1 A cuboid has a base area of $(7+4\sqrt{5})\text{cm}^2$ and a volume of $(16+18\sqrt{5})\text{cm}^3$. Find, without using a calculator, the height of the cuboid, in cm, in the form $(a+b\sqrt{5})$, where a and b are integers.

[3]

$$h=\frac{16+18\sqrt{5}}{7+4\sqrt{5}}\times\frac{7-4\sqrt{5}}{7-4\sqrt{5}}$$

$$=\frac{112-64\sqrt{5}+126\sqrt{5}-360}{-31}$$

$$=8-2\sqrt{5}$$

2 The curve $5x - xy = 20$ and the line $x - 2y - 3 = 0$ intersects at the points A and B.
Find the y −coordinate of A and of B. [3]

$sub\ x = 2y + 3\ into\ first\ equation,$

$5(2y + 3) - y(2y + 3) = 20$

$2y^2 - 7y + 5 = 0$

$(2y - 5)(y - 1) = 0$ or by quadratic formula

$$y = 2.5\ or\ 1$$

3 **(a)** Express $12x - 13 - 3x^2$ in the form $a(x+b)^2 + c$ and hence state the coordinates of the turning point of the curve $y = 12x - 13 - 3x^2$. [4]

$-3(x^2 - 4x) - 13$ or $-3(x^2 - 4x + \frac{13}{3})$ ------

$= -3[(x-2)^2 - 4] - 13$ or $= -3[(x-2)^2 - 4 + \frac{13}{3}]$

$= -3(x-2)^2 - 1$

Turning point $(2, -1)$

(b) State the range of k such that $y = k$ will intersect the curve $y = 12x - 13 - 3x^2$. [1]

$k \leq -1$

4 Integrate $\frac{4}{5x+1} - \frac{6}{x^3}$ with respect to x. [2]

$\int \frac{4}{5x+1} - 6x^{-3} dx = \frac{4\ln(5x+1)}{5} + \frac{3}{x^2} + c$

5 Express $\frac{7x^2-17x+1}{(x^2+1)(2-3x)}$ in partial fractions. [5]

$\frac{Ax+B}{x^2+1} + \frac{C}{2-3x}$ M1

$7x^2 - 17x + 1 = (Ax + B)(2 - 3x) + C(x^2 + 1)$

A = -4,

B = 3

C = -5

$\frac{3-4x}{x^2+1} - \frac{5}{2-3x}$-----------------

6 Given that $x^5 + ax^3 + bx^2 - 3 \equiv (x^2 - 1)Q(x) - x - 2$, where $Q(x)$ is a polynomial.

(a) State the degree of $Q(x)$. [1]

Degree of $Q(x)$ = 3

(b) Show that $a = -2$ and $b = 1$. [3]

Sub $x = 1$, $a + b = -1$ ------equation

Sub $x = -1$, $-a + b = 3$ -------equation

Solve simultaneous equations,

$a = -2$ and $b = 1$ ---------------

(c) Find the polynomial $Q(x)$. [3]

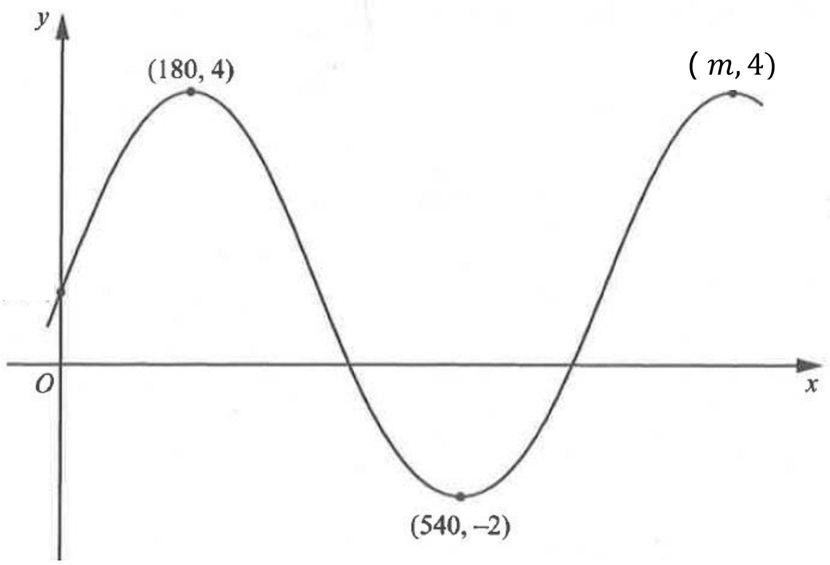
$Q(x)(x^2 - 1) = x^5 - 2x^3 + x^2 - 3 + x + 2$

