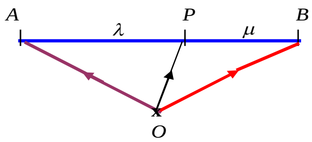
JC1 H2 Mathematics (9758)
Term 4 Revision Topical Quick Check
Chapter 5 Vectors
Chapter 6 3D Vector Geometry

Revision Guide Chapter 5 Page 2-3

Ratio Theorem



$$\overrightarrow{OP} = \frac{\mu\overrightarrow{OA} + \lambda\overrightarrow{OB}}{\mu + \lambda}$$

Unit Vector $\hat{\mathbf{v}}$

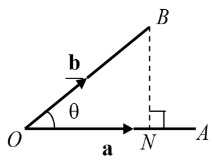
A unit vector is a vector whose magnitude is 1

$|\hat{\mathbf{v}}| = 1$ and $\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$

Scalar (Dot) Product

Definition: $\mathbf{a}\cdot\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \ (0^\circ \le \theta \le 180^\circ)$

Uses of Scalar Product



Proving two non-zero perpendicular vectors

$\mathbf{a} \perp \mathbf{b} \iff \mathbf{a}\cdot\mathbf{b} = 0$

Angle between Two Vectors

$\cos\theta = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}||\mathbf{b}|}$

Length of projection of b onto a:

$|\overline{ON}| = |\mathbf{b}\cdot\hat{\mathbf{a}}|$

Properties of scalar product:

1. $\mathbf{a}\cdot\mathbf{b} = \mathbf{b}\cdot\mathbf{a}$
2. $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c}) = \mathbf{a}\cdot\mathbf{b} + \mathbf{a}\cdot\mathbf{c}$
3. $\mathbf{a}\cdot\mathbf{a} = |\mathbf{a}|^2$
4. $\lambda\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\lambda\mathbf{b} = \lambda(\mathbf{a}\cdot\mathbf{b})$

Vector (Cross) Product

Definition: $\mathbf{a}\times\mathbf{b}=|\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$ $(0^\circ\le\theta\le180^\circ)$

So $|\mathbf{a}\times\mathbf{b}|=|\mathbf{a}||\mathbf{b}|\sin\theta$

Zero Vector

$\mathbf{a}\times\mathbf{b}=\mathbf{0}$

$\Rightarrow$ **a** is parallel to **b**

OR $\mathbf{a}=\mathbf{0}$

OR $\mathbf{b}=\mathbf{0}$

Normal Vector

$\mathbf{a}\times\mathbf{b}$ is vector perpendicular to both **a** and **b**

Properties of vector product:

1. $\mathbf{a}\times(\mathbf{c}+\mathbf{b})=\mathbf{a}\times\mathbf{c}+\mathbf{a}\times\mathbf{b}$
2. $\mathbf{c}\times\mathbf{c}=\mathbf{0}$ (zero vector)
3. $\mathbf{a}\times\mathbf{c}=-\mathbf{c}\times\mathbf{a}$
4. $\lambda\mathbf{a}\times\mathbf{c}=\mathbf{a}\times\lambda\mathbf{c}=\lambda(\mathbf{a}\times\mathbf{c})$

Let's Try Now!

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Relative to the origin *O*, the position vectors of points *A*, *B* and *C* are **a**, **b** and **c** respectively, where **b** is a unit vector. It is given that **b** and **b** − **a** are perpendicular and *C* lies on *AB* such that $AC:CB=3:1$.

(i) Show that $|\mathbf{a}|=\sec\theta$, where θ is the angle between **a** and **b**. [3]

By expressing **c** in terms of **a** and **b**,

(ii) find the value of $|\mathbf{c}\cdot\mathbf{b}|$ and state the geometrical interpretation of $|\mathbf{c}\cdot\mathbf{b}|$, [4]

(iii) find the value of $\dfrac{|\mathbf{b}\times\mathbf{c}|}{|\mathbf{a}\times\mathbf{b}|}$. [2]

(i)

<u>Method ①:</u>

Let θ be the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

Since $\underset{\sim}{b}$ and $\underset{\sim}{b}-\underset{\sim}{a}$ are perpendicular,

$$\underset{\sim}{b}\cdot\left(\underset{\sim}{b}-\underset{\sim}{a}\right)=0$$

$$\underset{\sim}{b}\cdot\underset{\sim}{b}-\underset{\sim}{b}\cdot\underset{\sim}{a}=0$$

$$\left|\underset{\sim}{b}\right|^2=\underset{\sim}{b}\cdot\underset{\sim}{a}=\underset{\sim}{a}\cdot\underset{\sim}{b}$$

