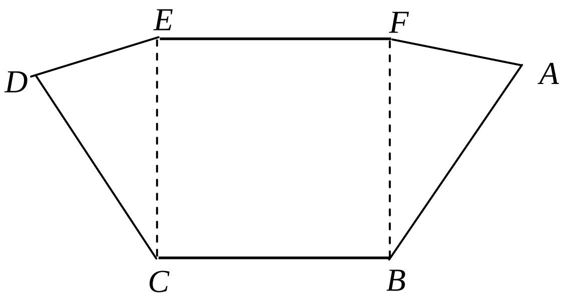
Anderson Junior College
Preliminary Examination 2012
H2 Mathematics Paper 2 (9740/02)

Section A: Pure Mathematics (40 marks)

1 The Environment Authority wishes to build a fence surrounding a park shown by the figure below. *BCEF* is a square with sides measuring 2 kilometres. *ECD* and *ABF* are two identical triangles with *CD* = *CE* and angle *DCE* = θ radians. The cost of building the fence is \$28000. Given that θ is sufficiently small for θ^2 and higher powers of θ to be neglected, show that the unit cost of building the fence is approximately $\$\,(a+b\theta)$ per kilometre, where *a* and *b* are constants to be determined. [5]



2 It is given that

$$\mathrm{f}(x)=\begin{cases} 4x & , \quad 0\le x<1 \\ (3-x)^2 & , \quad 1\le x\le 3 \end{cases}$$

and $\mathrm{f}(-x)=-\mathrm{f}(x)$. It is also known that $\mathrm{f}(x)=\mathrm{f}(x+6)$ for all real values of *x*.

(i) Show that f (4) = -1 . [2]

(ii) Sketch the graph of y =f(x) for $-6\le x\le 9$. [3]

(iii) Find the exact value of $\displaystyle\int_{-5}^{7}\left|\mathrm{f}(x)\right|\mathrm{d}x$. [3]

3 Relative to an origin *O*, the points *C* and *D* have position vectors $\underset{\sim}{c}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ and $\underset{\sim}{d}=\begin{pmatrix}3\\\alpha\\\beta\end{pmatrix}$ respectively, where α and $\beta\in\mathbb{R}$.

(i) The straight line *l*, passing through *C* and *D*, has cartesian equations $2-x=\dfrac{y-3}{3}$, $z=2$.

Find the values of α and β. [3]

(ii) Find the point of intersection of the line *l* and the *y*-*z* plane. [2]

(iii) The reflection of *C* in the *y*-*z* plane is C'. Find the position vector of C'. [3]

4 **(a)** By using the substitution $u = x - y$, solve the differential equation

$$\frac{dy}{dx} + 4(x-y)^2 \cos^2 x = \sin^2 x\,.$$ [5]

(b) A new drug for the treatment of diabetes is administered to a patient at a constant rate of R mg per day. The rate at which the drug is lost from the patient's body is proportional to the square of the amount x (mg) of the drug present in his body at time t (days).

(i) If the amount of drug in the patient remains constant at the instant when it is $2R$ (mg), show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{4R^2 - x^2}{4R}.$$

Given that, when $x = 0$ when $t = 0$, find x, in terms of R and t. [5]

(ii) Explain the significance of this result in the long run. [1]

5 The complex number z satisfies the equation

$$\sqrt{2}\, z^3 = 1 + \frac{2}{2a-1} i, \qquad \text{where } a \text{ is a real number.}$$

It is given that $\arg\left(z^3\right) = -\frac{\pi}{4}$.

(i) Show that $a = -\frac{1}{2}$. [2]

(ii) Hence solve the equation $\sqrt{2}\, z^3 = 1 + \frac{2}{2a-1} i$, expressing the solutions in polar form $r\left(\cos\theta + i\sin\theta\right)$, where $r > 0$ and $-\pi < \theta \le \pi$. Give r and θ in exact form. [3]

The complex number w satisfies the equation $\sqrt{2}\left(\frac{w}{1+i}\right)^3 = 1 - \mathrm{i}$. Describe how the point representing w can be obtained from the point representing z by a series of transformations. [3]

Section B: Statistics (60 marks)

6 Find the number of ways in which the letters of the word PERCEPTIVE can be arranged if

(i) all the vowels are separated; [2]

(ii) there are two letters between the two Ps, at least one of which must be an E. [3]

7 (a) Events A and B are such that $P(A) = 0.7$ and $P(B) = 0.8$. Show that

$$P(A \cap B) \geq 0.5$$ [2]

(i) If $P(A' \cup B) = 0.85$, find $P(A|B)$. State, with a reason, whether or not A and B are independent events. [3]

(ii) What is the relationship between event A and event B if $P(A \cap B) = 0.5$? [1]

(b) Eleven non-parallel lines are drawn in a plane. If no three lines intersect at a common point, show that there are 55 points of intersection between the lines.

