

RAFFLES INSTITUTION 2024 YEAR 6 PRELIMINARY EXAMINATION

CANDIDATE NAME	
CLASS	24

MATHEMATICS

Paper 2

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in. Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs. Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper. You may use the blank pages on page 22, 23 and 24 if necessary and you are reminded to indicate the question number(s) clearly. Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands. You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

For Examiner's Use Only													
н .	Q1	Q2		Q3		Q4		Q5	TOTAL				
Section <i>I</i> Pure Mat	/ 8		/ 5	17		/ 10		/ 10					
3: ats	Q6	Q7	Q8		Q9		Q10	Q11					
Section E Prob & Sta	/7	/ 8	/ 11		/ 10		/ 12	2 / 12	100				

This document consists of **21** printed pages and **3** blank pages.

© RI2024

RAFFLES INSTITUTION Mathematics Department 9758/02

3 hours

1 The function f is defined by
$$f: x \mapsto \frac{2x}{x-2}$$
, for $x \in \mathbb{R}$, $x \neq 2$.

(a) Sketch the graph of f and find its range.

[3]



Another function g is defined by $g: x \mapsto 3 + |x+2|$, for $x \in \mathbb{R}$.

(b) Show that the composite function fg exists. Find fg(x) and state the domain and range of fg.

2 The function f is defined by $f(z) = z^4 + Az^3 + Bz^2 + Cz + 45$, where A, B and C are real numbers. Given that 2+i is a root of f(z) = 0 and $(z-k)^2$ is a factor of f(z), where k is a positive real number, find the values of A, B, C and k. [5]

3505

(a) The points A, B and C on the plane π have position vectors **a**, **b** and **c** respectively. Show that a vector perpendicular to π is parallel to

$$\mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a} + \mathbf{a} \times \mathbf{b} \,. \tag{3}$$

- (b) **p** and **q** are non-zero vectors such that $\mathbf{p} = (\mathbf{p} \cdot \mathbf{q})\mathbf{q}$.
 - (i) Find the relationship between \mathbf{p} and \mathbf{q} .

[1]

(ii) Find $|\mathbf{q}|$.

(c) **u** is the position vector of a fixed point U relative to the origin O. A variable point V has position vector **v** relative to O.

5

Given that $\mathbf{v} \cdot (\mathbf{v} - \mathbf{u}) = 0$, describe geometrically the set of all possible positions of the point *V*. [2]

4 (a) Given that $y = e^{\sqrt{1+2x}}$, show that

$$(1+2x)\left(\frac{dy}{dx}\right)^2 = y^2$$
 and $(1+2x)\frac{d^2y}{dx^2} + \frac{dy}{dx} = y$. [3]

(b) Hence, or otherwise, obtain the series expansion for y in terms of x up to and including the term in x^3 . [3]

(c) Verify that the same series expansion for y in part (b) is obtained if the standard series expansions for e^x and $(1+x)^n$ are used. [4]



The diagram shows the curve C with parametric equations $(1 - 1)^2$

$$x = (1+t)^2$$
, $y = (3-t)^2$.

The curve C meets the axes at (16, 0) and (0, 16).

5

(a) Show that the line x = 16 meets C at the point P where t = -5. [1]

The normal to C at P is denoted by l.

(b) Find the cartesian equation of l.

[3]

(d) Show that the area (in units²) of the region bounded by C and l can be given by

$$\frac{266240}{81} + \int_{c}^{d} f(t) dt,$$

where f(t), and the constants c and d are to be determined. Hence find the value of this area. [3]

Section B: Probability and Statistics [60 marks]

- 6 Eleven cards each bears a single letter and together they can be made to spell the word COFFEEHOUSE. The 11 cards are arranged in a row.
 - (a) Find the number of different arrangements that can be made. [1]

(b) Find the number of different arrangements in which the 2 F's are next to each other and no E's are next to each other. [3]

(c) if the three cards all bear different letters, [1]

(d) if exactly two of the three cards bear the same letter.

[2]

The probability of obtaining a head when a particular coin is tossed is p. A fair cubical die has the number '1' on one face, number '2' on two faces and number '3' on three faces.

The coin and die are thrown simultaneously. The random variable X is defined as follows.

If the coin shows a head, then X is thrice the score on the die. If the coin shows a tail, then X is the score on the die.

(a) Show that $P(X = 3) = \frac{1}{2} - \frac{1}{3}p$, and find the probability distribution of X. [3]

(b) Given that E(X) = 5, find the exact value of p. [2]

(c) Using the value of p found in part (b), find the exact value of Var(X). [3]



(a) It is given that X is the number of times a student is late for school in a year and Y is the student's performance in the Mathematics Examination. The product moment correlation coefficient of a bivariate sample (x_i, y_i) , i = 1, 2, ..., n, for n students is r.

