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PRESBYTERIAN HIGH SCHOOL

ADDITIONAL MATHEMATICS **4049/01**
Paper 1

25 August 2021 Wednesday 2 hours 15 min

2021 SECONDARY FOUR EXPRESS
PRELIMINARY EXAMINATIONS

MARK SCHEME

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This question paper consists of **16** printed pages and **0** blank pages.

Answer all questions in the space provided.

1 Solve the following simultaneous equations.

$$2^{x+6y}=\frac{1}{32} \quad \text{and} \quad \left(9^x\right)\left(729^y\right)=243.$$

[4]

$2^{x+6y}=\frac{1}{32}=2^{-5} \qquad \Rightarrow x+6y=-5.....(1)$	M1
$\left(9^x\right)\left(729^y\right)=243$	
$\left(3^{2x}\right)\left(3^{6y}\right)=3^5 \qquad \Rightarrow 2x+6y=5.....(2)$	M1
$(2)-(1): \qquad x=10$	A1
$y=-\frac{5}{2}$	A1

2 **(i)** Express $y=-2x^2+6x-1$ in the form $a(x+b)^2+c$, where *a*, *b* and *c* are constants. [2]

$y=-2x^2+6x-1$	
$=-2\left(x^2-3x\right)-1$	
$=-2\left[x^2-3x+\left(-\frac{3}{2}\right)^2-\left(-\frac{3}{2}\right)^2\right]-1$	M1
$=-2\left[\left(x-\frac{3}{2}\right)^2-\left(\frac{9}{4}\right)\right]-1$	
$=-2\left(x-\frac{3}{2}\right)^2+\frac{9}{2}-1$	
$=-2\left(x-\frac{3}{2}\right)^2+\frac{7}{2}$	A1

(ii) Hence, determine with explanation, whether the curve $y=-2x^2+6x-1$ lies entirely below the *x*-axis. [2]

From part (i), the maximum point is $\left(\frac{3}{2},\frac{7}{2}\right)$, the curve does not lie entirely below *x*-axis.	M1 A1

3 If the line $x-ky=10$ is a tangent to the curve $x^2+y^2=20$, find the possible values of *k*. [5]

$x-ky=10 \quad \Rightarrow x=10+ky.......(1)$ $x^2+y^2=20........(2)$ Sub. into (1) into (2) $(10+ky)^2+y^2=20$ $100+20ky+k^2y^2+y^2-20=0$ $(k^2+1)y^2+20ky+80=0$ $b^2-4ac=(20k)^2-4(k^2+1)(80)=0$ $400k^2-320k^2-320=0$ $80k^2-320=0$ $80(k^2-4)=0$ $\therefore \quad k^2=4$ $k=2 \quad \text{or} \quad -2$	 M1 (Substitution) M1 (Expansion) M1 (Discriminant), M1 (=0) A1

4 Given that $\tan^2\theta = p$, where $90^o \le \theta \le 180^o$, express in terms of *p*,

(i) $\cos\theta$, [2]

(ii) $\operatorname{cosec} 2\theta$. [3]

(i) $\tan\theta=\sqrt{p}$ (rej) or $-\sqrt{p}$ $\cos\theta=-\dfrac{1}{\sqrt{p+1}}$	 M1 A1
(ii) $\operatorname{cosec} 2\theta=\dfrac{1}{\sin 2\theta}$ $=\dfrac{1}{2\sin\theta\cos\theta}$ $=\dfrac{1}{2\left(\dfrac{\sqrt{p}}{\sqrt{p+1}}\right)\left(-\dfrac{1}{\sqrt{p+1}}\right)}$ $=-\dfrac{p+1}{2\sqrt{p}}$	 M1 M1 (Either ratio) A1

5 **(i)** *PQR* is an equilateral triangle whose side is $\left(3\sqrt{3}-1\right)$ cm. Find the exact value of the area of the equilateral triangle *PQR*, in the form $a\sqrt{3}+b$ where a and b are rational numbers. [4]

