Anglo-Chinese School
(Barker Road)

Marking Scheme
Secondary 4 Express / 5 Normal (Academic)
Additional Mathematics Paper 2
Preliminary Examination 2023

1	(a)	$1000			
	(b)	$-\frac{1}{3}x^2+2x+1$			
		$=-\frac{1}{3}\left[x^2-6x-3\right]$			
		$=-\frac{1}{3}\left[\left(x-3\right)^2-\left(3\right)^2\right]+1$			
		$=-\frac{1}{3}\left(x-3\right)^2+3+1$			
		$=-\frac{1}{3}\left(x-3\right)^2+4$			
	(c)	3 months or August			
		$4000			
2	(a)	$\mathrm{f}(x)=ax^3+2bx^2-34x+12$			
		$\mathrm{f}(3)=0$			
		$a(3)^3+2b(3)^2-34(3)+12=0$			
		$27a+18b=90$			
		$3a+2b=10--(1)$			
		$\mathrm{f}(-1)=32$			
		$a(-1)^3+2b(-1)^2-34(-1)+12=0$			
		$-a+2b=-14--(2)$			
		Solving (1), (2)			
		$a=6$			
		$b=-4$			
	(b)	$y=kx-9 \qquad ---(1)$			
		$x=\sqrt{y-3x} \qquad ---(2)$			
		Sub. (1) into (2):			
		$x=\sqrt{(kx-9)-3x}$			
		$x^2=kx-9-3x$			
		$x^2-(k-3)x+9=0$			
		For the line to intersect the curve at two distinct points,			

		$b^2 - 4ac > 0$			
		$[-(k-3)]^2 - 4(1)(9) > 0$			
		$k^2 - 6k + 9 - 36 > 0$			
		$k^2 - 6k - 27 > 0$			
		$(k+3)(k-9) > 0$			
		$\therefore\ k < -3 \text{ or } k > 9$			
3	(a)	$\dfrac{2\cos^2\theta - \sin\theta\cos\theta + 1}{\sin^2\theta}$			
		$= \dfrac{2\cos^2\theta}{\sin^2\theta} - \dfrac{\sin\theta\cos\theta}{\sin^2\theta} + \dfrac{1}{\sin^2\theta}$			
		$= 2\cot^2\theta - \cot\theta + \cos\text{ec}^2\theta$			
		$=2\cot^2\theta - \cot\theta + \left(1 + \cot^2\theta\right)$			
		$=3\cot^2\theta - \cot\theta + 1$			
		Alternative Method			
		$\dfrac{2\cos^2\theta - \sin\theta\cos\theta + \cos^2\theta + \sin^2\theta}{\sin^2\theta}$			
		$\dfrac{3\cos^2\theta - \sin\theta\cos\theta + \sin^2\theta}{\sin^2\theta}$			
		$\dfrac{3\cos^2\theta}{\sin^2\theta} - \dfrac{\sin\theta\cos\theta}{\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta}$			
		$=3\cot^2\theta - \cot\theta + 1$			
	(b)	$3\cot^2\theta - \cot\theta + 1 = 1$			
		$3\cot^2\theta - \cot\theta = 0$			
		$\cot\theta\left(3\cot\theta - 1\right) = 0$			
		$\cot\theta = 0 \text{ or } \cot\theta = \dfrac{1}{3}$			
		$\dfrac{\cos\theta}{\sin\theta} = 0$			
		$\cos\theta = 0$, $\theta = -\dfrac{\pi}{2}, \dfrac{\pi}{2}$			
		When $\tan\theta = 3$, basic $\angle = 1.249$ $\theta = -1.89\ ,\ 1.25$			
		$\theta = -1.89\ , -\dfrac{\pi}{2},\ 1.25, \dfrac{\pi}{2}$			

