CANBERRA SECONDARY SCHOOL

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2024 Preliminary Examination

Secondary Four Express/Five Normal Academic

MATHEMATICS
4052/02

21 Aug 2024
2 hours 15 minutes
0800h – 1015h

Name: _____ ( ) **Class:** _____

READ THESE INSTRUCTIONS FIRST

Write your full name, class and index number on all work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
Calculators should be used where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 90.

FOR MARKER'S USE		
	Marks Awarded	Max Marks
Total		90

Errata
Q4. AED is a straight line.
Q9b. The second stage of the journey is 32 km longer than the first stage.

This question paper consists of 21 printed pages including the cover page.

Setter: Ms Sim Yi Lian

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved Surface area of a cone} = \pi rl$$

$$\text{Curved surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector Area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Solve the simultaneous equations.

$$4m + 6n = 58$$
$$3m + 5n = 46$$

Answer $m =$ .....

$n =$ ..... [3]

(b) Write as a single fraction in the simplest form $\frac{3x}{2x^2-50}-\frac{1}{x-5}$.

Answer ..... [3]

(c) $x=\sqrt[3]{\frac{25y}{7z+2}}$

(i) Find x when $y=25$ and $z=-1$.

Answer $x=$ ..... [1]

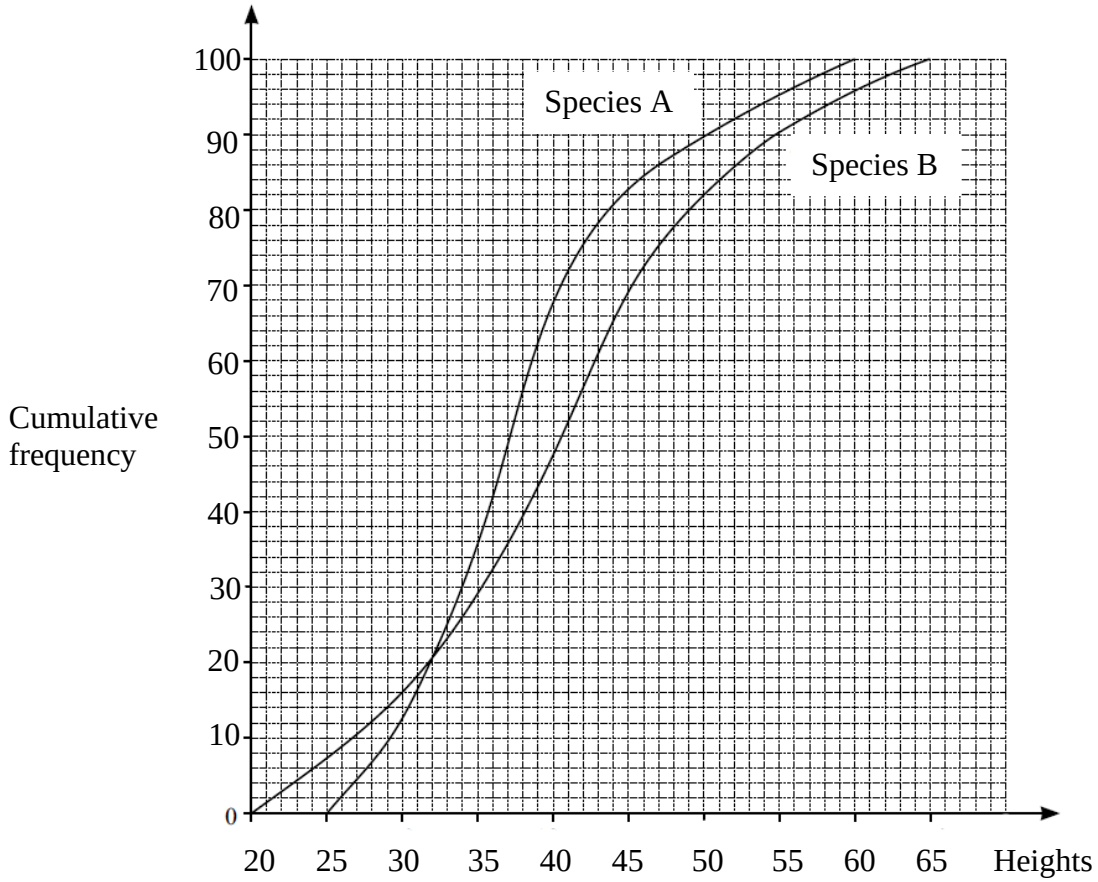
(ii) Rearrange the formula to make y the subject.

Answer $y=$ ..... [2]

(d) Solve the equation $\frac{2x+3}{(3x+1)(x-1)} - 2 = 0$.