$Q(x) = (x^5 - 2x^3 + x^2 - 3 + x + 2) \div (x^2 - 1)$---

By long division or comparing terms,

$$Q(x) = x^3 - x + 1$$

7



The sketch above shows part of the graph of $y = a\sin\left(\frac{x}{b}\right) + c$, where x is in degrees.

(a) Explain why $c = 1$. [2]

amplitude is 3.
So the maximum value of y is 4, 3+c =4, means that c = 1. for showing how to get c

(b) State the value of b. [1]

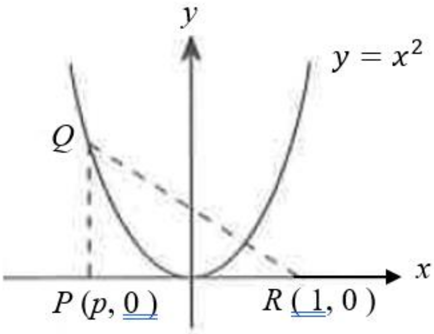
b = 2.

(c) Find the value of m and explain how you get it. [2]

The period is 720°. ----------------------

Hence m = 180 + 720 = 900

8 The figure shows the curve $y = x^2$ and the point $R(1,0)$. The variable point $P(p, 0)$ moves along the x axis and PQ is vertical. p is decreasing at the rate of 1.2 units per second.



(a) Show that he area of the triangle PQR, A units^2, is $A = \frac{1}{2}p^2 - \frac{1}{2}p^3$. [2]

getting y coordinate of Q = p^2 ----

A = $\frac{1}{2}(1-p)(p^2)$
= $\frac{1}{2}p^2 - \frac{1}{2}p^3$(Shown)

(b) Find the rate at which A is increasing at the instant when $p = -7$ units. [4]

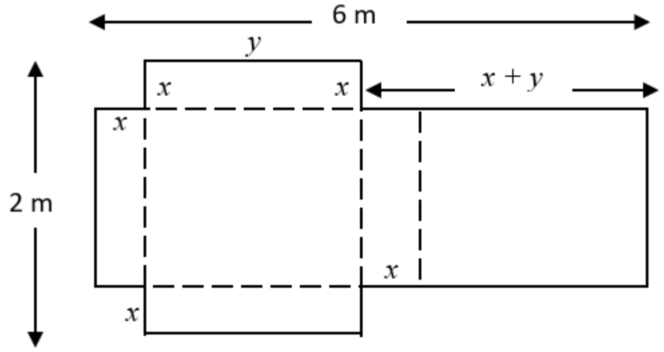
$$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$$

$\frac{dA}{dt} = p - \frac{3}{2}p^2$------

$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt}$-----

When p = -7,
$\frac{dA}{dt}$ = -80.5×-1.2
= 96.6 unit^2 per second

9 From a rectangular piece of metal of width 2m and length 6m, two squares of side *x* m and two rectangles of sides *x* m and (*x* + *y*) m are removed as shown. The metal is then folded about the dotted lines. To give a closed box with height *x* m.