Since $\underset{\sim}{b}$ is a unit vector, $\underset{\sim}{a}\cdot\underset{\sim}{b}=1$

$$\left|\underset{\sim}{a}\right|\left|\underset{\sim}{b}\right|\cos\theta=1$$

$$\left|\underset{\sim}{a}\right|\cos\theta=1$$

$$\left|\underset{\sim}{a}\right|=\frac{1}{\cos\theta}=\sec\theta$$

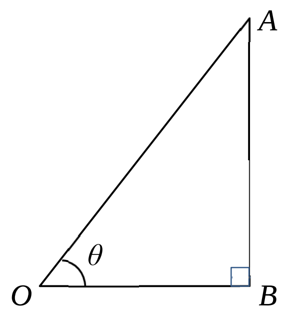
<u>Method ②:</u>

$$\cos\theta=\frac{OB}{OA}$$

$$=\frac{\left|\underset{\sim}{b}\right|}{\left|\underset{\sim}{a}\right|}$$

$$=\frac{1}{\left|\underset{\sim}{a}\right|}$$ since $\underset{\sim}{b}$ is a unit vector

$$\left|\underset{\sim}{a}\right|=\frac{1}{\cos\theta}=\sec\theta \qquad \text{(shown)}$$



(ii)

By Ratio Theorem,

$$\underset{\sim}{c}=\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}$$

$$\left|\underset{\sim}{c}\cdot\underset{\sim}{b}\right|=\left|\left(\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}\right)\cdot\underset{\sim}{b}\right|$$

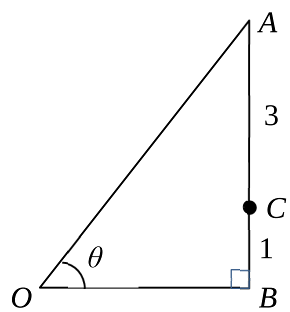
$$=\left|\frac{1}{4}\underset{\sim}{a}\cdot\underset{\sim}{b}+\frac{3}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}\right|$$

$$=\left|\frac{1}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}+\frac{3}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}\right|$$ since $\underset{\sim}{a}\cdot\underset{\sim}{b}=\underset{\sim}{b}\cdot\underset{\sim}{b}$

$$=\left|\underset{\sim}{b}\right|^2$$

$$=1$$

$\left|\underset{\sim}{c}\cdot\underset{\sim}{b}\right|$ is the length of projection of $\underset{\sim}{c}$ on $\underset{\sim}{b}$.



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Alternative:

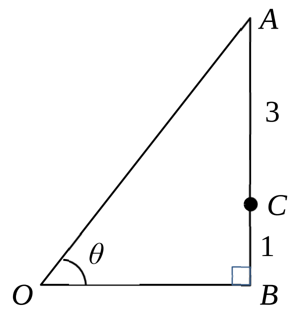
If Method 2 was use in (i), $\underset{\sim}{a}\cdot\underset{\sim}{b}=|\underset{\sim}{a}||\underset{\sim}{b}|\cos\theta=\sec\theta(1)\cos\theta=1$ since from (i)

$|\underset{\sim}{c}\cdot\underset{\sim}{b}|$ is the length of projection of $\underset{\sim}{c}$ on $\underset{\sim}{b}$.

By Ratio Theorem,

$$\underset{\sim}{c}=\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}$$

$$\begin{aligned}|\underset{\sim}{c}\cdot\underset{\sim}{b}|&=\left|\left(\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}\right)\cdot\underset{\sim}{b}\right|\\&=\left|\frac{1}{4}\underset{\sim}{a}\cdot\underset{\sim}{b}+\frac{3}{4}\underset{\sim}{b}\cdot\underset{\sim}{b}\right|\\&=\left|\frac{1}{4}(1)+\frac{3}{4}(1)\right|\\&=1\end{aligned}$$



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(iii)