4	(a)	$V=\frac{1}{3}\pi r^2\left(h+4\right)-\frac{1}{3}\pi(1)^2(4)$			
		Using similar triangles			
		$\frac{4}{4+h}=\frac{1}{r}$			
		$r=\frac{1}{4}\left(h+4\right)$			
		$\therefore V=\frac{1}{3}\pi\left(\frac{h+4}{4}\right)^2\left(h+4\right)-\frac{4\pi}{3}$			
		$=\frac{1}{3}\pi\left(\frac{1}{4}\right)^2\left(h+4\right)^2\left(h+4\right)-\frac{4\pi}{3}$			
		$=\frac{\pi}{48}\left(h+4\right)^3-\frac{4\pi}{3}$			
	(b)	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{\pi}{16}\left(h+4\right)^2$			
		Using chain rule			
		$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		$5-2=\frac{\pi}{16}\left(h+4\right)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		When h = 8, $3=\frac{\pi}{16}\left(8+4\right)^2\times\frac{\mathrm{d}h}{\mathrm{d}t}$			
		$\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{1}{3\pi}\text{cm/s}$			
5	(a)	$\sqrt{\left(3-0\right)^2+\left(9-k\right)^2}=\sqrt{\left(3-h\right)^2+\left(9-6\right)^2}$			
		$3^2+\left(9-k\right)^2=\left(3-h\right)^2+3^2$			
		$\left(9-k\right)^2=\left(3-h\right)^2$			
		$9-k=\pm\left(3-h\right)$			
		$9-k=3-h$ or $9-k=-\left(3-h\right)$			
		$h-k=3-9$ $9+3=h+k$			
		$h-k=-6$ (reject) $h+k=12$ (shown)			
		Since $h-k>0$			

	(b)	Midpoint of *PR* = $\left(\frac{0+7}{2},\frac{5+6}{2}\right)$			
		$=\left(\frac{7}{2},\frac{11}{2}\right)$			
		Gradient of *QS* = $\frac{9-\frac{11}{2}}{3-\frac{7}{2}}=-7$			
		Equation of *QS* : $y-\frac{11}{2}=-7\left(x-\frac{7}{2}\right)$			
		$y=-7x+30---(1)$			
		Sub (1) into $21y=7x-8$			
		$21(-7x+30)=7x-8$			
		$x=4\frac{1}{7}$			
		Subst. $x=4\frac{1}{7}$ into (1) : $y=1$			
		$S\left(4\frac{1}{7},1\right)$			
	(c)	Area = $\frac{1}{2}\begin{vmatrix} 3 & 0 & 4\frac{1}{7} & 7 & 3 \\ 9 & 5 & 1 & 6 & 9 \end{vmatrix}$			
		=. $\frac{1}{2}\left\{\left[(3)(5)+0+\left(4\frac{1}{7}\right)(6)+(7)(9)\right]-\left[0+(5)\left(4\frac{1}{7}\right)+(1)(7)+(6)(3)\right]\right\}$			
		$=\frac{1}{2}\left(\frac{720}{7}-\frac{320}{7}\right)$			
		$=28\frac{4}{7}$ or 28.6 (3sf)			
6	(a)	Given that *AE* = *EB* and *CF* = *FB*, <u>*E* is the midpoint of *AB* and *F* is midpoint *FB*.</u>			
		Hence by mid-point theorem, *AC* and *EF* are parallel			
	(b)	angle *AED* = angle *DCE* (tangent chord theorem)			
		angle *DCE* = angle *CEF*			