Area $=\frac{1}{2}\left(3\sqrt{3}-1\right)^2\sin 60^\circ$	M1
$=\frac{1}{2}\left(27-6\sqrt{3}+1\right)\frac{\sqrt{3}}{2}$	M1 (Expansion), B1 ($\frac{\sqrt{3}}{2}$ seen)
$=\frac{\sqrt{3}}{4}\left(28-6\sqrt{3}\right)$	
$=7\sqrt{3}-\frac{9}{2}$ cm^2	A1

(ii) A right prism with the equilateral triangle *PQR* as the cross-sectional base is to be made such that the volume of the prism is $8\left(3\sqrt{3}-1\right)$ cm^3. Find the height of the prism, giving your answer in surd form. [3]

Height $=\dfrac{8\left(3\sqrt{3}-1\right)}{\frac{1}{2}\left(3\sqrt{3}-1\right)^2\sin 60^\circ}$	M1
$=\dfrac{8}{\frac{1}{2}\left(3\sqrt{3}-1\right)\frac{\sqrt{3}}{2}}$	
$=\dfrac{32}{9-\sqrt{3}}\times\dfrac{9+\sqrt{3}}{9+\sqrt{3}}$	M1 (Conjugate)
$=\dfrac{32\left(9+\sqrt{3}\right)}{81-3}$	
$=\dfrac{32\left(9+\sqrt{3}\right)}{78}$	
$=\dfrac{16\left(9+\sqrt{3}\right)}{39}$ cm	A1

6 A man buys a precious gem. The value, V dollars, of the gem after t years is given by $V = N(0.97)^{kt}$, where N and k are constants. At the beginning, the value of the gem is \$12000.

(i) Find the value of N. [1]

$N = 12000$	B1

(ii) The value of the gem after 5 years is \$10000. Find the value of k. [4]

$10000 = 12000(0.97)^{5k}$	M1
$(0.97)^{5k} = \frac{5}{6}$	
$5k \lg 0.97 = \lg \frac{5}{6}$	M1 (Taking lg on both sides) M1 (Power Law)
$k = \frac{\lg \frac{5}{6}}{5 \lg 0.97} = 1.197 = 1.20$	A1

(iii) After 15 years, a gem dealer offers to pay the man \$5000 for the gem. Based on the given equation, would you advise him to sell it? Justify your answer. [2]

$V = 12000(0.97)^{1.197(15)}$ $= 6944.927$	M1
After 15 years, the value of the gem based on the equation is \$6944.93 which is more than the amount offered by the dealer. So, he should not sell it to the dealer.	A1(o.e.)

7 A curve has equation $y=\dfrac{3\tan^2 x}{e^x}$.

(i) Find the gradient of the curve when $x=\dfrac{\pi}{4}$, leaving your answer in the exact form. [5]

$y=\dfrac{3\tan^2 x}{e^x}$ $\dfrac{dy}{dx}=\dfrac{e^x\left(6\tan x\sec^2 x\right)-3\tan^2 x\left(e^x\right)}{e^{2x}}$	M1 (Quotient Rule), M2 (1 for each correct derivative)
When $x=\dfrac{\pi}{4}$, gradient $=\dfrac{6e^{\frac{\pi}{4}}\tan\dfrac{\pi}{4}\left(\dfrac{1}{\cos^2\dfrac{\pi}{4}}\right)-3e^{\frac{\pi}{4}}\left(\tan\dfrac{\pi}{4}\right)^2}{e^{\frac{\pi}{2}}}$	M1
$=\dfrac{3e^{\frac{\pi}{4}}\left[(2)(1)(2)-(1)\right]}{e^{\frac{\pi}{2}}}$ $=9e^{-\frac{\pi}{4}}$	A1

(ii) Given that x is increasing at a constant rate of 0.12 units per second, find the rate of change of y when $x=\dfrac{\pi}{4}$. [2]