Answer $x =$ ..... or ..... [3]

2 **(a)** The heights, in centimeters, of 100 plants of each of species A and species B are recorded.
The cumulative frequency curves show the distributions of their heights.



(i) Use the curve to estimate

(a) the median height for species B,

Answer ..... cm [1]

(b) the 40th percentile for species A,

Answer ..... cm [1]

(c) the interquartile range for species A.

Answer ..... cm [2]

(ii) Plants from species B are used for an event.
Only plants with height within 10% of 50 cm are used.
Find the percentage of plants from species B which are used.

Answer ..... % [2]

(b) A box has 20 pots of plants.
There are 5 pink pots, 9 blue pots and the remaining pots are green.

(i) Two pots are taken from the box at random, without replacement.
Find, as a fraction in its simplest form, the probability that one pot is pink and the other is blue.

Answer ..... [2]

(ii) These two pots are returned to the box.
Three pots are taken from the box at random, without replacement.
Find, as a fraction in its simplest form, the probability that only one pot is green.

Answer ..... [2]

3 **(a)** Wayne inherited a sum of money from his parents.

He decided to invest 60% of the money and shares $\frac{3}{4}$ of the **remaining** amount with his wife.
The other $20 000 are meant for their travelling expenses.
Calculate the sum of money he inherited.

Answer $ ..... [3]

(b) A bank offers two investment packages.

Package A	Package B
• Simple interest of 0.97% per annum. • One time payout of 10% of the amount of the investment.	• Compound interest of 3.78% per annum.

Wayne wants to invest $55 000 in one of the packages for 3 years.
Which package should he choose if he wishes to reap the most benefits from the investment?

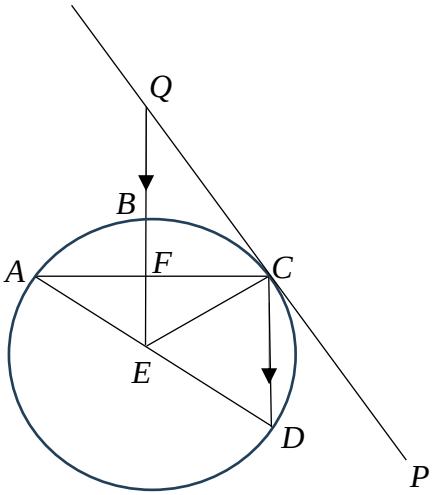
.....

..... [5]

(c) Wayne and his wife go on a seven days self-drive trip to Australia.
They pay 150 AUD per day for car hire and 380 AUD per night for hotel.
They pay using credit card and is charged with a fee of 2% for the currency conversion.
The exchange rate between Singapore dollars and Australian dollars is $1 = 0.91 AUD.
Calculate the total amount, including credit card fee, Wayne is charged for car hire and accommodation.
Give your answer in Singapore dollars, correct to the nearest dollar.

Answer $ ..... [3]

4 *A*, *B*, *C* and *D* are points on a circle.
Line *PCQ* is a tangent to the circle.
Line *QE* bisects angle *AEC* and is parallel to line *CD*.



(a) Show that $EC = ED$.
Give a reason for each statement you make.

.....

.....

..... [2]

(b) Show that triangle *AEF* is similar to triangle QCF.
Give a reason for each statement you make.

.....

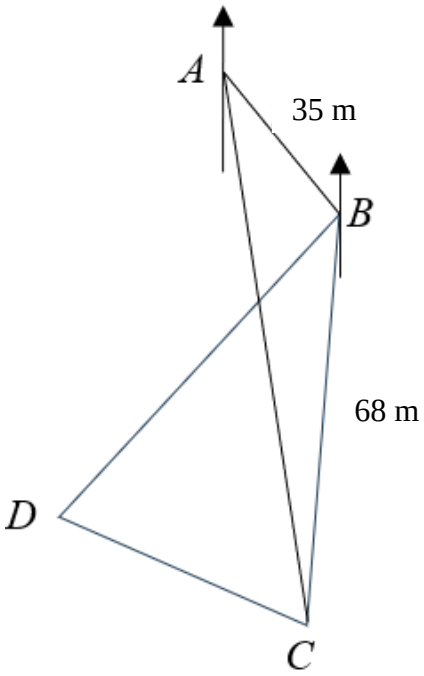
.....

..... [2]

(c) Diameter $AD = 28\,\text{cm}$ and angle $QCF = 61°$.
Calculate the area of segment CD.

Answer _____ cm^2 [4]

5 A, B, C and D are points on horizontal ground.
$AB = 35\,\text{m}$ and $BC = 68\,\text{m}$.
The bearing of B from A is 137° and the bearing of C from B is 192°.
Angle $ABD = 85°$ and angle $BCD = 70°$.