(a) Show that the volume of the box, V m^3, is given by $V = 2x^3 - 8x^2 + 6x$. [3]

Length = y , breadth = 2-2x , height = x

$x + y + x + y = 6$---→ $y = 3 - x$

V = $xy(2 - 2x)$
= $x(3 - x)(2 - 2x)$
= $2x^3 - 8x^2 + 6x$ ($shown$) $- - - -$

(b) Given that *x* can vary, find the stationary value of *V*. [4]

$\frac{dv}{dx} = 6x^2 - 16x + 6$ --

$$\frac{dv}{dx} = 0 \rightarrow 6x^2 - 16x + 6 = 0$$

Solve using quadratic formula (you need to show working)

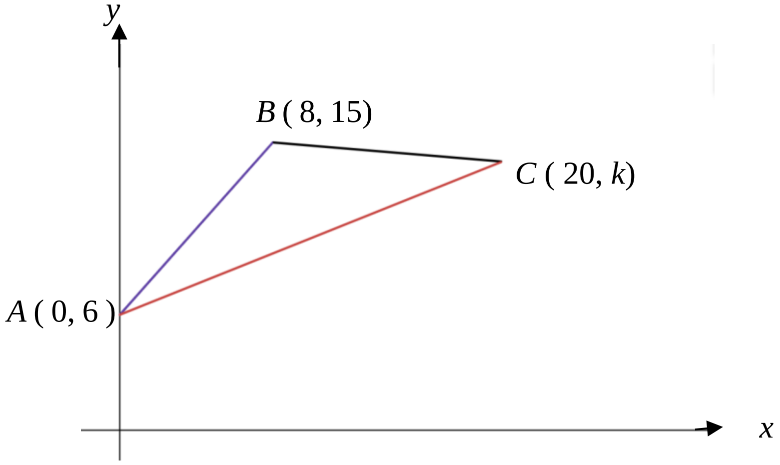
$$x = 2.215\ (\mathit{reject})\ or \quad 0.4514$$

Stationary value of V = 1.26 m^3

(c) Shows that this value of *V* is the maximum. [2]

Show either by first or second derivative test

10 The diagram shows a triangle ABC with vertices at A (0, 6) , B (8, 15) and C (20, k).



(a) Given that $AB = BC$, find the value of k. [4]

Since AB = BC, it means $8^2 + 9^2 = 12^2 + (15 - k)^2$----

$(15 - k\ )^2 = 1$

(15 – k) = 1 or (k – 15) = -1

$k\ = 14$ -----

(b) D is a point on the x axis such that $ABCD$ is a kite. Find the coordinates of D. [4]

$M_{AC} = \frac{2}{5}$, so $M_{BD} = -\frac{5}{2}$

Midpoint of AC = (10, 10) ------

Find Equation of BD : $y = -\frac{5}{2}x + 35$

Get D (14, 0)

(c) Hence, find the area of the kite *ABCD*. [2]

Shoelace method or sum of area of triangles -----

Area of kite ABCD = 174 units^2------------

11 (a) Prove the identity $\cot\theta + \frac{\sin\theta}{1+\cos\theta} = \operatorname{cosec}\theta$. [3]

$LHS = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{\cos\theta + cos^2\theta + sin^2\theta}{\sin\theta(1+\cos\theta)}$ ----------

$= \frac{cos\theta+1}{\sin\theta(1+\cos\theta)}$--------------

$= \frac{1}{\sin\theta}$

$= cosec\ \theta$

(b) Hence solve the equation $\cot 3\theta + \frac{\sin 3\theta}{1+\cos 3\theta} = -2$ for $-90° \leq \theta \leq 90°$. [4]

$cosec\ 3\theta = -2$------M1
$\sin 3\theta = -0.5$
Basic angle = 30

$3\theta = -30, -150, 210$
$\theta = -10, -50, 70$ ------------

12 (a) Solve $6^{2x-1} = 4^x \times 5$ [4]

$\frac{6^{2x}}{6} = 4^x \times 5$

$\frac{36^x}{4^x} = 30$
$9^x = 30$
$x = \log_9 30 \ \ or \ \ 1.548$

(b) Given that $\log_x 3 = p$, express $\log_3 \frac{27}{x}$ in terms of p. [2]