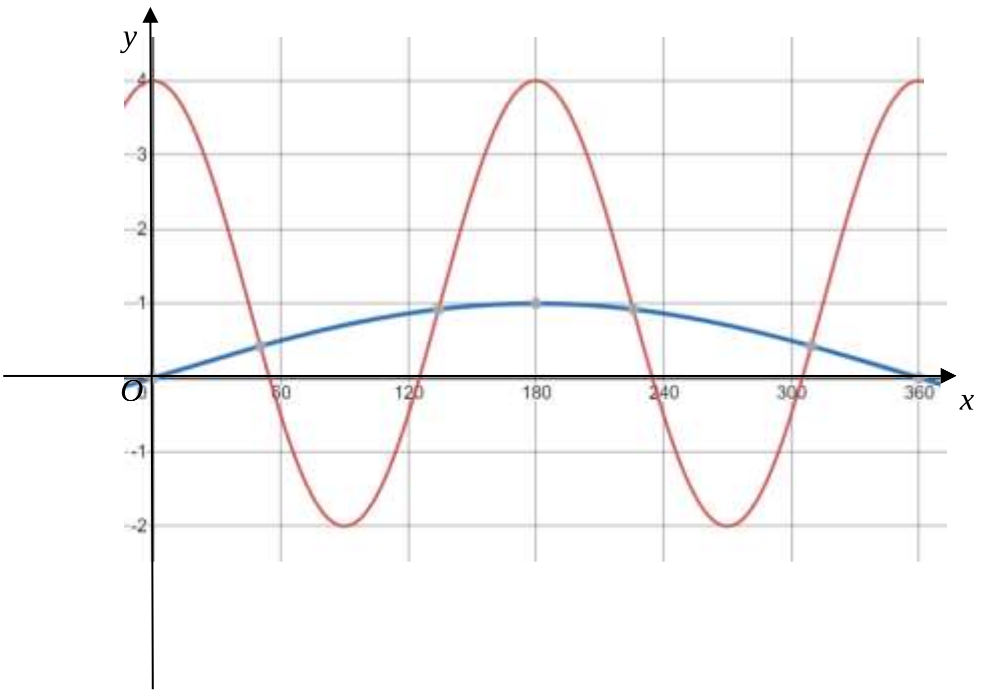
$\dfrac{dx}{dt}=0.12$ $\dfrac{dy}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ When $x=\dfrac{\pi}{4}$, $\dfrac{dy}{dt}=9e^{-\frac{\pi}{4}}\times 0.12$	M1
$=0.492$ unit/s	A1

8 The function $f(x) = 3\cos 2x + 1$ is defined for $x \geq 0°$.

(i) State the amplitude and period of f. [2]

Amplitude = 3 Period = 180°	B1 B1

(ii) Sketch on the same diagram below, the graphs of $f(x) = 3\cos 2x + 1$ and $g(x) = \sin\left(\frac{x}{2}\right)$ for $0° \leq x \leq 360°$. [4]



$f(x)=3\cos 2x+1$ $g(x)=\sin\left(\frac{x}{2}\right)$	G2 for each curve Deduct I mark for wrong period/amplitude/shape for each curve

(iii) Hence determine the value of *k* for which the equation $3\cos(2x)+1=\sin\left(\frac{x}{2}\right)+k$ has 3 solutions for $0^\circ \le x \le 360^\circ$. [1]

k = 3	B1

10 **(a)** **(i)** Find the first three terms in the expansion, in ascending powers of x, of $\left(2-\frac{x}{3}\right)^7$. [2]

$\left(2-\frac{x}{3}\right)^7 = 2^7 + \binom{7}{1}2^6\left(-\frac{x}{3}\right) + \binom{7}{2}2^5\left(-\frac{x}{3}\right)^2 + \ldots$	M1 (any two terms correct)
$= 128 - \frac{448}{3}x + \frac{224}{3}x^2 + \ldots$	A1

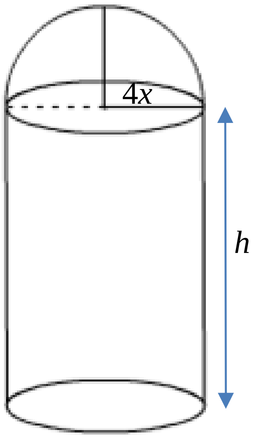
(ii) Hence find the value of p, where p is an integer, such that the coefficient of x^2 in the expansion of $(p+x)^2\left(2-\frac{x}{3}\right)^7$ is $-\frac{32}{3}p^2$. [3]