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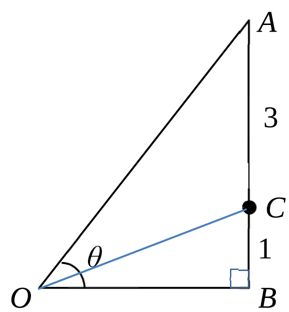
Method ①:

$$\begin{aligned}\frac{|\underset{\sim}{b}\times\underset{\sim}{c}|}{|\underset{\sim}{a}\times\underset{\sim}{b}|}&=\frac{\left|\underset{\sim}{b}\times\left(\frac{1}{4}\underset{\sim}{a}+\frac{3}{4}\underset{\sim}{b}\right)\right|}{|\underset{\sim}{a}\times\underset{\sim}{b}|}\\&=\frac{\left|\frac{1}{4}(\underset{\sim}{b}\times\underset{\sim}{a})\right|}{|\underset{\sim}{a}\times\underset{\sim}{b}|},\text{ where }\underset{\sim}{b}\times\underset{\sim}{b}=\underset{\sim}{0}\\&=\frac{1}{4}\frac{|\underset{\sim}{a}\times\underset{\sim}{b}|}{|\underset{\sim}{a}\times\underset{\sim}{b}|},\text{ where }|\underset{\sim}{b}\times\underset{\sim}{a}|=|\underset{\sim}{a}\times\underset{\sim}{b}|\\&=\frac{1}{4}\end{aligned}$$



Method ②:

$$\begin{aligned}\frac{|\underset{\sim}{b}\times\underset{\sim}{c}|}{|\underset{\sim}{a}\times\underset{\sim}{b}|}&=\frac{\frac{1}{2}|\underset{\sim}{b}\times\underset{\sim}{c}|}{\frac{1}{2}|\underset{\sim}{a}\times\underset{\sim}{b}|}\\&=\frac{\text{area of }\Delta OBC}{\text{area of }\Delta OAB}\\&=\frac{\frac{1}{2}\times BC\times h}{\frac{1}{2}\times AB\times h}\\&=\frac{1}{4}\end{aligned}$$

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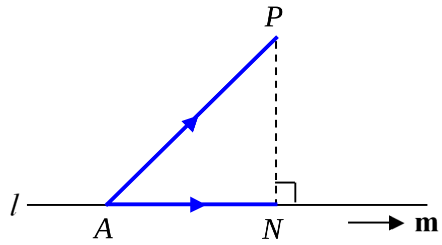
Foot of the Perpendicular from a Point to a Line



Step 1: Since N lies on l,

$\overrightarrow{ON} = \mathbf{a} + \lambda\mathbf{m}$ for some $\lambda \in \mathbb{R}$

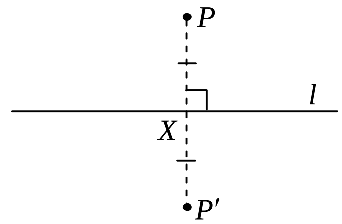
(λ is unknown, to be found later)

Step 2: Find $\overrightarrow{PN}$, using $\overrightarrow{PN} = \overrightarrow{ON} - \overrightarrow{OP}$.

Step 3: $\overrightarrow{PN} \perp l$ implies $\overrightarrow{PN} \bullet \mathbf{m} = 0$.

Use this to find λ.

Reflection of a point *P* about a line ℓ



Step 1:

Find the position vector of X, the foot of the perpendicular from P to l

Step 2:

Use the Ratio Theorem to obtain the position vector of the reflection $\overrightarrow{OP'}$.

$$\overrightarrow{OX} = \frac{\overrightarrow{OP} + \overrightarrow{OP'}}{2} \quad \text{or} \quad \overrightarrow{PX} = \overrightarrow{XP'}$$

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The line L has vector equation

$$L: \mathbf{r} = \begin{pmatrix} 6 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix}, \lambda \in \mathbb{R}.$$

(i) Find the position vector of the point N, the foot of perpendicular from the point A with coordinates $(3, 6, 1)$ to the line L. [3]

(ii) Find the position vector of the point A', the reflection of point A in the line L. [2]

(iii) Find the coordinates of the points on the line L that are $3\sqrt{3}$ units away from point A. [2]

(i)

Since N lies on L, $\overrightarrow{ON} = \begin{pmatrix} 6-2\lambda \\ \lambda \\ 1-\lambda \end{pmatrix}$, for some $\lambda \in \mathbb{R}$

$\therefore \overrightarrow{AN} = \begin{pmatrix} 6-2\lambda \\ \lambda \\ 1-\lambda \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix}$

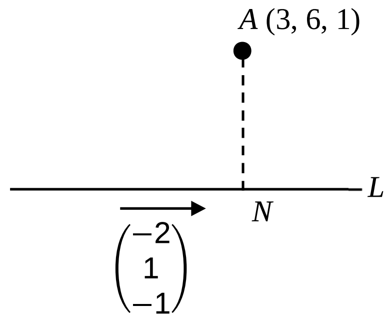
$\overrightarrow{AN} \bullet \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = 0$

$\begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix} \bullet \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} = 0$

$-6+4\lambda-6+\lambda+\lambda = 0$

$\lambda = 2$

$\overrightarrow{ON} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$



(ii)

$\overrightarrow{OA'} = 2\overrightarrow{ON} - \overrightarrow{OA}$

$= 2\begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} 3 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$

(iii)

Let D denote the point(s) that are $3\sqrt{3}$ units away from A on line L.

