]

$\left[x = \pm\sqrt{2}, -\frac{3}{2}\right]$

(ii) solve the equation $2 + ay + by^2 - 6y^3 = 0$. [2]

$\left[y = \pm\frac{\sqrt{2}}{2}, -\frac{2}{3}\right]$

4. (a) Solve $2\sin x = \tan x$ for which $0^\circ \le x \le 360^\circ$. [4]

$[0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ]$

(b) Find all the angles between 0 and 6 for which
$2\tan y(\tan y + 1) = 3(2 - \sec^2 y)$. [4]

$[0.540, 2.36, 3.68,$
$5.50, \frac{3\pi}{4}, \frac{7\pi}{4}]$

5. (a) Given that $\cos A = -\frac{3}{5}$ where $\pi < A < \frac{3\pi}{2}$. Find , without using a calculator, the value of

(i) $\sin(\pi - A)$, [1]

[$-\frac{4}{5}$]

(ii) $\cot A$, [1]

[$\frac{3}{4}$]

(iii) $\cos\frac{A}{2}$. [2]

[$-\frac{\sqrt{5}}{5}$]

(b) Prove the identity

$$\frac{2-\sec^2 x}{\sec^2 x+2\tan x}\equiv\frac{\cos x-\sin x}{\cos x+\sin x}.$$ [4]

8 Two points A and B have coordinates $(1, 2)$ and $(-3, 6)$ respectively. A point $P\,(x\,, y)$ is such that AP and BP are perpendicular.

(a) Show that P lies on the circumference of a circle. [2]

Find

(b) the coordinates of the centre of the circle and the radius of the circle, [4]

$\left[\begin{array}{l} C(-1,4) \\ R = 2\sqrt{2} \end{array}\right]$

(c) the equation of the circle. [1]

(d) Explain why the tangents to the circle at A and B are parallel. [2]

9 (a) A curve has the equation $y = \dfrac{2x+11}{x+1}, x \neq -1$. Show that y is a decreasing function. [3]

(b) Find the coordinates of the stationary point(s) of the curve $y = xe^{-2x}$ and determine the nature of the stationary point(s). [6]

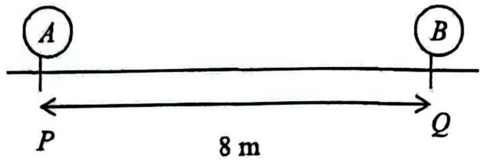
[max $\left(\frac{1}{2}, \frac{1}{2e}\right)$]

11 A particle A moves in a straight line, so that, t seconds after leaving a fixed point P, its velocity, v_A ms^{-1}, is given by $v_A = 2t - 6$.

(a) Find the distance travelled by the particle A before it comes to instantaneous rest. [3]

[9m]

A second particle B moves along the same horizontal line as A, and starts from Q, a point 8 m away from P, at the same instant that A begins to move. Particle B moves with a velocity of 5 ms^{-1} and decelerates at 1 ms^{-2}.



(b) Calculate the distance between particle A and particle B when A comes to instantaneous rest. [3]

[27.5m]

(c) Find the time during the interval $0 \leq t \leq 5$ when the distance between particle A and particle B is at its maximum. Calculate this maximum distance. [3]

$\left[t = \frac{11}{3}, \text{ distance} = 28\frac{1}{6}\text{ m}\right]$

(d) Find the range of values of t for which both particles are moving in the same direction. [1]

$[3 < t < 5]$

END OF PAPER