$(p+x)^2\left(2-\frac{x}{3}\right)^7 = \left(p^2+2px+x^2\right)\left(128-\frac{448}{3}x+\frac{224}{3}x^2+\ldots\right)$	
Coefficient of $x^2 = 128 + 2p\left(-\frac{448}{3}\right) + \frac{224}{3}p^2 = -\frac{32}{3}p^2$	M1 (any two terms correct)
$256p^2 - 896p + 384 = 0$	M1
$2p^2 - 7p + 3 = 0$	
$(p-3)(2p-1) = 0$	
$\therefore p = 3$ or $p = \frac{1}{2}$ (rejected)	A1

(b) Explain why there is no independent term in the expansion of $\left(x^2-\frac{1}{2x}\right)^{17}$. [4]

General Term $= \binom{17}{r}\left(x^2\right)^{17-r}\left(-\frac{1}{2x}\right)^r$	M1
Independent Term $\Rightarrow 34 - 2r - r = 0$	M1 (Correct power) B1 (Power =0)
$3r = 34$	
$r = \frac{34}{3}$	
Since $r = \frac{34}{3}$ is not an integer, there is no independent term.	AG1

11 [The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and the surface area is $4\pi r^2$.]
Mr Lim wants to make a solid cylinder with a hemisphere on top as shown in the diagram below. The cylinder has radius $4x$ cm and height h cm.
The volume of the entire solid is 896π cm^3.



(i) Show that $h = \frac{56}{x^2} - \frac{8}{3}x$. [2]

$\text{Vol } = \pi(4x)^2 h + \frac{1}{2}\left(\frac{4}{3}\pi(4x)^3\right) = 896\pi$	M1 (Any attempt to have two vols)
$16x^2\pi h = 896\pi - \frac{128}{3}\pi x^3$ $h = \frac{896\pi}{16x^2\pi} - \frac{128}{3(16x^2\pi)}\pi x^3$ $= \frac{56}{x^2} - \frac{8}{3}x$ (Shown)	AG1

(ii) Show that the total surface area of the solid is given by $A = \frac{80}{3}\pi x^2 + \frac{448\pi}{x}$ cm^2. [3]

Total Surface Area, $A = \pi(4x)^2 + 2\pi(4x)h + \frac{1}{2}\left(4\pi(4x)^2\right)$	M1 (Any attempt to have two SA)
$= 48\pi x^2 + 8\pi x\left(\frac{56}{x^2} - \frac{8}{3}x\right)$	M1
$= 48\pi x^2 + \frac{448\pi}{x} - \frac{64}{3}x^2$ $= \frac{80}{3}\pi x^2 + \frac{448\pi}{x}$ (Shown)	AG1

(iii) Find the value of x for which A is stationary. [2]

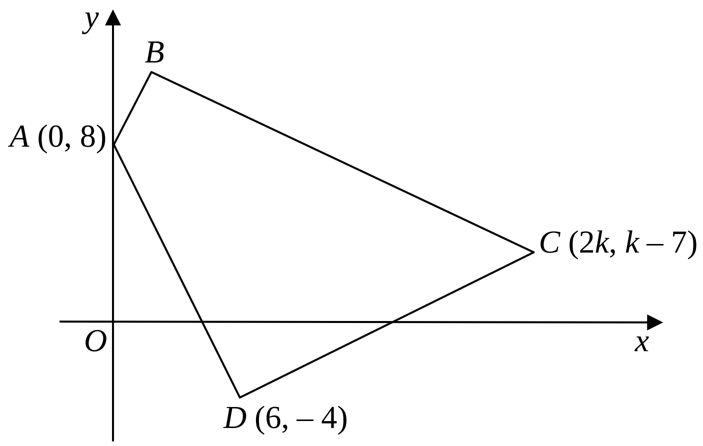
$\frac{dA}{dx}=\frac{160}{3}\pi x-\frac{448\pi}{x^2}=0$ $\frac{160}{3}\pi x=\frac{448\pi}{x^2}$ $x^3=\frac{42}{5}$ $x=2.0327=2.03$	M1 (At least one correct derivative) A1

(iv) The solid is to be painted completely. The cost of painting is \$0.70 per cm^2.
Using the value of x in **(iii)**, calculate the cost of painting the solid and determine whether Mr Lim would be pleased with the cost. [3]