		(alternate angles, *AC* is parallel to *EF*)			
		Hence angle *AED* = angle *CEF*			
	(c)	In triangle *AED* and triangle *CEF*			
		Let angle CFE = x			
		Angle *CDE* = $180-x$ (angles in opposite segment)			
		Angle *ADE* = $180-(180-x)=x$ (adjacent angles on a a straight line)			
		Angle *CFE* = Angle *ADE*			
		or Angle *CFE* = Angle *ADE* (exterior angle of cyclic quadrilateral)			
		or angle *DAE* = angle *FEB* (corresponding angles, *AC* parallel *EF*)			
		Angle *FEB* = angle *ECF* (tangent chord theorem)			
		Angle *DAE* = angle *ECF*			
		From part (a) angle *AED* = angle *CEF*			
		Triangle *AED* is similar to triangle *CEF* (AA similarity)			
7	(a)	$\lg V = \lg pq^t$			
		$\lg V = \lg p + t\lg q$			
		$\lg V = \lg qt + \lg p$			

t	0	10	20	30	40
V	8 000	17 500	38 000	83 000	190 000
$\lg V$	3.90	4.24	4.58	4.92	5.28

		Plot a straight line graph of $\lg V$ against *t*.			
	(b)	$\lg V$ intercept: $\lg p = 3.90$ $p = 10^{3.90}$ $p = 7940$ (to 3 s.f.)			
		Gradient: $\lg q = \dfrac{5.3-4.06}{35-5} = 0.041333$			
		$q = 10^{\frac{5.3-4.06}{35-5}}$ $= 1.10$ (3s.f)			

	(c)	$V = pq^t$			
		$10^6 = 10^{3.90}\left(10^{0.041333t}\right)$			
		$\dfrac{10^6}{10^{3.90}} = 10^{0.041333t}$			
		$\lg\dfrac{10^6}{10^{3.90}} = 0.041333t$			
		$t = 50.81$ years			
		1970 + 50.81 = 2020			
	(d)	2020 is outside range of values in the initial table. House prices may not grow in the same way after 2010. 2020 would be using extrapolation, so is not likely to be reliable.			
8	(a)	$\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$			
		$4-x^2>0$			
		$(x-2)(x+2)<0$			
		$-2<x<2$			
		$x-1>0$			
		$x>1$			
		$-2<x<2$ and $x>1$			
		$1<x<2$			
	(b)	$\log_3\left(4-x^2\right)-\log_{\sqrt{3}}(x-1)=1$			
		$\log_3\left(4-x^2\right)-\dfrac{\log_3(x-1)}{\log_3\sqrt{3}}=1$			
		$\log_3\left(4-x^2\right)-\log_3(x-1)^2=1$			
		$\log_3\dfrac{\left(4-x^2\right)}{(x-1)^2}=1$			
		$\dfrac{\left(4-x^2\right)}{(x-1)^2}=3$			
		$4x^2-6x-1=0$			
		$x=\dfrac{6\pm\sqrt{36-4(-4)}}{8}$			

		$x = 1.65$ or -0.151(rejected)			

9	(a)	$h = \frac{400}{\pi r^2}$			
		Total surface area $= 2\pi r^2 + 2\pi r h$			
		$= 2\pi r^2 + 2\pi r\left(\frac{400}{\pi r^2}\right)$			
		$= 2\pi r^2 + \frac{800}{r}$			
		$C = 0.03\left(2\pi r^2\right) + 0.025\left(\frac{800}{r}\right)$			
		$= \frac{3}{50}\pi r^2 + \frac{20}{r}$			
	(b)	$\frac{\mathrm{d}C}{\mathrm{d}r} = \frac{3}{25}\pi r - \frac{20}{r^2}$			
		For stationary cost			
		$\frac{\mathrm{d}C}{\mathrm{d}r} = 0$			
		$\frac{3}{25}\pi r - \frac{20}{r^2} = 0$			
		$\frac{3}{25}\pi r = \frac{20}{r^2}$			
		$r^3 = \frac{500}{3\pi}$			
		$r = 3.7575$			
		$r = 3.76$ (3s.f)			
	(c)	$\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = \frac{3}{25}\pi + \frac{40}{r^3}$			
		When $r^3 = \frac{500}{3\pi}$, $\frac{\mathrm{d}^2C}{\mathrm{d}r^2} = 1.13097 > 0$			
		Since the cost is minimum, the company should choose *r* found in (b) .			