Then $\overrightarrow{OD} = \begin{pmatrix} 6-2\lambda \\ \lambda \\ 1-\lambda \end{pmatrix}$ and $\overrightarrow{AD} = \begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix}$. (can be taken from (i)).

$\left| \begin{pmatrix} 3-2\lambda \\ -6+\lambda \\ -\lambda \end{pmatrix} \right| = 3\sqrt{3}$

$(3-2\lambda)^2 + (-6+\lambda)^2 + (-\lambda)^2 = 27$

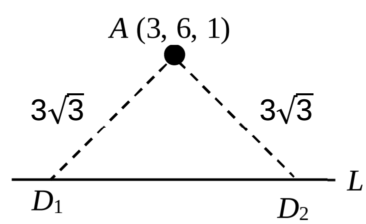
$6\lambda^2 - 24\lambda + 18 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda-3)(\lambda-1) = 0$

$\lambda = 1$ or $\lambda = 3$

$\therefore D(4,1,0)$ or $D(0,3,-2)$



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Equation of a plane (π) in three forms:

Vector equation form:
$\mathbf{r} = \mathbf{a} + \lambda \mathbf{m}_1 + \mu \mathbf{m}_2, \quad \lambda, \mu \in \mathbb{R}$

Note:
$\mathbf{n} = \mathbf{m}_1 \times \mathbf{m}_2$,

position vector of a <u>general</u> point on

position vector of a <u>particular</u> point

$\mathbf{m}_1$ and $\mathbf{m}_2$ are two non-parallel vectors that are ...lel

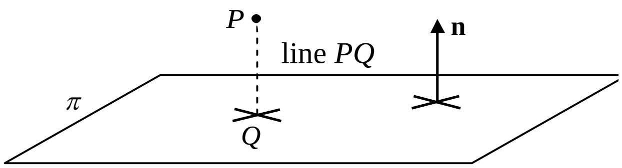
real parameters

Scalar product form:
$\pi : \mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$
$\pi : \mathbf{r} \cdot \mathbf{n} = d$, where d is a scalar constant.

if the point P lies on π, then the position vector of P will satisfy $\overrightarrow{OP} \cdot \mathbf{n} = d$.

Cartesian form:
Set $\mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \mathbf{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \mathbf{a} \cdot \mathbf{n} = d$,
$\pi : ax + by + cz = d$

Foot of the Perpendicular from a Point to a Plane



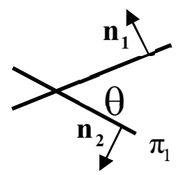
Step 1: Form the equation of the line *PQ*

Note that the line *PQ*
- passes through the point *P*, and
- is parallel to $\mathbf{n}$ (the normal vector of π)

Equation of line PQ $\mathbf{r} = \overrightarrow{OP} + \lambda \mathbf{n}, \quad \lambda \in \mathbb{R}$

Step 2:
Solve for the point of intersection $\overrightarrow{OQ}$ of the line PQ and the plane π .

Angle between two planes



Acute angle between π_1 and π_2

$$\cos\theta = \left| \frac{\mathbf{n_1} \cdot \mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|} \right|$$

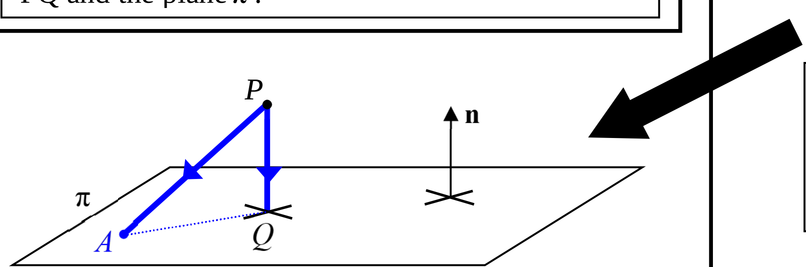
Length of Projection and Shortest distance