(a) Show that angle $ABC = 125°$.

[2]

(b) Calculate AC.

Answer ..... m [3]

(c) Calculate DC.

Answer ..... m [2]

A man travels in a straight line from D to C.
X, is the position of the man where it is the shortest distance from B during this journey.

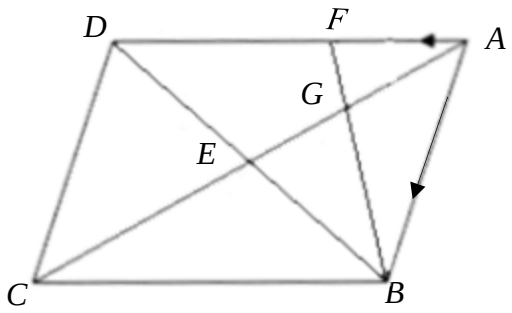
(d) Show that $BX = 63.9\,\text{m}$.

[2]

(e) A drone is hovering vertically above B.
The angle of elevation of the drone from C is $40.5°$.
Calculate the greatest angle of elevation of the drone from a point on DC.

Answer ..... [3]

6 In the diagram *ABCD* is a parallelogram.
The diagonal *AC* and *BD* intersect at *E*.
$\overrightarrow{AF} = \mathbf{2p}$ and $\overrightarrow{AB} = \mathbf{6p} + 4\mathbf{q}$.
F is a point on *AD* such that $\frac{AF}{AD} = \frac{1}{3}$.
G is the mid-point of *AE*.



(a) Express $\overrightarrow{DE}$ in terms of **p** and **q**, as simply as possible.

Answer _____ [2]

(b) Express $\overrightarrow{AG}$ in terms of **p** and **q**, as simply as possible.

Answer _____ [2]

(c) Show that $\overrightarrow{BG} = \mathbf{-3p - 3q}$.

[2]

(d) Explain why *B*, *G* and *F* lie on a straight line.

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.....

.....

..... [2]

(e) Calculate $\frac{\text{the area of triangle } ABF}{\text{the area of parellogram } ABCD}$.

Answer ..... [2]

7 **(a)** Complete the table of values for $y = \frac{x^3}{4} - 2x + 1$.

x	–3	–2	–1	0	1	2	3
y	0.25	3	2.75	1	–0.75		1.75

[1]

(b) On the grid opposite, draw the graph of $y = \frac{x^3}{4} - 2x + 1$ for $-3 \le x \le 3$. [3]

(c) Explain how your graph shows that there is only one solution of the equation $\frac{x^3}{4} - 2x + 1 = k$ for some values of k.

.....

..... [1]