State, giving a reason, whether each of the following statements is true or false.

(i) When the value of r is zero, it can be implied that the variables X and Y are not related. [1]

- (ii) When the value of r is -1, it can be implied that late-coming causes poor performance in the subject. [1]
- (b) The table below shows the daily sales of cups of iced coffee in a week by a shop and the maximum daily temperature.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Temperature (°C), t	30.5	31.3	31.9	34.8	25.9	28.5	29.2
Daily sales, y	95	101	115	79	81	86	88

(i) Sketch a scatter diagram of y against t, labelling the axes clearly. [2]

One of the values of y appears to be incorrect.

(ii) Indicate the corresponding point on your diagram by labelling it *P*. Omitting *P*, find the equation of the least squares regression line of $\frac{1}{y}$ on *t*, and the value of the product moment correlation coefficient between $\frac{1}{y}$ and *t*. Comment on this value. [5]

Use an appropriate regression line to give an estimate of the daily sales (iii) when the temperature is 20.4 $^{\circ}C$.

State, with a reason, whether the estimate is reliable. [2]

21 SON

- (a) The masses of a randomly chosen bolt and a randomly chosen nut are denoted by M grams and W grams respectively. M and W are independent random variables with the distributions N(272, 8²) and N(98, 5²) respectively.
 - (i) Find the range of values of a for which P(a < M < 275) > 0.2024. [3]

(ii) Calculate the probability that twice the mass of a randomly chosen bolt differs from the total mass of 5 randomly chosen nuts by less than 80 grams.
[3]

(b) Bolts are manufactured to fit into holes in steel plates. The bolts have diameters, in cm, that follow the distribution N(2.65, 0.03²) and the diameters of the holes, in cm, follow the distribution N(2.72, 0.02²). A manufacturer sells boxes of twenty pairs of these bolts and steel plates, where each pair consists of one randomly selected bolt and one randomly selected steel plate. A pair is acceptable only if the diameter of the hole in the steel plate is at least 0.02 cm larger, but no more than 0.15 cm larger, than the diameter of the bolt. Use a normal distribution to estimate the probability that the average number of acceptable pairs in 50 boxes is more than 18.

- 10 (a) A company has a machine designed to fill bags with, on average, μ_0 kg of salt. The mass of salt in a randomly chosen bag has a normal distribution with population standard deviation denoted by σ kg. The production manager wishes to investigate if the machine is adjusted correctly. He takes a random sample of *n* bags and carries out a hypothesis test at the 1% level of significance.
 - (i) State null and alternative hypotheses for the manager's test, defining any parameters you use. [2]

(ii) Find, in terms of μ_0 , σ and *n*, the critical region(s) for this test. [4]

(b) The company has a different machine which fills bags with, on average, 25 kg of low sodium salt. One of the company's production supervisor has reported that some of the workers suspect the machine is no longer set correctly, and the average mass of low sodium salt in the bags may in fact be more than 25 kg. The production supervisor decides to carry out a hypothesis test at the 0.5% level of significance with a random sample of 80 bags of low sodium salt. Summary data for the mass, y kg, of low sodium salt in these bags is as follows.

$$n = 80$$
 $\sum (y-25) = 27.2$ $\sum (y-25)^2 = 85.1$

(i) Carry out the test and state the conclusion of the test in the context of the question. [5]



(ii) State, giving a reason, whether any assumptions about the population are needed in order for the test to be valid. [1]

- 11 Two brothers Kai and Leo are duathlon (running and cycling) athletes who train regularly. For each day, the probability that Kai cycles is $\frac{3}{4}$, the probability that he runs is $\frac{3}{5}$ and the probability he does both is p.
 - (a) Write down, in terms of p, the probability that, on one day, Kai either runs or cycles but not both. [1]

Sol

(b) Find the range of possible values of p.

[2]

On average, Leo cycles 5 out of 7 days in a week. The probability that Leo cycles when Kai cycles is 0.9.

(c) Find the probability that Leo cycles when Kai does not. [3]

(d) State, in context, two assumptions needed for the number of days Leo cycles over a period of 5 weeks to be well modelled by a binomial distribution. [2]

Assume now that the number of days Leo cycles in 5 weeks has a binomial distribution.

(e) Find the probability that, in the 5 weeks, Leo cycles on at least 20 days but fewer than 30 days. [2]

(f) Find the probability that, in the 5 weeks, there are exactly 5 days in which both brothers do not cycle and they both cycle on the other days. [2]

BLANK PAGE

You may continue your working on this page if necessary, indicating the question number(s) clearly.

BLANK PAGE

You may continue your working on this page if necessary, indicating the question number(s) clearly.

BLANK PAGE

You may continue your working on this page if necessary, indicating the question number(s) clearly.