Shortest ($\perp$) distance from a point P to plane

$$PQ = \left| \overrightarrow{AP} \bullet \hat{\mathbf{n}} \right|$$

Length of projection of *AP* onto the plane

$$AQ = \left| \overrightarrow{AP} \times \hat{\mathbf{n}} \right|$$



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Intersection of Two Non-Parallel Planes

Method 1: Using GC

Equation of Planes:

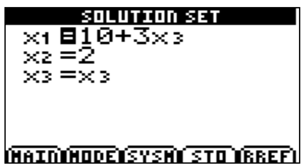
$\pi_1 : \mathbf{r}\cdot\mathbf{n}_1 = p_1$ and $\pi_2 : \mathbf{r}\cdot\mathbf{n}_2 = p_2$.

Step 1: Convert π_1 and π_2 into Cartesian Form

Step 2: Use GC.

- Press **APPS** and select **PlySmlt2**
- Select **2: SIMULT EQN SOLVER**

Step 3: Interpret the solutions given by the GC



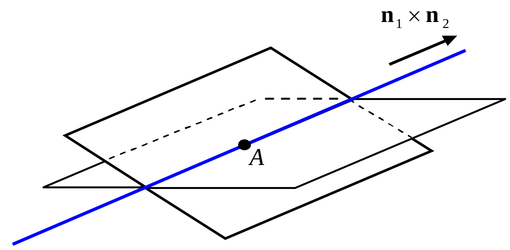
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10+3z \\ 2 \\ z \end{pmatrix} = \begin{pmatrix} 10+3\lambda \\ 2 \\ \lambda \end{pmatrix} \quad \text{By letting } z=\lambda$$

The equation of line of intersection is l:

$$\mathbf{r} = \begin{pmatrix} 10 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}, \ \lambda \in \mathbb{R}.$$

Method 2: Using $\mathbf{n}_1 \times \mathbf{n}_2$

Given a point A that lies on both planes π_1 and π_2,



Step 1:

Find the direction vector of the line of intersection using

$\mathbf{n}_1 \times \mathbf{n}_2$

Step 2:

Form the equation of the line

$\ell : \mathbf{r} = \mathbf{a} + \lambda\mathbf{m}, \lambda \in \mathbb{R}$

Note: $\mathbf{n}_1 \times \mathbf{n}_2$ is also known as the vector parallel to both π_1 and π_2.

Let's Try Now!

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The planes P_1 and P_2 have equations $\mathbf{r}=\begin{pmatrix}4\\-1\\3\end{pmatrix}+s\begin{pmatrix}3\\-1\\7\end{pmatrix}+t\begin{pmatrix}1\\-3\\12\end{pmatrix}$ and $x+3y=1$ respectively, where s and t are parameters.

(i) Find the line of intersection of P_1 and P_2. [3]

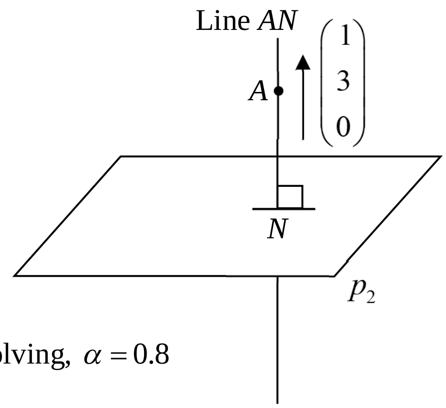
(ii) Find the acute angle between P_1 and P_2. [2]

The point A has position vector $5\mathbf{i}-4\mathbf{j}+15\mathbf{k}$ and the point B has position vector $\mathbf{i}-2\mathbf{k}$.

(iii) Find the foot of perpendicular from A to P_2. [3]

(iv) Find the length of projection of AB onto P_2. [3]