If three of these points are chosen at random, find the probability that they are all on one of the eleven lines. [4]

8 At the drinks stall in a canteen, both hot and cold drinks are sold. The number of hot drinks sold per minute follows a Poisson distribution with mean 0.4. The number of cold drinks sold per minute follows an independent Poisson distribution with mean 0.5.

(i) Show that the probability that more than 4 drinks are sold altogether in a given 5-minute period is 0.468, correct to 3 significant figures. [2]

(ii) In a given 5-minute period, at most 4 drinks were sold. Find the probability that they are all cold drinks. [3]

(iii) A random sample of twenty 5-minute periods was chosen. If there is a probability of less than 0.4 that more than 4 drinks were sold in at least n of the periods, find the least value of n. [3]

(iv) Using a suitable approximation, find the probability that, in a 30-minute period, the number of hot drinks sold is more than the number of cold drinks sold. [4]

9 The owner of Eng Choo Farm claims that the eggs produced at his farm has an average cholesterol content of μ_o mg per egg.

The Health Promotion Agency took a random sample of 16 eggs from the farm and measured the cholesterol content, x mg , of each egg. The results are summarized as follows:

$$\sum x = 2848 \text{ and } \sum x^2 = 509884.$$

a) Find the set of values of μ_o for which the owner's claim will not be rejected at the 5% significance level. [4]

b) A second random sample of 36 eggs from the farm is taken and the cholesterol content of each egg is measured. The sample mean and the sample variance are found to be 185 mg and 120 mg^2 respectively.

Combining the two samples into a single sample, find unbiased estimates of the population mean μ mg and population variance σ^2 mg^2 of the cholesterol content of the eggs produced at the farm. [3]

Using this combined sample, the null hypothesis $\mu = 180$ is tested against the alternative hypothesis $\mu > 180$ at 5% significance level. Find the *p*-value of the test and explain the meaning of this *p*-value in the context of the question. [3]

10 In a small town, there are two clinics, Lee's Clinic and Hope Clinic. At Lee's Clinic, the waiting time of a patient follows a normal distribution with mean 25 minutes and standard deviation 8 minutes. The waiting time for patient at Hope Clinic follows a normal distribution with mean 37 minutes and standard deviation 4 minutes.

(i) A random sample of five patients at Lee's Clinic is selected. Find the probability that the mean waiting time of these five patients differs from the waiting time of a randomly selected patient at Hope Clinic by at least five minutes. [4]

State an assumption needed for your calculations to be valid. [1]

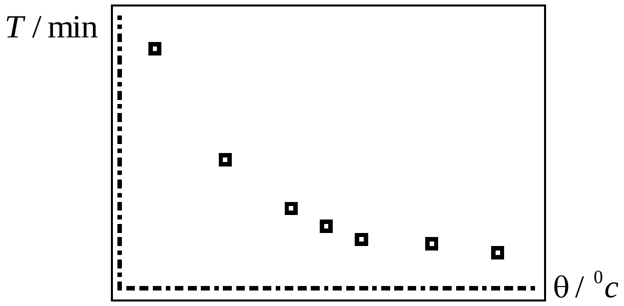
(ii) On a particular day, n (where $n > 40$) patients were treated at Lee's Clinic. By using a suitable approximation, find the least value of n such that the probability that at most 40 of the patients have waiting times of more than 25 minutes is less than 0.95. [5]

(iii) At Lee's Clinic, on average, one in five patients is treated for influenza. Sixty samples of 20 patients are chosen. Find the probability that the average number of patients treated for influenza per sample is more than 3.5. [3]

(iv) State, with a reason, whether a normal model is likely to be appropriate for the waiting time of a randomly selected patient from the combined group of patients from Lee's Clinic and Hope Clinic. [1]

11 In an experiment, the time taken, T minutes, for a 1-kg block of ice to melt completely under 7 pre-determined temperatures, $\theta\,^{o}$C, is recorded in a table.

Albert analysed the data and made the following conclusion:



Product Moment Correlation Coefficient, $r = -0.88316$

Regression line of T on θ: $T = 35.857 - 0.51429\theta$.

Conclusion: Since the value of r is close to -1, the linear model $T = 35.9 - 0.514\theta$ is appropriate.

When Albert attempted to explain his analysis to Betty, he realised that he had accidentally erased a value in his table.

Temperature ($\theta°C$)	20	30	40	45	50	60	70
Time taken (T minutes)	33	18	11	?	7	6	5

(i) Show that the missing value is 9, correct to the nearest integer. [2]

(ii) Calculate the estimated time when the temperature is $70\,°C$. Give 2 reasons to explain why Albert's linear model may not be appropriate. [3]

Betty suggested using the model $\theta(T-a) = b$.

(iii) Calculate the least squares estimates of a and b and estimate the temperature at which it will take 30 minutes to melt a 1-kg ice block completely. [4]

END OF PAPER