10	(a)	$y = 3\sin\frac{x}{2}$			
		$\frac{dy}{dx} = 3\left(\frac{1}{2}\right)\cos\frac{x}{2}$ $= \frac{3}{2}\cos\frac{x}{2}$			
		At $x = \frac{4\pi}{3}$,			
		$\frac{dy}{dx} = \frac{3}{2}\cos\frac{2\pi}{3}$			
		$= -\frac{3}{4}$ (gradient of tangent)			
		Gradient of normal = $= \frac{4}{3}$			
		At $x = \frac{4\pi}{3}, y = \frac{3\sqrt{3}}{2}$			
		Eqn of normal: $\frac{3\sqrt{3}}{2} = \frac{4}{3}\left(\frac{4\pi}{3}\right) + c$			
		$y = \frac{4}{3}x - \frac{16\pi}{9} + \frac{3\sqrt{3}}{2}$			
		$y = 0$ $\frac{16\pi}{9} - \frac{3\sqrt{3}}{2} = \frac{4}{3}x$			
		$x = \frac{3}{4}(\frac{16\pi}{9} - \frac{3\sqrt{3}}{2})$			
		$x = \frac{4\pi}{3} - \frac{9\sqrt{3}}{8}$			
		$Q(\frac{4\pi}{3} - \frac{9\sqrt{3}}{8}, 0)$			
	(b)	Shaded area $= \int_0^{\frac{4\pi}{3}} 3\sin\frac{x}{2}\ dx - \frac{1}{2} \times \frac{3\sqrt{3}}{2} \times \frac{9\sqrt{3}}{8}$			
		$= \left[-6\cos\frac{x}{2}\right]_0^{\frac{4\pi}{3}} - 2\frac{17}{32}$			
		$= -6\cos\frac{2\pi}{3} - (-6\cos 0) - 2\frac{17}{32}$			

		$= 3 + 6 - 2\frac{17}{32}$				
		$= 6\frac{15}{32}$ or 6.47 units^2				
11	(a)	At B , $v = 0$				
		$10e^{-0.1t} - 5 = 0$				
		$e^{-0.1t} = \frac{1}{2}$				
		$-0.1t = \ln\frac{1}{2}$				
		$0.1t = \ln 2$				
		$t = 10\ln 2$				
	(b)	$s = \int v \, dt$				
		$= \int 10e^{-0.1t} - 5 \, dt$				
		$= -100e^{-0.1t} - 5t + c$				
		when $t = 0$, $s = 0$, 4 $c = 100$				
		$s = -100e^{-0.1t} - 5t + 100$				
		when $t = 10 \ln 2$,				
		$s = -100e^{-\ln 2} - 50\ln 2 + 100$				
		$= -15.3\text{m}$				
		Distance $AB = 15.3$ m				
		Alternative Method:				
		Distance $AB = \int_0^{10\ln 2} \left(10e^{-0.1t} - 5\right) dt$				
		$= \left[-100e^{-0.1t} - 5t\right]_0^{10\ln 2}$				
		$= (-100e^{-\ln 2} - 50\ln 2) - (-100 - 0)$				
		$= -50 - 50\ln 2 + 100$				
		$= 15.3$ m (3s.f)				
	(c)	$a = \frac{\mathrm{d}v}{\mathrm{d}t} = -e^{-0.1t}$				
		When $t = 3$, $a = -e^{3} = -0.741 \text{ m/s}^2$ (3s.f)				
	(d)	$s = -100e^{-0.1t} - 5t + 100$				

		when t = 15, $s=-100e^{-0.1(15)}-5(15)+100=2.69\text{m}$			
		when t = 16, $s=-100e^{-0.1(16)}-5(16)+100=-0.1897\text{m}$			
		Since the displacement changes from positive 2.69m at t = 15 to negative 0.1897 m at t = 16,			
		4 the particle was at A again during the sixteenth second.			