(i)	For p_1: Consider $\begin{pmatrix}3\\-1\\7\end{pmatrix}\times\begin{pmatrix}1\\-3\\12\end{pmatrix}=\begin{pmatrix}9\\-29\\-8\end{pmatrix}$. Then $\mathbf{r}.\begin{pmatrix}9\\-29\\-8\end{pmatrix}=\begin{pmatrix}4\\-1\\3\end{pmatrix}.\begin{pmatrix}9\\-29\\-8\end{pmatrix}=41$. For p_2: $\mathbf{r}.\begin{pmatrix}1\\3\\0\end{pmatrix}=1$ For p_1: $9x-29y-8z=41$ -----------Equation 1 For p_2: $x+3y=1$ -----------------Equation 2 Using GC, let $z=\mu$ Then $\mathbf{r}=\begin{pmatrix}19/7\\-4/7\\0\end{pmatrix}+\mu\begin{pmatrix}3/7\\-1/7\\1\end{pmatrix}=\frac{1}{7}\begin{pmatrix}19\\-4\\0\end{pmatrix}+\lambda\begin{pmatrix}3\\-1\\7\end{pmatrix}$, where $\lambda\in\mathbb{R}$.
(ii)	For p_1: $\mathbf{r}.\begin{pmatrix}9\\-29\\-8\end{pmatrix}=41$. For p_2: $\mathbf{r}.\begin{pmatrix}1\\3\\0\end{pmatrix}=1$ Acute angle between p_1 and p_2: $\cos\theta=\dfrac{\left|\begin{pmatrix}1\\3\\0\end{pmatrix}\bullet\begin{pmatrix}9\\-29\\-8\end{pmatrix}\right|}{\sqrt{10}\sqrt{986}}=\dfrac{\lvert-78\rvert}{\sqrt{9860}}=\dfrac{78}{\sqrt{9860}}$ $\therefore\ \theta=38.2^\circ$ (nearest to 0.1°)

(iii)	Let the foot of perpendicular from *A* to p_2 be *N*. Line *AN*: $\mathbf{r} = \begin{pmatrix}5\\-4\\15\end{pmatrix} + \alpha\begin{pmatrix}1\\3\\0\end{pmatrix}$ where $\alpha \in \mathbb{R}$  $\overrightarrow{ON} = \begin{pmatrix}5+\alpha\\-4+3\alpha\\15\end{pmatrix}$ for a value of α Since the point *N* lies on p_2, $\begin{pmatrix}5+\alpha\\-4+3\alpha\\15\end{pmatrix}\bullet\begin{pmatrix}1\\3\\0\end{pmatrix} = 1 \Rightarrow 5+\alpha-12+9\alpha = 1$. Solving, $\alpha = 0.8$ $\therefore \overrightarrow{ON} = \begin{pmatrix}5.8\\-1.6\\15\end{pmatrix}$. The coordinates of the foot of perpendicular from *A* to p_2 = $(5.8, -1.6, 15)$.
(iv)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix}-4\\4\\-17\end{pmatrix}$ Length of projection of *AB* onto p_2 $= \frac{1}{\sqrt{10}}\left\lvert \overrightarrow{AB} \times \begin{pmatrix}1\\3\\0\end{pmatrix}\right\rvert$ $= \frac{1}{\sqrt{10}}\left\lvert \begin{pmatrix}-4\\4\\-17\end{pmatrix} \times \begin{pmatrix}1\\3\\0\end{pmatrix}\right\rvert = \frac{1}{\sqrt{10}}\left\lvert \begin{pmatrix}51\\-17\\-16\end{pmatrix}\right\rvert$ = 17.7 units (to 3 s.f.)

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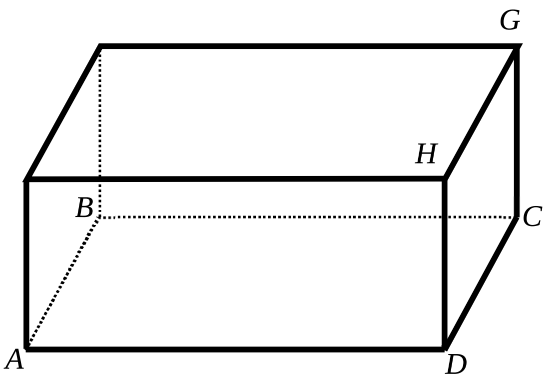


Fig. 1

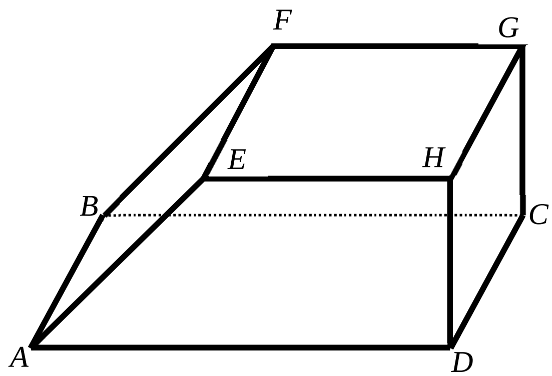


Fig. 2

Fig. 1 shows a rectangular prism. Fig. 2 shows the prism with a removed part that was cut along the plane *ABFE*. The resulting object has a base *ABCD*, a top *EFGH*, a side *CDHG*, and a slanted side *ABFE*. The following information is given.