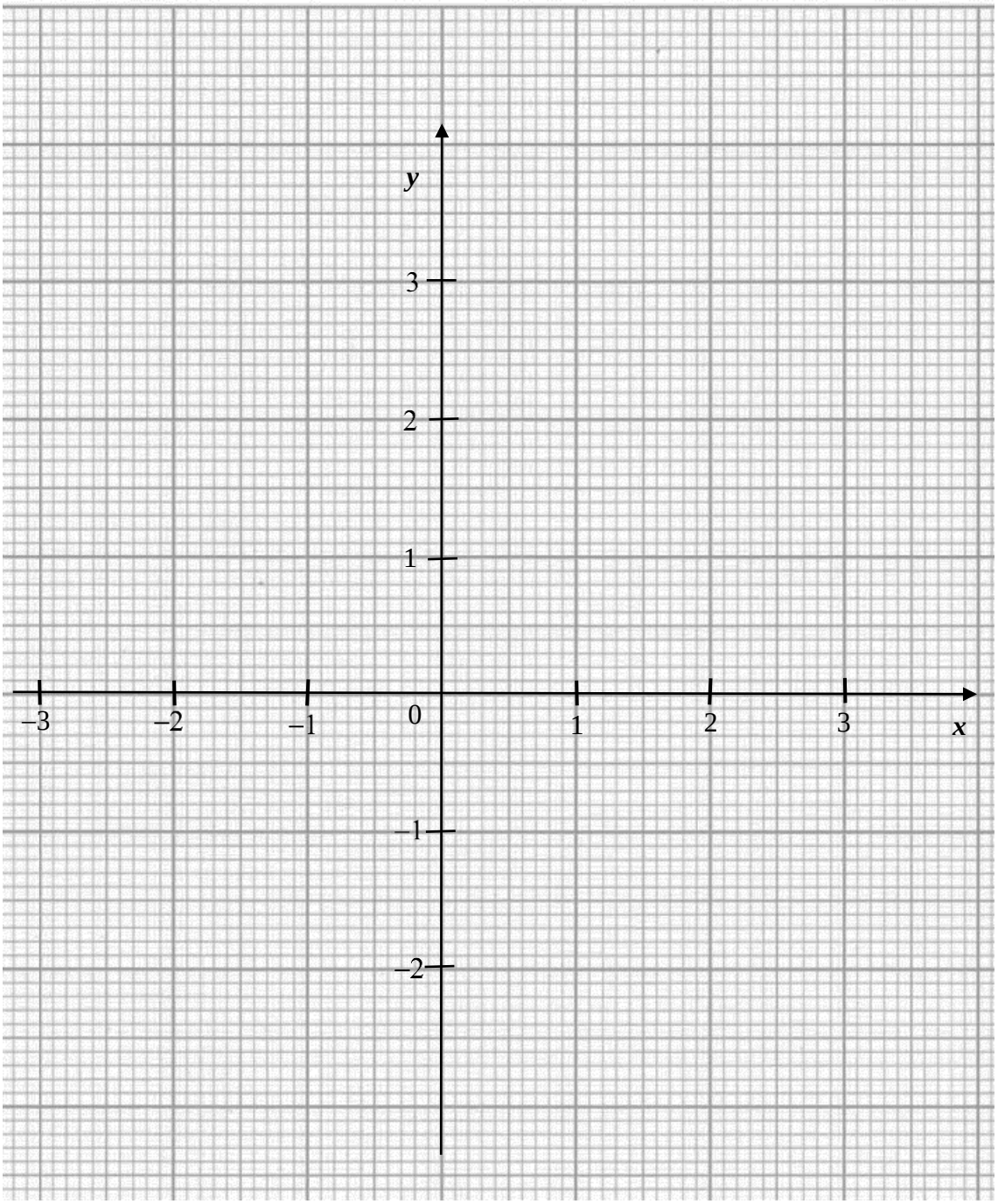
(d) The equation $x^3 - 9x - 4 = 0$ can be solved by drawing a suitable straight line on the grid.

(i) Find the equation of the straight line.

Answer ..... [2]

(ii) By drawing this straight line, solve the equation $x^3 - 9x - 4 = 0$.

Answer $x =$ ..... or ..... [2]


y
3
2
1
−3
−2
−1
0
1
2
3
x
−1
−2

8 The first four terms in a sequence of numbers are given below.

$$T_1 = (1\times 3)+8 = 11$$
$$T_2 = (2\times 4)+12 = 20$$
$$T_3 = (3\times 5)+16 = 31$$
$$T_4 = (4\times 6)+20 = 44$$

(a) Find T_5.

Answer ..... [1]

(b) Show that the n^{th} term of the sequence, T_n is given by n^2+6n+4.

[2]

(c) T_{p-1} and T_p are consecutive terms in the sequence.
Find and simplify an expression, in terms of p, for the sum of T_{p-1} and $T_{p.}$

Answer ..... [3]

(d) Find the value of p when the sum of T_{p-1} and T_p is 303.

Answer $p=$ ..... [2]

9 Jim, his wife and their child visited Hokkaido, Japan.

The tables below give information that can be used to work out some of their expenses.

	Prices of flight tickets for a return trip			
Payment mode	Cash (Includes tax)		Redemption with miles (Excludes tax of $115 per pax)	
Class	Economy	Business	Economy	Business
Adult	$1920	$5500	90 000 miles	140 000 miles
Child	$1350	$4000		

	Amount of fuel used (litres/100 km)		
	Type of car		
Type of driving	Sedan car	Sport Utility Vehicle (SUV)	Mini van
City	7.8	9.3	10.8
Out of city	5.7	7.5	8.9
Combined	6.2	8.8	9.5

	Daily rates of driver	
Day(s) of hire	8 hours	Additional time
1	$350	$45 per hour
2 or more	$300	$40 per hour

(a) They travelled in business class and Jim had 150 000 miles to offset some of the cost of their flight tickets.
Calculate the amount of money he paid for the tickets.

Answer $ ..... [2]

(b) Jim engaged a driver for 10 hours and his family travelled in a SUV for one day during their trip.
The driver picked them from their accommodation in the city to attractions outside the city.

The driver drove the first stage of the journey at an average speed of 70 km/h.
The second stage of the journey is 32 km longer than the second stage.
The driver drove at an average speed of 80 km/h.
The journey takes a total of 4 hours and 9 minutes.

Fuel price for the vehicle was $1.70 per litre.
Jim paid $550 to the driver.
He thinks that he included a 20% tips in the amount paid to the driver.

Is he correct?
Justify your decision with calculations.

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..... [8]

End of paper