$A=\frac{80}{3}\pi\left(2.032\right)^2+\frac{448\pi}{2.032}$ $=1038.5$ Cost $=\$1038.5\times 0.7$ $=\$726.95$ $\frac{d^2A}{dx^2}=\frac{160}{3}\pi+\frac{896\pi}{x^3}>0$ Mr Lim would be pleased as the cost is minimum.	M1 A1 A1

12 The diagram below shows a quadrilateral $ABCD$ in which coordinates of A, C and D are $(0,8)$, $(2k,k-7)$ and $(6,-4)$ respectively.

The equation of BC is $19y=-7x+197$ and the line $y=8$ bisects angle BAD.



(i) Show that $k = 10$. [2]

$19y=-7x+197$ $19(k-7)=-7(2k)+197$ $19k+14k=197+133$ $33k=330$ $\therefore k=10$ (Shown)	M1 AG1

(ii) Explain with working why angle *ADC* is 90°. [2]

$m_{AD}=\dfrac{8+4}{-6}=-2$ $m_{CD}=\dfrac{3+4}{20-6}=\dfrac{1}{2}$ $m_{AD}\times m_{CD}=-2\times\dfrac{1}{2}=-1$ $\Rightarrow \angle ADC=90^\circ$ (Shown)	M1 (Either gradient) AG1

(iii) Find the equation of *AB*. [2]

$m_{AD}=-2\Rightarrow m_{AB}=2$	B1
Equation of AB is $y=2x+8$	B1

(iv) Calculate the area of quadrilateral *ABCD*. [4]

$y=2x+8$.....(1) $19y=-7x+197$.....(2) Sub. (1) into (2): $19(2x+8)=-7x+197$ $38x+7x=197-152$ $45x=45$ $x=1$ $y=10$ $\therefore B(1,10)$ $\text{Area}=\dfrac{1}{2}\begin{vmatrix}0 & 6 & 20 & 1 & 0\\ 8 & -4 & 3 & 10 & 8\end{vmatrix}$ $=\dfrac{1}{2}\left[(18+200+8)-(3-80+48)\right]$ $=\dfrac{1}{2}(255)=127\dfrac{1}{2}\text{ unit}^2$	M1 A1 M1 A1

13 A particle moves in a straight line so that t seconds after leaving a fixed point O, its velocity, v m/s is given by $v = kt^2 + 12t - 16$ where k is a constant. When $t = 1$, the acceleration of the particle is 8 m/s^2.

(i) Show that $k = -2$. [2]

$v = kt^2 + 12t - 16$ $a = \frac{dv}{dt} = 2kt + 12$ When $t = 1,\ a = 8$ $2k + 12 = 8$ $2k = -4$ $\therefore\ k = -2$ (Shown)	M1 AG1

(ii) Find the value of t when the velocity of the particle is equal to its initial velocity. [2]

Initial $v = -16$ $v = -2t^2 + 12t - 16 = -16$ $-2t(t-6) = 0$ $t = 0$ (N.A.) or $t = 6$	B1 A1

(iii) Find the values of t when the particle is instantaneously at rest. [2]

$v = -2t^2 + 12t - 16 = 0$ $-2\left(t^2 - 6t + 8\right) = 0$ $(t-2)(t-4) = 0$ $t = 2$ or $t = 4$	M1 A1

(iv) Find the distance travelled by the particle during the first 5 seconds. [4]

$v=-2t^2+12t-16$	
$s=\int v\,dt$	
$=-\frac{2t^3}{3}+\frac{12t^2}{2}-16t+c$	M1
When $t=0,\ s=0 \Rightarrow c=0$	M1
$s=-\frac{2t^3}{3}+6t^2-16t$	
When $t=2,\ s=-13\frac{1}{3}$ When $t=4,\ s=-10\frac{2}{3}$ When $t=5,\ s=-13\frac{1}{3}$	M1 (Any one)
Total distance travelled $=13\frac{1}{3}+(13\frac{1}{3}-10\frac{2}{3})(2)$	
$=18\frac{2}{3}$ m	A1