The top *EFGH* is a part of the plane with equation $4x+y-z=-6$.

The base *ABCD* is a part of the plane with equation $4x+y-z=7$.

The slanted side *ABFE* is a part of the plane with equation $3x+4y+9z=15$.

(i) Find the acute angle between the base and the slanted side. [2]

(ii) Find the height of the object, measured in the direction perpendicular to the base. [3]

(iii) Find a vector equation of the line of intersection between the base and the slanted side. [2]

Point *S* with coordinates $(2,11,12)$ lies on *CD*.

(iv) Find the position vector of the foot of the perpendicular from point *S* to the line found in part **(iii)**. [3]

(v)

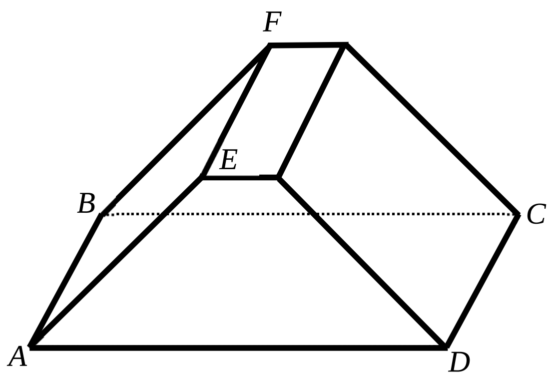


Fig. 3

A part of the object is to be further removed so that the remaining object is now symmetrical about a plane p that is parallel to CD. Fig. 3 shows the remaining object after the removal. Find the cartesian equation of p. [4]

4	**TMJC Promo 9758/2022/Q10**
(i)	Base: $\mathbf{r}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=7$ Slanted: $\mathbf{r}\cdot\begin{pmatrix}3\\4\\9\end{pmatrix}=15$ $\cos\theta=\left\lvert\dfrac{\begin{pmatrix}4\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}3\\4\\9\end{pmatrix}}{\left\lvert\begin{pmatrix}4\\1\\-1\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}3\\4\\9\end{pmatrix}\right\rvert}\right\rvert$ $=\left\lvert\dfrac{12+4-9}{\sqrt{16+1+1}\sqrt{9+16+81}}\right\rvert$ $=\dfrac{7}{\sqrt{18}\sqrt{106}}$ $\theta=80.8^\circ$ (1d.p.)

(ii)

Method 1 (Using concept of distance between 2 planes)

Top: $\mathbf{r}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=-6$

Base: $\mathbf{r}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=7$

Distance between 2 planes $=\left|\dfrac{d_1-d_2}{|\mathbf{n}|}\right|$

Height of object $=\left|\dfrac{7-(-6)}{\sqrt{4^2+1^2+1^2}}\right|$

$=\dfrac{13}{\sqrt{18}}=\dfrac{13}{6}\sqrt{2}$

Method 2 (Using concept of length of projection)

The point $P(0,0,6)$ lies on the top plane because $\begin{pmatrix}0\\0\\6\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=-6$.

The point $Q(0,7,0)$ lies on the base plane because $\begin{pmatrix}0\\7\\0\end{pmatrix}\cdot\begin{pmatrix}4\\1\\-1\end{pmatrix}=7$.

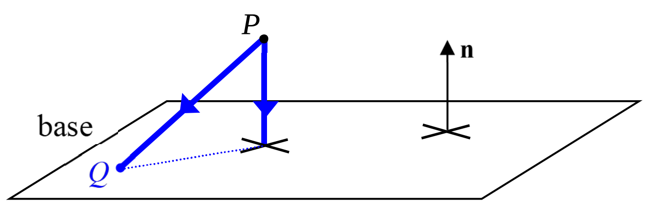
$\overrightarrow{PQ}=\begin{pmatrix}0\\7\\0\end{pmatrix}-\begin{pmatrix}0\\0\\6\end{pmatrix}=\begin{pmatrix}0\\7\\-6\end{pmatrix}$

Height of object = Distance between top and base

= Length of projection of $\overrightarrow{PQ}$ onto normal of base

$=\dfrac{\left|\begin{pmatrix}0\\7\\-6\end{pmatrix}\bullet\begin{pmatrix}4\\1\\-1\end{pmatrix}\right|}{\sqrt{4^2+1^2+1^2}}$

$=\dfrac{13}{\sqrt{18}}=\dfrac{13}{6}\sqrt{2}$



(iii)

Base: $4x+y-z=7$

Slanted: $3x+4y+9z=15$

Using GC, the line of intersection is $\mathbf{r}=\begin{pmatrix}1\\3\\0\end{pmatrix}+\lambda\begin{pmatrix}1\\-3\\1\end{pmatrix},\lambda\in\mathbb{R}.$

When solving for system of linear equations with numerical coefficients, please just use GC. It is very efficient to use GC to help in calculations.