$\log_3 27 - \log_3 x$----------
$=3 - \frac{1}{p}$ ----------

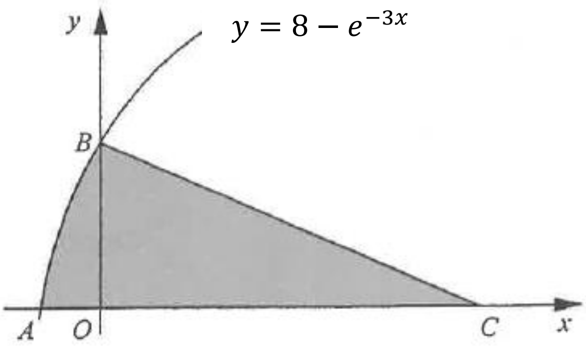
(c) Solve the equation $\lg x - 1 = \lg(x - 1)$ [3]

$$\lg x - \lg 10 = \lg(x - 1)$$

$$\frac{x}{10} = x - 1$$

$$x = \frac{10}{9}$$

13 The diagram shows part of the curve $y = 8 - e^{-3x}$ which crosses the axes at *A* and at *B*.



(a) Show that the *x* coordinate of *A* is $-\ln 2$. [3]

$when\ y = 0, e^{-3x} = 8$---
$-3x = \ln 8$
$x = -\frac{1}{3}\ln 8$----
$x = -\ln 8^{1/3}$
$x = -\ln 2$---------

(b) The normal to the curve at *B* meets the *x* axis at *C*. Find the coordinates of *C*. [4]

$\frac{dy}{dx} = 3e^{-3x}$---

At B, x = 0, so dy/dx = 3-----
Find Gradient of normal at B = -1/3

Find B (0, 7) -----------

Equation of normal : $y = -\frac{1}{3}x + 7$

So C (21, 0) ---------------

(c) Find the area of the shaded region. [4]

Area of OAB = $\int_{-\ln 2}^{0} 8 - e^{-3x} dx$

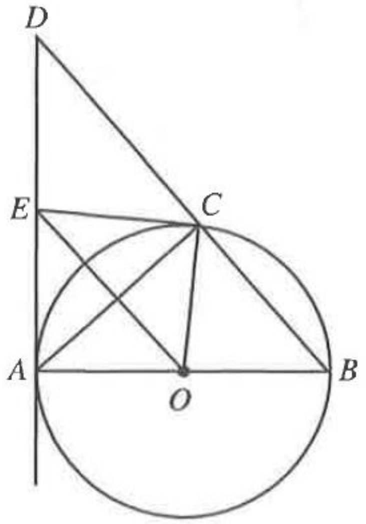
$= \left[8x + \frac{e^{-3x}}{3}\right]_{-\ln 2}^{0}$ ---------------

$=8\ln 2 - \frac{7}{3}$-----------------

Area of triangle OBC = 0.5 x 21 x 7 = 73.5 -----

Area of shaded region = 76.7 units^2 ----------------

14



The diagram shows a circle, centre *O*, with diameter *AB*. The point *C* lies on the circle. The tangent to the circle at *A* meets *BC* extended at *D*. The tangent to the circle at *C* meets the line *AD* at *E*.

(a) Prove that triangles *EOA* and *EOC* are congruent. [3]

AE = CE (tangents from external point are equal)
AO = OC (radius)
OE = OE (common side) ---------------------------

Hence triangle EOA is congruent to triangle EOC (SSS) ------------

(b) Prove that triangles *ADC* and *BAC* are similar. [3]

Angle ACB = 90 (angle in a semi circle)

Angle DCA = 90 (sum of angles on a straight line)

Hence angle ACB = angle DCA (A) -----

Angle CAD = Angle CBA (A)
(angles in alternate segment/ tangent chord theorem) ----

Hence triangles *ADC* and *BAC* are similar. (AA) -----------

(c) Hence prove that $AC^2 = CD \times BC$. [2]

$Since\ they\ are\ similar\ , \frac{AC}{BC} = \frac{DC}{AC}$----

$AB^2 = BC \times DC$------------

- End of Paper -