(iv)

Let N be the foot of perpendicular from S to the line.

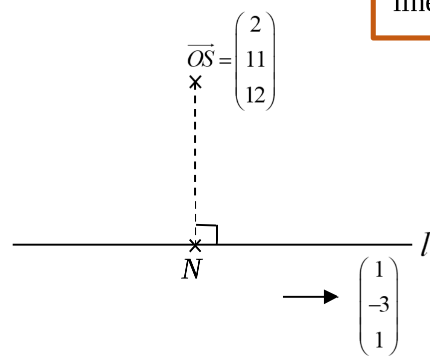
> When introducing variables, make sure that it is not already used in the question. So in this question, for example, you should not use F again here.

Since N lies on the line, $\overrightarrow{ON} = \begin{pmatrix} 1+\lambda \\ 3-3\lambda \\ \lambda \end{pmatrix}$ for some $\lambda \in \mathbb{R}$.

$$\overrightarrow{SN} \bullet \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = 0$$

> Note from the diagram that SN is perpendicular to the line l.

$$\left[\begin{pmatrix} 1+\lambda \\ 3-3\lambda \\ \lambda \end{pmatrix} - \begin{pmatrix} 2 \\ 11 \\ 12 \end{pmatrix} \right] \bullet \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = 0$$



$$\begin{pmatrix} \lambda-1 \\ -3\lambda-8 \\ \lambda-12 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} = 0$$

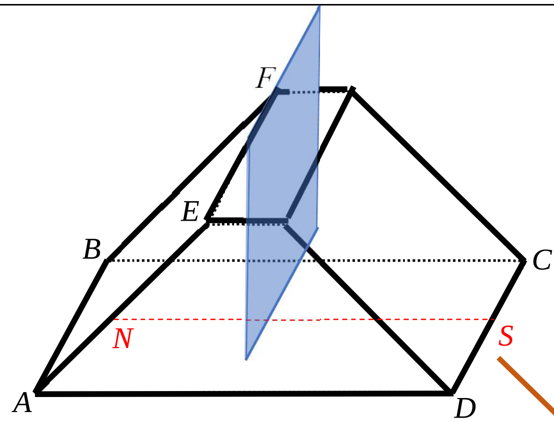
$$\lambda - 1 + 9\lambda + 24 + \lambda - 12 = 0$$

$$11\lambda = -11$$

$$\lambda = -1$$

Thus $\overrightarrow{ON} = \begin{pmatrix} 1-1 \\ 3-3(-1) \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix}$

(v)



Can use ratio theorem to find the position vector of the midpoint of $\overrightarrow{SN}$. Note that $\frac{1}{2}\overrightarrow{SN}$ is NOT the mid point of $\overrightarrow{SN}$.

Method 1

$$\overrightarrow{SN} = \begin{pmatrix} 2 \\ 11 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 6 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix}$$

Observe from the diagram that $\overrightarrow{SN}$ is perpendicular to the required plane.

Equation of the plane of symmetry is

$$\mathbf{r} \bullet \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 17 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix}$$

$$= \frac{1}{2}(4+85+143)$$

$$= 116$$

The cartesian equation is $2x+5y+13z=116$.

Be careful to answer the question. Question needed "cartesian equation".

Method 2

Normal of the plane of symmetry is

$$\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \times \begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix}$$

Observe from the diagram that the plane is parallel to both $\begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}$, the direction vector of *AB* and *CD*, and $\begin{pmatrix} 4 \\ 1 \\ -1 \end{pmatrix}$, the normal vector of the base and top planes.

Equation of the plane of symmetry is

$$\mathbf{r} \bullet \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 17 \\ 11 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 5 \\ 13 \end{pmatrix}$$

$$= \frac{1}{2}(4+85+143)$$

$$= 116$$

The cartesian equation is $2x+5y+13z=116$.